

# CHAPTER 4

## SETS

### More Questions for Practice

- Which of the following collections are sets?
  - A collection of rich people of India.
  - A collection of students of your class with age exceeding 12 years.
  - A collection of languages spoken in India.
  - A collection of positive integers less than 5.
- Which of the following are sets?
  - $\{x \mid x = \frac{0}{p}, p \in \mathbb{N}\}$
  - $\{x \mid x = \frac{p}{0}, p \in \mathbb{N}\}$
- Represent the following sets using the *Roster-listing method*:
  - $\{x \mid x \in \mathbb{W} \text{ and } 2x - 3 < 5\}$
  - $\{x \mid x = 2n + 1 \text{ and } n \text{ is a natural number less than } 7\}$
  - $\{x \mid x \text{ is a prime number and } x \leq 11\}$
  - $\{x \mid x = \frac{1}{n}, n \in \mathbb{N} \text{ and } n \leq 4\}$
  - $\{x \mid x = p^2, p \in \mathbb{N} \text{ and } 3 \leq p \leq 5\}$
  - $\{x \mid x = 3y, y \in \mathbb{N}, 3 \leq y \leq 7\}$
  - $\{x \mid x \text{ is a two-digit number, the sum of whose digits is } 5\}$
  - $\{x \mid x \text{ is a multiple of } 4 \text{ but less than } 24\}$
  - $\{x \mid x \text{ is a composite number such that } 17 < x \leq 27\}$
  - $\{x \mid x \in \mathbb{N}, x \text{ is divisible by } 3 \text{ and } 5; x \leq 30\}$
  - $\{x \mid x = \frac{45}{p}, p \in \mathbb{N} \text{ and } x \in \mathbb{N}\}$
  - $\{x \mid x = 3^n, 2 \leq n < 7, n \in \mathbb{N}\}$
  - $\{x \mid x = q^2 - 1, -3 < q < 3, q \in \mathbb{I}\}$
  - $\{x \mid x = 5q, q \in \mathbb{I} \text{ and } x^2 < 400\}$
  - $\{x \mid x = \frac{2p-1}{p+2}, p \in \mathbb{W} \text{ and } p < 4\}$
- Represent the following sets using the *Set-builder method*:
  - $\{\text{Atlantic, Arctic}\}$
  - $\{\text{Spring, Autumn, Winter, Summer}\}$
  - $\{1\}$
  - $\left\{1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{9}\right\}$
  - $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots\right\}$
  - $\{16, 25, 36, 49, 64, 81, 100\}$
  - $\{2, 3, 5, 7, 11, 13, 17\}$

5. Which of the following sets are *finite* and which are *infinite*?
- The set of solution(s) of the equation  $x + 4 = 6$ .
  - The set of all points on a line.
  - The set of circles in a plane passing through the origin.
  - $\{x \mid x \in \mathbb{I}, x < 4\}$
  - $\{x \mid x < 3p - 5, p < 15, p \in \mathbb{I}\}$
  - $\{x \mid x = 7n, n \in \mathbb{I} \text{ and } x < 10\}$
6. Which of the following pairs of sets are *equivalent*?
- $A = \{x \mid x \text{ is a letter in the word EYE}\}$
  - $C = \{x \mid x \in \mathbb{N}, 9 < x < 14\}$
  - $B = \{x \mid x \text{ is a letter in the word EAR}\}$
  - $D = \{x \mid x \in \mathbb{I}, -2 < x < 3\}$
7. Which of the following pairs of sets are *equal*?
- $A = \{x \mid x < 5 \text{ and } x \in \mathbb{N}\}$
  - $C = \{x \mid x = p^2 + 1, 1 < p < 4, p \in \mathbb{N}\}$
  - $B = \{1, 4 - 2, 2^3 - 5, 3^2 - 5\}$
  - $D = \{x \mid x = 5m, m \leq 2, m \in \mathbb{N}\}$
8. State whether the following statements are *True* or *False*:
- A null set is a subset of every set.
  - Every subset of an infinite set is finite.
  - If  $n(A) = n(B)$ , then  $A = B$ .
  - $\phi$  has no proper subset.
  - Infinite sets are always equivalent sets.
  - The set of all lines parallel to a given line is a finite set.
  - Every subset of a finite set is finite.
9. If  $A = \{1, 2, 3, 4\}$  and  $B = \{x \mid x \in \mathbb{N} \text{ and } 4 \leq x \leq 8\}$ , then find (i)  $A \cup B$ , and (ii)  $A \cap B$ . Also, state whether the two sets  $A$  and  $B$  are disjoint or not.
10. If  $A = \{x \mid x \in \mathbb{N} \text{ and } x \text{ is a factor of } 12\}$  and  $B = \{1, 3, 7, 12\}$ , then represent each of the following sets by the *Roster-listing method*:
- $A - B$
  - $B - A$
  - $A \cup B$
  - $A \cap B$
11. Let  $U = \{x \mid x \leq 25, x \in \mathbb{N}\}$ ,  $A = \{x \mid x \leq 12, x \in \mathbb{N}\}$ ,  $B = \{x \mid 9 \leq x \leq 18, x \in \mathbb{N}\}$ . Represent each of the following sets by the *Roster-listing method*:
- $A'$
  - $B'$
  - $(A \cap B)'$
  - $A' \cup B'$
- Hence, show that  $(A \cap B)' = A' \cup B'$ .
12. Let  $U = \{x \mid x \in \mathbb{W} \text{ and } x \leq 10\}$ ,  $A = \{0, 1, 3, 5\}$  and  $B = \{y \mid y \in \mathbb{N} \text{ and } 2 \leq y \leq 7\}$ . Verify the following:
- $A - B = A \cap B'$
  - $B - A = B \cap A'$
  - $(A - B) \cup B = A \cup B$
  - $(A - B) \cap B = \phi$
13. Given:  $U = \{x \mid 1 \leq x \leq 20, x \in \mathbb{N}\}$   
 $A = \{x \mid x \text{ is multiple of } 2\}$   
 $B = \{x \mid x \text{ is multiple of } 3\}$   
 $C = \{x \mid x \text{ is multiple of } 7\}$
- Verify the following relations:
- $(A \cup B)' = A' \cap B'$
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

14. If  $U = \{4, 3, \frac{-3}{2}, \frac{-2}{3}, -1, 0, \frac{1}{2}, 2\}$

$$A = \{x \mid \frac{-3}{2} \leq x \leq 1\}; B = \{y \mid y < 0 \text{ and } y = \frac{p}{2}, \text{ where } p \in \mathbb{Z}\}, \text{ verify that}$$

(i)  $A \cap B = B$

(ii)  $A' \cup B' = (A \cap B)'$ .

15. Given  $U = \{x \mid x \in \mathbb{N}, 1 \leq x \leq 12\}$ ,  $A = \{1, 9, 10\}$ ,  $B = \{3, 4, 6, 11, 12\}$  and  $C = \{2, 5, 6\}$ , find  $A \cup (B \cap C)$  and  $(A \cup B) \cap (A \cup C)$  and verify that  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ .

16. How many elements are there in the set

$$S = \{x \mid x \in \mathbb{N}, 6 < x < 7\}?$$

17. If  $X = \{a, b, c, d, e\}$  and  $Y = \{d, e, f, g\}$ , find:

$$(X - Y) \cup (Y - X)$$

18. If  $A = \{x \mid x \text{ is a letter in the word APPLE}\}$ ;  $B = \{x \mid x \text{ is a letter in the word PLANE}\}$ ; then find if  $A = B$ .

19. A and B are two sets such that  $A \cap B = \phi$ .

(i) Draw Venn diagram to represent the relation between A and B and shade  $(A \cup B)'$ .

(ii) Redraw Venn diagram and shade  $A' \cap B'$ .

(iii) Write the relation between  $(A \cup B)'$  and  $A' \cap B'$ .

20. Fill up the Venn diagram (Fig. 4.1) with the elements of the following sets:

$$U = \{2, 4, 6, 8, 10, 12, 14\}$$

$$P = \{2, 4, 6, 8\}$$

$$Q = \{8, 10, 12\}$$

Using the filled up Venn diagram, find the following pairs of sets and hence show that they are equal.

(i)  $(P \cup Q)'$  and  $P' \cap Q'$

(ii)  $(P \cap Q)'$  and  $P' \cup Q'$

(iii)  $P - Q$  and  $P \cap Q'$

(iv)  $Q - P$  and  $Q \cap P'$

21. Fill up the Venn diagram (Fig. 4.2) with the elements of the following sets:

$$U = \{x : x \leq 8, x \in \mathbb{N}\}$$

$$A - B = \{3, 4, 5\}$$

$$A' = \{6, 7, 8\}$$

Using the filled up Venn diagram, find the elements of the following sets:

(i) A

(ii) B

(iii)  $B - A$

(iv)  $A' - B$

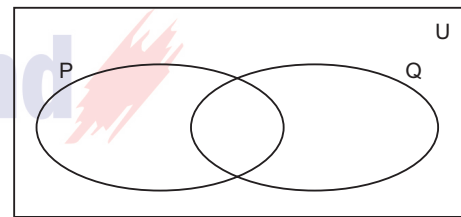


Fig. 4.1

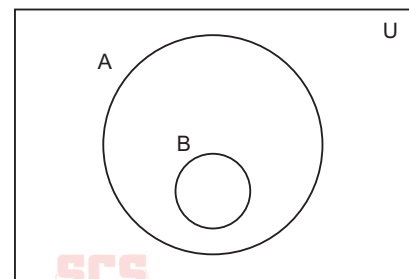


Fig. 4.2

22. Using the Venn diagram (Fig. 4.3), list the elements of the following sets:

- (i)  $(A \cup B) - C$     (ii)  $(B \cap C) - A$     (iii)  $A' \cap B$     (iv)  $C' - A$     (v)  $A' \cup B'$

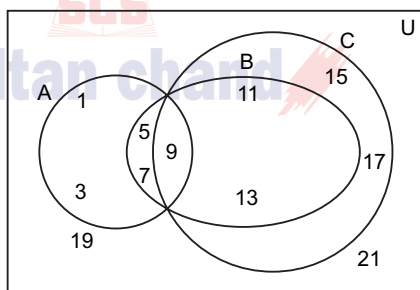


Fig. 4.3

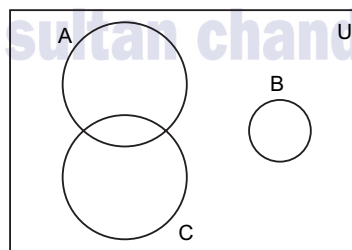


Fig. 4.4

23. Shade the given Venn diagram (Fig. 4.4) to describe the following sets:

- (i)  $A' \cup (B \cup C)$                                   (ii)  $(A - C) \cup B$   
(iii)  $(A \cup B) \cap C$                                   (iv)  $(A \cap C) \cup B$

24. In a class, 24 boys study Mathematics and 18 boys study Science. Of these, 9 boys study both Mathematics and Science. How many boys study either Mathematics or Science or both?

25. Of a group of 30 students, 15 are interested in Music, 10 are interested in Photography and 5 students like both. Find out the number of students who like neither Music nor Photography.

26. In a group of 256 students, 225 passed in Geography, 220 passed in History and 11 failed in both the subjects. Find:

- (i) the number of students who passed in both the subjects.  
(ii) the number of students who passed in Geography only.

27. In an examination 56% of the candidates failed in English and 48% failed in Mathematics. If 18% failed in both English and Mathematics, find the percentage of those who passed in both the subjects.

28. 40 students of a class study either Physics or Chemistry or both. 10 students study both the subjects and 17 students study only Chemistry. How many students study Physics?

29. In a group of children, 40 play cricket and 25 play football. If there are 45 students in total, how many students play both the games?

30. Of a group of 50 students, 27 are interested in music, 17 in photography and 7 like both. How many students do not like both the hobbies?

31. In an examination, 65% of the candidates failed in English, 50% failed in Mathematics. If 15% failed in both the subjects, what percentage of students passed in both the subjects?

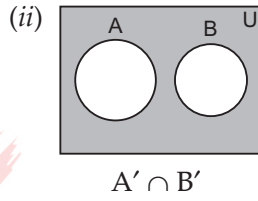
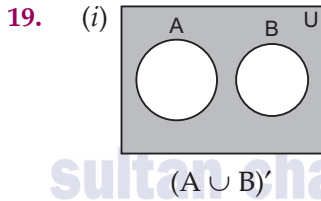
32. In a group of 200 people, 120 can speak English and 110 can speak Hindi. Find:

- (i) How many people can speak both the languages?  
(ii) How many speak only Hindi?  
(iii) How many speak only English?

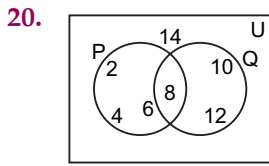
33. In a class, four-tenths of the students like to read English magazines only, three-tenths of the students like to read Hindi magazines only. If 3 students like to read none of these kinds of magazines and 12 like to read magazines of both the languages, find the total number of students in the class (Use Venn diagram).
34. Draw a Venn diagram using the sets  $U, P, Q$  and  $R$  such that
- $$P \subseteq U, Q \subseteq U, R \subseteq U,$$
- $$P \cap Q = \phi, Q \cap R \neq \phi \text{ and } P \cap R = \phi.$$

## ANSWERS

1. (ii), (iii) and (iv) are sets. 2. (i) is a set.
3. (i)  $\{0, 1, 2, 3\}$  (ii)  $\{3, 5, 7, 9, 11, 13\}$  (iii)  $\{2, 3, 5, 7, 11\}$
- (iv)  $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$  (v)  $\{9, 16, 25\}$  (vi)  $\{9, 12, 15, 18, 21\}$
- (vii)  $\{14, 23, 32, 41, 50\}$  (viii)  $\{4, 8, 12, 16, 20\}$  (ix)  $\{18, 20, 21, 22, 24, 25, 26, 27\}$
- (x)  $\{15, 30\}$  (xi)  $\{45, 15, 9, 5, 3, 1\}$  (xii)  $\{9, 27, 81, 243, 729\}$
- (xiii)  $\{3, 0, -1\}$  (xiv)  $\{-15, -10, -5, 0, 5, 10, 15\}$  (xv)  $\left\{-\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, 1\right\}$
4. (i)  $\{x \mid x \text{ is a name of ocean beginning with A}\}$  (ii)  $\{x \mid x \text{ is a season of the year}\}$
- (iii)  $\{x \mid x \text{ is neither a prime nor a composite number}\}$  (iv)  $\left\{x \mid x = \frac{1}{n}, n \in \mathbb{N}, 1 \leq n \leq 9\right\}$
- (v)  $\left\{x \mid x = \frac{p}{p+1}, p \in \mathbb{N}\right\}$  (vi)  $\{x \mid x = p^2, p \in \mathbb{N} \text{ and } 4 \leq p \leq 10\}$
- (vii)  $\{x \mid x \text{ is a prime number less than } 18\}$
5. (i) is a finite set and (ii), (iii), (iv), (v), (vi) are infinite sets. 6. (ii) 7. Both
8. (i) True (ii) False (iii) False (iv) True (v) False (vi) False
- (vii) True.
9. (i)  $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$  (ii)  $A \cap B = \{4\}$ . The two sets are not disjoint.
10. (i)  $A - B = \{2, 4, 6\}$  (ii)  $B - A = \{7\}$  (iii)  $A \cup B = \{1, 2, 3, 4, 6, 7, 12\}$
- (iv)  $A \cap B = \{1, 3, 12\}$
11. (i)  $A' = \{13, 14, 15, 16, 17, \dots, 25\}$  (ii)  $B' = \{1, 2, 3, \dots, 8, 19, 20, 21, \dots, 25\}$
- (iii)  $(A \cap B)' = \{1, 2, 3, \dots, 8, 13, 14, 15, \dots, 25\}$  (iv)  $A' \cup B' = \{1, 2, 3, \dots, 8, 13, 14, 15, \dots, 25\}$
15.  $A \cup (B \cap C) = \{1, 6, 9, 10\}$ ;  $(A \cup B) \cap (A \cup C) = \{1, 6, 9, 10\}$ .
16. zero 17.  $\{a, b, c, f, g\}$  18. No



(iii)  $(A \cup B)' = A' \cap B'$

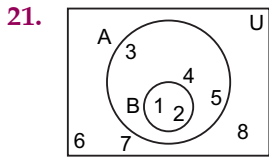


(i)  $(P \cup Q)' = P' \cap Q' = \{14\}$

(ii)  $(P \cap Q)' = P' \cup Q' = \{2, 4, 6, 10, 12, 14\}$

(iii)  $P - Q = P \cap Q' = \{2, 4, 6\}$

(iv)  $Q - P = Q \cap P' = \{10, 12\}$



(i)  $A = \{1, 2, 3, 4, 5\}$

(ii)  $B = \{1, 2\}$

(iii)  $B - A = \{ \}$

(iv)  $A' - B = \{6, 7, 8\}$

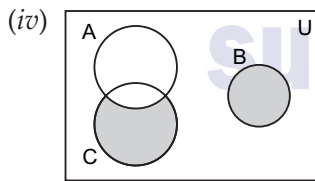
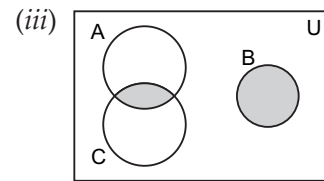
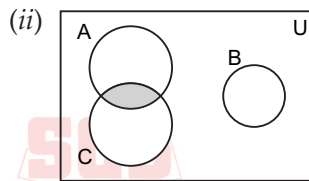
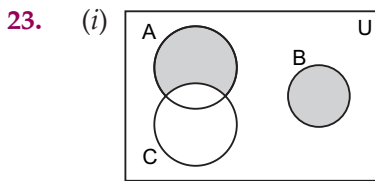
22. (i)  $(A \cup B) - C = \{1, 3, 5, 7\}$

(ii)  $(B \cap C) - A = \{11, 13\}$

(iii)  $A' \cap B = \{11, 13\}$

(iv)  $C' - A = \{19, 21\}$

(v)  $A' \cup B' = \{15, 17, 19, 21\}$



24. 33

25. 10

26. (i) 189

(ii) 36

27. 14%.

28. 23

29. 20

30. 13

31. 0%

32. (i) 30

(ii) 80

(iii) 90

33. 50 students

