

# CHAPTER 10

## ALGEBRAIC EXPRESSIONS

### More Questions for Practice

1. Add the following algebraic expressions:

(i)  $-\frac{2pq}{3} + \frac{3p^2q}{4} + \frac{4pq^2}{5}$ ,  $\frac{pq}{3} + \frac{2p^2q}{5} + \frac{3pq^2}{5}$  and  $\frac{pq}{4} - \frac{p^2q}{5} + \frac{pq^2}{5}$ .

(ii)  $5mn^2 + 6m^2n - 11mn$ ,  $-7mn - mn^2 + 7m^2n$  and  $11m^2n - 2mn + 4mn^2$ .

2. Subtract:

(i)  $x^6y^2 - 7x^5y + 8xy^4 - 9x^2y^3$  from  $2xy^4 + x^5y - 3x^2y^3$ .

(ii)  $5m^3 - 4m^2n + 9mn^2 - n^3$  from  $1 - m^3 + n^3$ .

3. Subtract the sum of  $\frac{1}{3}a^3 - 2a^2b - \frac{3}{2}b^3$ ,  $\frac{3}{2}a^2b - \frac{3}{4}ab^2 + 2b^3$  and  $-\frac{3}{2}a^3 + ab^2 + \frac{1}{2}b^3$  from  $\frac{1}{6}a^3 + \frac{1}{2}ab^2 + \frac{1}{3}b^3$ .

4. Simplify:

(i)  $(2x^5 - x^4y - 4xy^4 + 5x^2y^3 - 1) + (3x^2y^3 - 4 + 11x^5)$

(ii)  $(p^3 - 11q^3 + 9r^3) + (9p^3 - 11r^3 - q^3 - 2pqr) - (9pqr - 11r^3 - 3p^3)$

(iii)  $(2x^2y - 4y^2x + 3xy - 4) - (13y^2x - 2x^2y + 1) - (xy - 11x^2y - 4)$

5. Find the following products:

(i)  $(2x^2 + 3x - 7)(3x^2 - 5x + 4)$

(ii)  $(3x - 2)(4x^3 - 3x^2 + 7x - 1)$

6. Find the following products:

(i)  $(x^2 + x - y^2 + y)(x^2 + x + y^2 - y)$

(ii)  $(3x^3 + 2x^2 - x + 1)(x^3 - x^2 + x + 3)$

(iii)  $(1 + y^3 - y^2 + 2y)(4y^3 - 4y^2 - y - 1)$

(iv)  $(2x^2y - xy + y^2 - 1)(3y^2x - yx + x^2 - 1)$

7. Simplify:

(i)  $x^2y(x - y^2) + xy^2(4xy - 2x^2) - x^3y(1 - 2y)$

(ii)  $x^2y(x^3 - x + 1) - xy(x^4 - 2x^2 + 2x) - y(x^3 - x^2 - 1)$

8. Divide  $4y^3 - 3y + 8y^2 + 5$  by  $2y^2 - y + 1$  and write the quotient.

9. Divide  $6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$  by  $2y^3 + 1$  and write the remainder.

10. Divide  $15y^4 + 16y^3 + \frac{10}{3}y - 9y^2 - 6$  by  $3y - 2$  and write the quotient.

11. Simplify:

$$(i) 7[x - 2\{4x - x(3 - \overline{7 - 2}) - 3x\} - 2x] \quad (ii) 6[2a - 3\{5a - 4a - \overline{7a}\} - 2(a - 3)]$$

$$(iii) 2y - [5x - \{2x - 2(2x - 3y) - 5(y - \overline{10x - 8y})\}]$$

12. Simplify the following using BODMAS rule:

$$(i) (19x - 8x \div 4) - \left(12x + 8x \div 4 \text{ of } \frac{1}{2}\right) \quad (ii) 10x + 4 \left[3x \text{ of } \frac{1}{3} - 7x + 2\{4x \times 2 - 2x\}\right]$$

$$(iii) 16x + 16x \div 4 - 3x \times 5 + 10x \text{ of } \frac{1}{5} \quad (iv) 2x \text{ of } 8 \div 4 - 3x + 9x \div 3 \times 3 + 5x$$

$$(v) 4 \left[ a - 3\{2a - a \times 5 + 8a \div 4\} - 6a \text{ of } \frac{1}{2} \right]$$

13. If  $a = 9$  and  $b = 16$ , find the value of  $\frac{a + 2\sqrt{ab} + b}{\sqrt{a} + \sqrt{b}} + \frac{a - 2\sqrt{ab} + b}{\sqrt{a} - \sqrt{b}}$ .

14. If  $a = 1$ ,  $b = -1$  and  $c = 2$ , find the value of  $\sqrt{3a^3(b - c) + 3b^3(c - a) + 3c^3(a - b)}$ .

15. Add the following:

$$(i) 2a - b + (2c - 3d), 3a + 4c - 2(b - d), 4(b - d) + c - 5a \text{ and } b - (3c - 2a).$$

$$(ii) x - (1 - \overline{a + b}), x - (1 + \overline{a - b}), x - (1 - \overline{a - b}) \text{ and } 4 - (a + b + 3x).$$

$$(iii) a(x + y - z) - 6(x^2 - y^2) + b, -5(x^2 - y^2) + 3b, -a(x + y - z) + 2(x^2 - y^2) \text{ and } -(x^2 - y^2).$$

16. Divide:

$$(i) x^4 - 16 \text{ by } x - 2$$

$$(ii) x^4 - 64y^4 \text{ by } x^2 + 8y^2$$

17. (i) If  $x = 6$ ,  $y = 4$  and  $z = 3$ , find the value of  $\frac{x^2 - (y^2 - z^2) - y(x - y + z)}{\sqrt[3]{xyz^2}}$ .

(ii) If  $p = 3$  and  $x = -2$ , find the value of  $\frac{p^2 + px + x^2}{p^2 - px + x^2} + \frac{2p + x}{3p - 5x} + \frac{2p - x}{7p + x}$ .

18. Solve:  $2x - 2 - \frac{1}{3}\{3x - 3 - \frac{1}{4}(7x + 7 - \overline{5x - 5})\} = 7$

19. Find the following special products:

$$(i) \left(t - \frac{1}{2}\right)\left(t + \frac{3}{2}\right)$$

$$(ii) \left(\frac{a}{2} - 4\right)\left(\frac{a}{2} + 3\right)$$

$$(iii) (2 + a)(3 + a)$$

$$(iv) (a - 8b)(a - 4b)$$

$$(v) (x^2 - 1)(x^2 + 4)$$

$$(vi) (r^2 - 8)(r^2 + 7)$$

20. Find the following special products:

(i)  $(xy + 2)(xy - 2)$

(ii)  $(3x^2 - y^2)(3x^2 + y^2)$

(iii)  $(1.1x + 0.2y)(1.1x - 0.2y)$

(iv)  $\left(\frac{7p}{4q} - 1\right)\left(\frac{7p}{4q} + 1\right)$

(v)  $\left(\frac{3m^2}{5} + \frac{n^2}{7}\right)\left(-\frac{3m^2}{5} + \frac{n^2}{7}\right)$

(vi)  $\left(\frac{m^2px}{4} + \frac{2}{y^2}\right)\left(-\frac{m^2px}{4} + \frac{2}{y^2}\right)$

21. Use the special product  $(x + y)(x - y) = x^2 - y^2$  to evaluate the following:

(i)  $77 \times 83$

(ii)  $46 \times 54$

(iii)  $19 \times 21$

(iv)  $207 \times 193$

(v)  $91 \times 89$

(vi)  $45 \times 55$

(vii)  $93 \times 87$

(viii)  $67 \times 73$

22. Simplify:

(i)  $\left(\frac{x^2}{y} - \frac{4y}{x^2}\right)\left(\frac{x^2}{y} + \frac{4y}{x^2}\right) + \frac{x^4}{y^2} - \frac{16y^2}{x^4}$

(ii)  $\left(3x^2 - \frac{y^2p}{5}\right)\left(3x^2 + \frac{y^2p}{5}\right) - 9x^4 - \frac{y^4p^2}{25}$

23. Expand the following:

(i)  $\left[\sqrt{3}\left(a + \frac{2}{b}\right)\right]^2$

(ii)  $\left(\sqrt{3}p + \frac{1}{\sqrt{2}q}\right)^2$

(iii)  $\left[\sqrt{5}\left(\frac{p}{q} + \sqrt{3}m\right)\right]^2$

24. Expand the following:

(i)  $\left[\sqrt{3}\left(\frac{m}{p} - \frac{p}{m}\right)\right]^2$

(ii)  $\left(\frac{2x}{y} - \frac{y}{2x}\right)^2$

(iii)  $\left[\sqrt{5}\left(\frac{1}{mn} - \sqrt{2}\right)\right]^2$

25. (i) If  $2p + \frac{1}{2p} = 3$ , find the values of  $\left(4p^2 + \frac{1}{4p^2}\right)$  and  $\left(16p^4 + \frac{1}{16p^4}\right)$ .

(ii) If  $3x - \frac{1}{3x} = 8$ , find the values of  $\left(9x^2 + \frac{1}{9x^2}\right)$  and  $\left(81x^4 + \frac{1}{81x^4}\right)$ .

26. (i) What should be added to  $225x^2 + 121y^2$  to make it a perfect square?

(ii) What should be added to  $289p^2 + 374pq$  to make it a perfect square?

(iii) What should be added to  $\frac{x^2}{4} - \frac{xy}{6}$  to make it a perfect square?

(iv) What should be subtracted from  $1.21x^2 + 4.6xy + 4y^2$  to make it a perfect square?

27. Simplify:

(i)  $\left(\frac{a}{2} + \frac{2}{a}\right)^2 + \left(\frac{a}{2} - \frac{2}{a}\right)^2$

(ii)  $(m^2 + n^2)(m^2 + n^2) - m^4 - 2m^2n^2 - n^4$

(iii)  $(p^3 - q^2p)^2 + 2p^4q^2$

(iv)  $\left(4x^2 - \frac{y^2}{5}\right)^2 - \left(4x^2 + \frac{y^2}{5}\right)^2$

28. Expand the following:

$$(i) [\sqrt{2}(a-b+1)]^2 \quad (ii) \left(4-2x+\frac{y}{3}\right)^2 \quad (iii) \left(1-\frac{a}{2}-\frac{b}{3}\right)^2$$

29. Expand the following:

$$(i) (3x^2 - y^2)^2 \quad (ii) (0.1x - 0.2y)^2 \quad (iii) \left(5 - x + \frac{2}{x}\right)^2 \quad (iv) (2p - 3q + r)^2$$

$$(v) \left(2r - \frac{q}{2} - p\right)^2$$

30. Evaluate:

$$(i) \frac{198 \times 198 - 102 \times 102}{96} \quad (ii) \frac{8.37 \times 8.37 - 1.63 \times 1.63}{0.674}$$

31. Find the value of the following using expansions:

$$(i) a^2 + b^2 \text{ and } (a-b)^2, \text{ when } a+b=10 \text{ and } ab=4.$$

$$(ii) a^2 + b^2 \text{ and } (a+b)^2, \text{ when } a-b=2 \text{ and } ab=63.$$

$$(iii) x^4 + y^4 \text{ and } (x^2 - y^2)^2, \text{ when } x^2 + y^2 = 5 \text{ and } xy = 2.$$

$$(iv) ab \text{ when } a-b=2 \text{ and } a^2 + b^2 = 36.$$

$$(v) x^2 + \frac{1}{x^2} \text{ and } \left(x - \frac{1}{x}\right)^2, \text{ when } x + \frac{1}{x} = 3.$$

$$(vi) p^3 - q^3, \text{ when } p - q = -8 \text{ and } pq = -12.$$

$$(vii) 64x^3 - 125z^3, \text{ when } 4x - 5z = \frac{3}{5} \text{ and } xz = 6.$$

32. If  $3x + \frac{1}{3x} = 3$ , find the value of: (i)  $81x^4 + \frac{1}{81x^4}$  (ii)  $27x^3 + \frac{1}{27x^3}$ .

## ANSWERS

$$1. (i) -\frac{pq}{12} + \frac{19}{20}p^2q + \frac{8}{5}pq^2$$

$$(ii) 8mn^2 + 24m^2n - 20mn$$

$$2. (i) -6xy^4 + 6x^2y^3 - x^6y^2 + 8x^5y$$

$$(ii) 2n^3 - 9mn^2 + 4m^2n - 6m^3 + 1$$

$$3. \frac{4}{3}a^3 + \frac{1}{4}ab^2 - \frac{2}{3}b^3 + \frac{1}{2}a^2b$$

$$4. (i) 13x^5 - x^4y - 4xy^4 + 8x^2y^3 - 5 \quad (ii) 13p^3 - 12q^3 + 9r^3 - 11pqr \quad (iii) 15x^2y - 17y^2x + 2xy - 1$$

$$5. (i) 6x^4 - x^3 - 28x^2 + 47x - 28 \quad (ii) 12x^4 - 17x^3 + 27x^2 - 17x + 2$$

$$6. (i) x^4 + 2x^3 + x^2 - y^2 + 2y^3 - y^4 \quad (ii) 3x^6 - x^5 + 13x^3 + 4x^2 - 2x + 3$$

$$(iii) 4y^6 - 8y^5 + 11y^4 - 4y^3 - 5y^2 - 3y - 1$$

$$(iv) 6x^3y^3 - 2x^3y^2 + 2x^4y - 2x^2y^3 + 2x^2y^2 - x^3y + 2xy + 3xy^4 - y^3x - 3y^2x - y^2 - x^2 + 1$$

7. (i)  $3x^2y^3$  (ii)  $y$       8.  $2y + 5$       9.  $4y^2 + 25y + 4$
10.  $5y^3 + \frac{26}{3}y^2 + \frac{25}{9}y + \frac{80}{27}$       11. (i)  $-49x$  (ii)  $36 - 144a$  (iii)  $43x - 37y$
12. (i)  $x$  (ii)  $34x$  (iii)  $7x$  (iv)  $15x$  (v)  $4a$
13. 6      14. 6      15. (i)  $2a + 2b + 4c - 5d$  (ii) 1 (iii)  $-10x^2 + 10y^2 + 4b$
16. (i) Quotient =  $x^3 + 2x^2 + 4x + 8$ , Remainder = 0  
(ii) Quotient =  $x^2 - 8y^2$ , Remainder = 0
17. (i)  $\frac{3}{2}$  (ii) 1      18.  $x = 6$
19. (i)  $t^2 + t - \frac{3}{4}$  (ii)  $\frac{a^2}{4} - \frac{a}{2} - 12$  (iii)  $6 + 5a + a^2$   
(iv)  $a^2 - 12ab + 32b^2$  (v)  $x^4 + 3x^2 - 4$  (vi)  $r^4 - r^2 - 56$
20. (i)  $x^2y^2 - 4$  (ii)  $9x^4 - y^4$  (iii)  $1.21x^2 - 0.04y^2$  (iv)  $\frac{49p^2}{16q^2} - 1$   
(v)  $\frac{n^4}{49} - \frac{9m^4}{25}$  (vi)  $\frac{4}{y^4} - \frac{m^4p^2x^2}{16}$
21. (i) 6391 (ii) 2484 (iii) 399 (iv) 39951 (v) 8099 (vi) 2475  
(vii) 8091 (viii) 4891      22. (i)  $\frac{2x^4}{y^2} - \frac{32y^2}{x^4}$  (ii)  $\frac{-2y^4p^2}{25}$
23. (i)  $3\left(a^2 + \frac{4a}{b} + \frac{4}{b^2}\right)$  (ii)  $3p^2 + \frac{2\sqrt{3p}}{\sqrt{2q}} + \frac{1}{2q^2}$  (iii)  $5\left(\frac{p^2}{q^2} + 2\sqrt{3}\frac{pm}{q} + 3m^2\right)$
24. (i)  $3\left(\frac{m^2}{p^2} - 2 + \frac{p^2}{m^2}\right)$  (ii)  $\frac{4x^2}{y^2} - 2 + \frac{y^2}{4x^2}$  (iii)  $5\left(\frac{1}{m^2n^2} - \frac{2\sqrt{2}}{mn} + 2\right)$
25. (i) 7, 47 (ii) 66, 4354      26. (i)  $\pm 330xy$  (ii)  $121q^2$  (iii)  $\frac{y^2}{36}$  (iv)  $0.2xy$
27. (i)  $\frac{a^2}{2} + \frac{8}{a^2}$  (ii) 0 (iii)  $p^6 + q^4p^2$  (iv)  $-\frac{16x^2y^2}{5}$
28. (i)  $2(a^2 + b^2 + 2a - 2b - 2ab + 1)$  (ii)  $16 + 4x^2 + \frac{y^2}{9} - 16x - \frac{4xy}{3} + \frac{8y}{3}$   
(iii)  $1 + \frac{a^2}{4} + \frac{b^2}{9} - a + \frac{ab}{3} - \frac{2b}{3}$
29. (i)  $9x^4 - 6x^2y^2 + y^4$  (ii)  $0.01x^2 - 0.04xy + 0.04y^2$   
(iii)  $25 + x^2 + \frac{4}{x^2} - 10x - 4 + \frac{20}{x}$   
(iv)  $4p^2 + 9q^2 + r^2 - 12pq - 6qr + 4pr$  (v)  $4r^2 + \frac{q^2}{4} + p^2 - 2rq + pq - 4rp$
30. (i) 300 (ii) 100      31. (i) 92; 84 (ii) 130; 256 (iii) 17; 9  
(iv) 16 (v) 7; 5 (vi) -224 (vii)  $216\frac{27}{125}$       32. (i) 47 (ii) 18.