SOLUTIONS TO COMPETENCY-BASED PRACTICE QUESTIONS

Unit I: Commercial Mathematics (Chapters 1–3)

I: Multiple Choice Questions (1 mark each)

1. (*c*) List price of the article = ₹ 2500

Rate of GST = 5%

∴ GST paid by the retailer to the wholesaler

= 5% of ₹ 2500

= ₹ 125

Price marked up by the retailer = 20%

∴ Sale price of the article for the retailer = ₹ 2500 + 20% of ₹ 2500

= ₹ 2500 + ₹ 500

= ₹ 3000

∴ GST collected by the retailer = 5% of ₹ 3000

= ₹ 150

Now, GST paid at the time of purchase will be set off against GST collected at the time of sale and balance amount is to be deposited into the government account as payable GST.

∴ Payable GST = ₹ 150 – ₹ 125 = ₹ 25

Since this is an intra-state transaction, the payable GST splits into two equal halves—CGST and SGST.

Therefore, the tax liability of the retailer to the central government, *i.e.*, CGST = $\frac{1}{2}$ of ₹ 25, *i.e.*, ₹ 12.50.

2. (*b*) M.P. of the electrical fan = ₹ 800

Rate of GST charged = 5%

∴ GST charged on the sale of the fan = 5% of ₹ 800

= ₹ 40

This GST is splits into two equal halves—CGST and SGST.

∴ SGST charged on the sale of the fan = $\frac{1}{2}$ of ₹ 40, *i.e.*, ₹ 20.

3. (*d*) In a recurring deposit (R.D.) account, interest calculated is simple interest for one month.

4. (*b*) Here, P = ₹ 1000, r = 5% p.a., n = 1 year, *i.e.*, 12 months

$$\therefore \text{Interest (I)} = \frac{\text{P} \times n\,(n+1) \times r}{12 \times 2 \times 100}$$

$$= ₹\ \frac{1000 \times 12 \times 13 \times 5}{12 \times 2 \times 100} = ₹\ 325$$

Let Anwesha deposit ₹ P monthly for 1 year at 4% p.a. to obtain the same interest, *i.e.*, ₹ 325. Therefore, using the above formula we have

$$₹\ 325 = ₹\ \frac{P \times 12 \times 13 \times 4}{12 \times 2 \times 100}$$

$\Rightarrow$ $325 = \frac{26P}{100}$ $\Rightarrow$ P = ₹ 1250

Thus, Anwesha must deposit ₹ 1,250 monthly to receive the same interest.

5. (*d*) Nominal value (N.V.) of each share of Company A = ₹ 100

So, dividend received on 1 share of Company A = 12% of ₹ 100 = ₹ 12

Market value (M.V.) of each share of Company A = ₹ 60

$\therefore$ Rate of return on 1 share of Company A = $\frac{12}{60} \times 100\% = 20\%$

Similarly,

Nominal value (N.V.) of each share of Company B = ₹ 50

So, dividend received on 1 share of Company B = 16% of ₹ 50
= ₹ 8

Market value (M.V.) of each share of Company B = ₹ 40

$\therefore$ Rate of return on 1 share on Company B = $\frac{8}{40} \times 100\% = 20\%$

Thus, both Mr. Das and Mr. Singh have the same rate of return of 20%.

6. (*b*) Nominal value of (N.V.) of each share = ₹ 100

So, income received from 1 share = 7.5% of ₹ 100 = ₹ 7.50

Therefore, income received from 10 shares = 10 × ₹ 7.50 = ₹ 75

$\therefore$ Rate of return = $\frac{\text{Total income from 10 shares}}{\text{Market value of 10 shares}} \times 100$

$$10 = \frac{75}{\text{Market value of 10 shares}} \times 100$$

$\Rightarrow$ Market value of 10 shares = $\frac{75}{10} \times 100 = 750$

Thus, money invested by Amit to purchase 10 shares is ₹ 750.

7. (*a*) Assertion (A) is true:

According to the given information,

Rate of return from Company A = $\frac{7}{120} \times 100\% = 5.83\%$ (approx)

Rate of return from Company B = $\frac{80}{1620} \times 100\% = 4.94\%$ (approx)

So, investment in Company A is better than Company B.

Therefore, Assertion (A) is true.

Reason (R) is true:

The statement given in Reason (R) is true and the correct explanation for Assertion (A).

Hence, option (*a*) is the correct answer.

II: Short Answer Questions-1 (3 marks each)

8. Marked price of the pressure cooker = ₹ 1800

Rate of GST = 12%

$\therefore$ Sale price of the cooker including GST

= ₹ 1800 + 12% of ₹ 1800

= ₹ (1800 + 216) = ₹ 2016

Reduced price of the cooker = ₹ 1792

∴ Discount given by the shopkeeper = ₹ 2016 – ₹ 1792
= ₹ 224

Discount per cent = $\frac{224}{2016} \times 100\% = 11.11\%$

Thus, the shopkeeper must give 11.11% discount to the customer.

9. Let the man deposit ₹ P per month in his R.D. account.

Then,

Monthly deposit, P = ₹ P; Time, n = 2 years, *i.e.*, 24 months; Interest, I = ₹ P;

Rate of interest, r = ?

Using R.D. formula, we have

$$\text{Interest, I} = ₹\text{ P} \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow \quad ₹\text{ P} = ₹\text{ P} \times \frac{24 \times 25}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow \quad 1 = \frac{24 \times 25}{24} \times \frac{r}{100}$$

$$\Rightarrow \quad r = \frac{24 \times 100}{24 \times 25} = 4$$

Thus, the bank was paying 4% p.a. interest on the recurring deposit account.

10. Par value or Nominal value of 1 share = ₹ 50

Number of shares purchased = 350

So, dividend received from 350 shares = 350 × 14% of ₹ 50
= 350 × ₹ 7 = ₹ 2450

$$\therefore \text{ Rate of return} = \frac{\text{Total dividend received}}{\text{Market value of 350 shares}} \times 100$$

$$\Rightarrow \quad 10 = \frac{2450}{\text{Market value of 350 shares}} \times 100$$

$$\Rightarrow \quad \text{Market value of 350 shares} = \frac{2450 \times 100}{10} = 24500$$

∵ Market value of 350 shares = ₹ 24500

$$\therefore \text{ Market value of 1 share} = ₹\ \frac{24500}{350} = ₹\ 70$$

Hence, market value of each share is ₹ 70.

III: Short Answer Questions-2 (4 marks each)

11. Following is the bill with marked price and GST rate charged on various items.

Grow Shree Groceries				
S. No.	**Item**	**Marked Price (₹ per kg)**	**Quantity**	**Rate of GST**
1	Wheat Flour (Unpacked)	35.00	5 kg	x%
2	Basmati Rice (Branded & Packed)	180.00	5 kg	5%
3	Surf Excel Quick Wash Detergent	220.00	y kg	18%

(*a*) Total price of 5 kg of wheat flour at ₹ 35 per kg = ₹ (5 × 35) = ₹ 175

GST charged on wheat flour = x% of ₹ 175 = ₹ $\frac{175x}{100}$

$$= ₹\ \frac{7}{4}x$$

Total price of 5 kg of basmati rice at ₹ 180 per kg = ₹ (5 × 180)

= ₹ 900

GST charged on basmati rice = 5% of ₹ 900 = $\frac{5}{100}$ × ₹ 900

= ₹ 45

Given that total GST on wheat flour and basmati rice is ₹ 45.

∴ ₹ $\frac{7}{4}x$ + ₹ 45 = ₹ 45

⇒ ₹ $\frac{7}{4}x = 0$ ⇒ $x = 0$

Thus, value of x is 0, *i.e.*, rate of GST on wheat flour is 0% or nil.

(*b*) Here, CGST paid for detergent powder is ₹ 39.60. Since rate of GST on detergent powder is 18%, half of this GST, *i.e.*, 9% will be deposited in CGST account and other half in SGST account.

Now,

Marked price of y kg of detergent powder at ₹ 220 per kg = ₹ (y × 220) = ₹ 220 y

∴ CGST charged on detergent power = 9% of ₹ 220y

= $\frac{9}{100}$ × ₹ 220y = ₹ 19.8y

Given that CGST paid for detergent power is ₹ 39.60.

∴ $19.8y = 39.60$

⇒ $y = \frac{39.60}{19.8} = 2$

Thus, value of y is 2, *i.e.*, 2 kg of detergent power is purchased.

(*c*) Total marked price of all the three items

= ₹ (5 × 35 + 5 × 180 + 2 × 220) = ₹ 1515

Total GST charged on the three items

= ₹ (0% of ₹ 175 + 5% of ₹ 900 + 18% of ₹ 440)

= ₹ (45 + 79.20) = ₹ 124.20

Thus, total amount to be paid (including GST)

= ₹ 1515 + ₹ 124.20 = ₹ 1639.20.

12. Here, monthly deposit, P = ₹ 600; Rate of interest, r = 12% p.a.

(*a*) Let n be the number of monthly instalments. Then,

Interest received, I = P × $\frac{n(n+1)}{2\times 12} \times \frac{r}{100}$

= ₹ 600 × $\frac{n(n+1)}{24} \times \frac{12}{100}$ = ₹ $3n(n+1)$

Also, maturity value, M.V. = P × n + I

⇒ $11826 = 600n + 3n(n+1)$ [∵ M.V. = 11826]

⇒ $3942 = 200n + n^2 + n$

⇒ $n^2 + 201n - 3942 = 0$

⇒ $n^2 + 219n - 18n - 3942 = 0$

⇒ $n(n + 219) - 18(n + 219) = 0$

⇒ $(n + 219)(n - 18) = 0$

⇒ $n = 18$ or $n = -219$

⇒ $n = 18$ [∵ n cannot be negative, $n \neq -219$]

Thus, Amit deposited ₹ 600 in his R.D. account for 18 months.

(*b*) Total interest paid by the bank = ₹ $3n(n + 1)$

= ₹ 3 × 18 (18 + 1)

= ₹ 54 × 19 = ₹ 1026

(*c*) Total amount deposited by Amit = ₹ $600n$

= ₹ 600 × 18 = ₹ 10800

13. Market value of ₹ 100 shares = ₹ 120

∴ Total investment in 500, ₹ 100 shares = ₹ (500 × 120) = ₹ 60,000

(*a*) Aman has 500, ₹ 100 shares, paying 10% dividend.

He sells these shares when the share price rises to ₹ 200.

∴ Total selling price of 500, ₹ 100 share = ₹ (500 × 200)

= ₹ 100000

Thus, the sale proceeds is ₹ 1,00,000.

(*b*) Aman invest half of the sale proceeds in ₹ 10, 12% share at ₹ 25.

∴ Total investment in ₹ 10, 12% shares at ₹ 25 = ₹ 100000 ÷ 2

= ₹ 50,000

(*c*) Original income can be calculated from 500, ₹ 100 shares, paying 10% dividend.

∴ Original income received from these shares

= 500 × 10% of ₹ 100

= ₹ (500 × 10) = ₹ 5000

(*d*) Out of total sale proceeds, Aman invest half in ₹ 10, 12% shares at ₹ 25, and the remaining in ₹ 400, 9% shares at ₹ 500.

∴ Number of ₹ 10 at ₹ 25 shares purchased = $\frac{50000}{25} = 2000$

Number of ₹ 400 at ₹ 500 shares purchased = $\frac{50000}{500} = 100$

So, income received from 2000, ₹ 10, 12% shares = ₹ (2000 × 10 × 12%)

= ₹ 2400

and income received from 100, ₹ 400, 9% shares

= ₹ (100 × 400 × 9%)

= ₹ 3600

∴ Total income received from these shares = ₹ 2400 + ₹ 3600

= ₹ 6000

So, total income from new shares is ₹ 6000.

Thus, change in income = total income from new shares – original income

= ₹ 6000 – ₹ 5000 = ₹ 1000.

IV: Long Answer Questions (5 marks each)

14. The stationary bill handed over to Chetan is:

	Pen	**Pencil**	**Rainbow Cover Notebook**
Price	₹ 5 each	₹ 7 each	₹ 200 each
Discount	5% on a dozen pens	10% on 20 pencils	...
Premium	...	...	₹ 50 on each notebook
Items purchased	1 dozen	20 pencils	5
GST	18%	12%	12%

(*a*) (*i*) Marked price of 1 dozen pens at ₹ 5 each

$= 12 \times ₹\ 5 = ₹\ 60$

Discount offered = 5% of ₹ 60 = ₹ 3

∴ Selling price of 1 dozen pens = ₹ 60 – ₹ 3 = ₹ 57

[∵ Selling price = Marked price – Discount]

Marked price of 20 pencils at ₹ 7 each

$= 20 \times ₹\ 7 = ₹\ 140$

Discount offered = 10% of ₹ 140 = ₹ 14

∴ Selling price of 20 pencils = ₹ 140 – ₹ 14 = ₹ 126

Marked price of 5 notebooks at ₹ 200 each

$= 5 \times ₹\ 200 = ₹\ 1000$

Premium charged on notebooks at ₹ 50 each = 5 × ₹ 50 = ₹ 250

∴ Selling price of 5 notebooks = ₹ 1000 + ₹ 250 = ₹ 1250

On adding selling prices of all the items, we have

Total selling price = ₹ 57 + ₹ 126 + ₹ 1250 = ₹ 1433

Thus, total selling price as per the offers displayed on the board is ₹ 1,433.

(*ii*) Applying correct GST on selling prices of all the items, we have

GST charged at the rate 18% on pens = 18% of ₹ 57 = ₹ 10.26

GST charged at the rate 12% on pencils = 12% of ₹ 126

= ₹ 15.12

GST charged at the rate 12% on notebooks = 12% of ₹ 1250

= ₹ 150

∴ Total GST charged on all the items = ₹ 10.26 + ₹ 15.12 + ₹ 150

= ₹ 175.38

Thus,

Total amount to be paid by Chetan including GST with correct rates =

Total selling price + Total GST charged = ₹ 1433 + ₹ 175.38 = ₹ 1608.38

Hence, Chetan had to pay ₹ 1,608.38 for purchasing all the items.

(*iii*) It is given that the shopkeeper charged a uniform 18% GST on all the items. It means instead of 12%, the shopkeeper mischarged 18% on pencils and notebooks as well. Since GST on pens was already 18%, we recalculate GST on these two items only.

GST charged at the rate of 18% on pencils = 18% of ₹ 126

= ₹ 22.68

GST charged at the rate of 18% on notebooks = 18% of ₹ 1250

= ₹ 225

∴ Total GST charged on all the items = ₹ 10.26 + ₹ 22.68 + ₹ 225

= ₹ 257.94

Thus,

Total amount of all the items including incorrect GST

= Total selling price + Total GST charged

= ₹ 1433 + ₹ 257.94

= ₹ 1690.94

On this amount, the shopkeeper further gave a discount of 2%.

∴ After additional 2% discount, actual bill = ₹ 1690.94 – 2% of ₹ 1690.94
= ₹ 1690.94 – ₹ 33.82
= ₹ 1657.12

Hence, the actual amount charged by shopkeeper is ₹ 1,657.12

(*b*) Yes, the shopkeeper overcharged Chetan.

Justification:

Total amount of the bill including GST with correct rates
= ₹ 1608.38

Actual amount charged by the shopkeeper = ₹ 1657.12

∴ Amount overcharged by the shopkeeper = ₹ 1657.12 – ₹ 1608.38
= ₹ 48.74

Hence, the shopkeeper charged ₹ 48.74 more than the amount of correct bill.

Unit II: Algebra (Chapters 4–10)

I: Multiple Choice Questions (1 mark each)

1. (*c*) The given inequation is $-3 \le -4x + 5,\ x \in W$. Therefore,

$$-3 \le -4x + 5$$
$$\Rightarrow \quad -3 - 5 \le -4x + 5 - 5 \qquad \text{[Subtracting 5 from both sides]}$$
$$\Rightarrow \quad -8 \le -4x$$
$$\Rightarrow \quad 8 \ge 4x \qquad \text{[Multiplying both sides by } -1\text{]}$$
$$\text{or} \quad 4x \le 8 \quad \text{or} \quad x \le 2$$

$\therefore$ The solution set is {0, 1, 2}.

2. (*c*) Given, $-4x > 8y$

$$\Rightarrow \quad -x > 2y \qquad \text{[Dividing both sides by 4]}$$
$$\Rightarrow \quad x < -2y. \qquad \text{[Multiplying both sides by } -1\text{]}$$

3. (*d*) The given quadratic equation is $2x^2 - kx + k = 0$.

The roots of the given equation will be equal, if $b^2 - 4ac = 0$.

$$\Rightarrow \quad (-k)^2 - 4 \times 2 \times k = 0 \quad \Rightarrow \quad k^2 - 8k = 0$$
$$\Rightarrow \quad k(k - 8) = 0$$
$$\Rightarrow \quad k = 0 \ \text{ or } \ k = 8$$

Thus, for $k = 0$ and $k = 8$, the roots of the given equation will be equal.

4. (*b*) Given that $x = -2$ is one of the solutions of the quadratic equation $x^2 + 3a - x = 0$.

$\therefore$ Putting $x = -2$ in the given equation, we have

$$(-2)^2 + 3a - (-2) = 0$$
$$\Rightarrow \quad 4 + 3a + 2 = 0 \quad \Rightarrow \quad 3a + 6 = 0$$
$$\Rightarrow \quad 3a = -6$$
$$\text{or} \quad a = -2$$

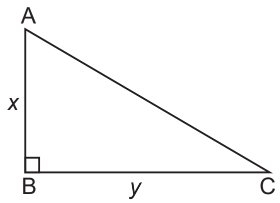
Thus, the required value of a is -2.

5. (*d*) Given value of the variable $x = 233.356$

$\therefore$ Solution rounded to two significant figures is 230.

[$\because$ All zeroes after a non-zero digit are not significant.]

6. (*c*) In right triangle ABC,



$$\text{Area of } \Delta ABC = \frac{1}{2} \times AB \times BC$$
$$\Rightarrow \quad 30 \text{ cm}^2 = \frac{1}{2} \times x \times y \text{ cm}^2$$
$$\Rightarrow \quad xy = 60 \qquad \ldots(1)$$

Also, we have

$$x - y = 7 \qquad \ldots(2)$$
$$\text{or} \quad x = 7 + y$$

Putting $x = 7 + y$ in eq (1), we have

$$(7 + y)\, y = 60$$
$$\Rightarrow \quad y^2 + 7y - 60 = 0$$
$$\Rightarrow \quad y^2 + 12y - 5y - 60 = 0$$
$$\Rightarrow \quad (y + 12) - 5(y + 12) = 0$$
$$\Rightarrow \quad (y + 12)\,(y - 5) = 0$$

$\Rightarrow \quad y = -12 \quad \text{or} \quad y = 5$

$\Rightarrow \quad y = 5$ [$\because$ y cannot be –ve, $y = -12$ is not an admissible value.]

From eq (2), $x = 7 + 5 = 12$

$\therefore$ In right triangle ABC, AB = 12 cm and BC = 5 cm

Now, by Pythagoras' Theorem, $AC^2 = AB^2 + BC^2$

$$= (12)^2 + (5)^2$$

$$= 144 + 25 = 169$$

$\therefore$ AC = 13 cm

7. (*d*) Given, p, q and r are in continued proportion. Then,

$$\frac{p}{q} = \frac{q}{r} \quad \text{or} \quad q^2 = pr$$

$\therefore$ From option (*d*), $\frac{p}{r} = \frac{p^2}{q^2}$

$$\Rightarrow \quad \frac{p}{r} = \frac{p^2}{pr} \quad \Rightarrow \quad \frac{p}{r} = \frac{p}{r},\ \text{which is true.}$$

Thus, option (*d*) is the correct option.

8. (*d*) Let $p(x) = 2x^3 + 3x^2 - 2x - 3$

Since the polynomial $p(x)$ is completely divisible by $(2x + a)$,

$(2x + a)$ is a factor of $p(x)$.

$\therefore$ By Factor Theorem, $p\left(\frac{-a}{2}\right) = 0$.

Now,

$$p\left(\frac{-a}{2}\right) = 0 \quad \Rightarrow \quad 2\left(\frac{-a}{2}\right)^3 + 3\left(\frac{-a}{2}\right)^2 - 2\left(\frac{-a}{2}\right) - 3 = 0$$

$$\Rightarrow \quad \frac{-a^3}{4} + \frac{3a^2}{4} + a - 3 = 0$$

$$\Rightarrow \quad -a^3 + 3a^2 + 4a - 12 = 0$$

$$\Rightarrow \quad a^3 - 3a^2 - 4a + 12 = 0$$

$$\Rightarrow \quad a^2(a - 3) - 4(a - 3) = 0$$

$$\Rightarrow \quad (a - 3)(a^2 - 4) = 0$$

$$\Rightarrow \quad a = 3 \quad \text{or} \quad a^2 = 4$$

$$\Rightarrow \quad a = 3 \quad \text{or} \quad a = \pm 2$$

Since one of the values of a is 3, option (*d*) is the correct option.

Alternatively: By division algorithm,

Dividend = Divisor × Quotient

$$\therefore \quad 2x^3 + 3x^2 - 2x - 3 = (2x + a)(x^2 - 1)$$

$$\Rightarrow \quad 2x^3 + 3x^2 - 2x - 3 = 2x^3 + ax^2 - 2x - a$$

Equating the coefficient of x^2 or constant terms on both sides, we have $a = 3$.

9. (*c*) The given polynomial is $p(x) = x^3 + 5x^2 - kx - 24$.

Putting $k = 2$, we have

$$p(x) = x^3 + 5x^2 - 2x - 24$$

$$= x^3 + 4x^2 + x^2 + 4x - 6x - 24$$

$$= x^2(x+4) + x(x+4) - 6(x+4)$$
$$= (x+4)(x^2+x-6)$$
$$= (x+4)(x^2+3x-2x-6)$$
$$= (x+4)[x(x+3)-2(x+3)]$$
$$= (x+4)(x+3)(x-2)$$

Since $(x + 4)$ is one of the factors in the given options, option (*c*) is the correct answer.

Alternatively:

Putting $x = 2$ in $p(x) = x^3 + 5x^2 - 2x - 24$, we have

$$p(2) = (2)^3 + 5(2)^2 - 2(2) - 24$$
$$= 8 + 20 - 4 - 24 = 28 - 28 = 0$$

Thus, $x - 2$ is a factor of $p(x)$.

∴ By division algorithm,

$$\begin{array}{r|l} & x^2 + 7x + 12 \\ x-2 & x^3 + 5x^2 - 2x - 24 \\ & \underline{_{-}x^3 \mp 2x^2} \\ & 7x^2 - 2x - 24 \\ & \underline{_{-}7x^2 \mp 14x} \\ & 12x - 24 \\ & \underline{_{-}12x \mp 24} \\ & 0 \end{array}$$

So, we have $p(x) = (x^2 + 7x + 12)(x - 2)$

$$= (x^2 + 4x + 3x + 12)(x - 2)$$
$$= [x(x + 4) + 3(x + 4)](x - 2)$$
$$(x + 4)(x + 3)(x - 2)$$

Hence, of the given options, $(x + 4)$ is the required factor.

10. (*d*) Here, A is 1×2 matrix and B is 2×1 matrix.

Since number of columns is A is equal to number of rows in B, the product matrix AB is possible and AB is 1×1 matrix. Similarly, number of columns in B is equal to the number of rows in A, the product matrix BA is also possible but BA is 2×2 matrix.

Thus, both matrix AB and BA are possible but AB ≠ BA.

Verification: We have, $AB = [a \quad b]\begin{bmatrix} c \\ d \end{bmatrix}$

$$= [ac + bd]$$

and
$$BA = \begin{bmatrix} c \\ d \end{bmatrix}[a \quad b] = \begin{bmatrix} ca & cb \\ da & db \end{bmatrix}$$

Clearly, AB is a 1×1 matrix whereas BA is a 2×2 matrix, therefore, AB ≠ BA.

11. (*b*) Given, $A = \begin{bmatrix} 6 & 9 \\ -4 & k \end{bmatrix}$ and $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\therefore \quad A^2 = A.A = \begin{bmatrix} 6 & 9 \\ -4 & k \end{bmatrix}\begin{bmatrix} 6 & 9 \\ -4 & k \end{bmatrix}$$

$$= \begin{bmatrix} 36-36 & 54+9k \\ -24-4k & -36+k^2 \end{bmatrix} = \begin{bmatrix} 0 & 54+9k \\ -24-4k & -36+k^2 \end{bmatrix}$$

Since $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ we have

$$\begin{bmatrix} 0 & 54+9k \\ -24-4k & -36+k^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$
$$54 + 9k = 0 \quad \Rightarrow \quad k = -6$$
$$-24 - 4k = 0 \quad \Rightarrow \quad k = -6$$
$$-36 - k^2 = 0 \quad \Rightarrow \quad k = \pm 6$$

Clearly, $k = -6$ is the common value of the above equations, the required value of k is -6.

12. (*b*) We have, $S_n = n^2 - n$

We know that nth term of an A.P. is given by $a_n = S_n - S_{n-1}$

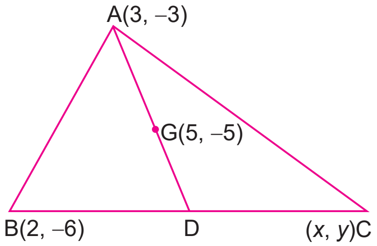
Putting $n = 3$ in above relation, we have

$$\begin{aligned} a_3 &= S_3 - S_2 \\ &= [(3)^2 - 3] - [(2)^2 - 2] \qquad [\because S_n = n^2 - n] \\ &= (9 - 3) - (4 - 2) \\ &= 6 - 2 = 4 \end{aligned}$$

Thus, third term of the series is 4.

13. (*d*) The common ratio for the sequence 2, 0, 4, 0, 8, 0 is not same, *i.e.*, $\frac{0}{2} \neq \frac{4}{0}$. Therefore, sequence given in option (*d*) is not a G.P. It can be verified that the sequences given in other options form a G.P.

14. (*c*) Given, G is the centroid of ΔABC, therefore



$$(5, -5) = \left(\frac{3+2+x}{3}, \frac{-3-6+y}{3}\right)$$

$$\Rightarrow \quad (5, -5) = \left(\frac{5+x}{3}, \frac{-9+y}{3}\right)$$

$$\Rightarrow \quad \frac{5+x}{3} = 5 \quad \text{and} \quad \frac{-9+y}{3} = -5$$

$$\Rightarrow \quad 5 + x = 15 \quad \text{and} \quad -9 + y = -15$$

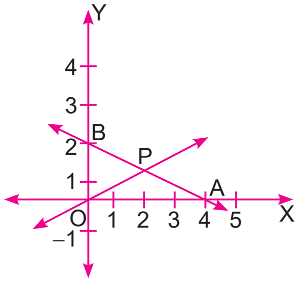
$$\Rightarrow \quad x = 10 \quad \text{and} \quad y = -6$$

Thus, coordinates of vertex C are (10, –6).

Since AD is one of the medians of ΔABC, D is the mid-point of BC.

$$\therefore \text{ D} = \left(\frac{2+10}{2}, \frac{-6-6}{2}\right) = (6, -6).$$

15. (*b*) In the given diagram,



$$A = (4, 0) \text{ and } B = (0, 2)$$

Since P is the mid-point of AB

$$P = \left(\frac{4+0}{2}, \frac{0+2}{2}\right) = (2, 1)$$

Thus, coordinates of P are (2, 1).

Given O is the origin, therefore coordinates of O are (0, 0).

Now, the line OP is passing through O and P. Therefore,

Equation of OP is given by

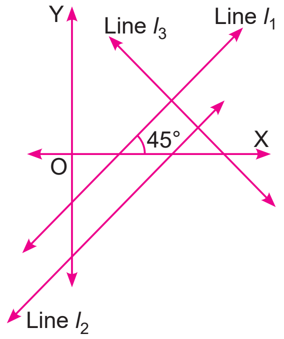
$$y - 0 = \frac{1-0}{2-0}(x - 0) \qquad \left[\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

$$\Rightarrow \quad y = \frac{1}{2}x \quad \Rightarrow \quad 2y = x$$

Thus, the equation of line OP is $2y = x$.

16. (*c*) In the figure, line l_1 is making an angle of 45° with positive direction of x-axis.

Therefore, slope (m_1) of line l_1 = tan 45° = 1



Line l_2 is parallel to line l_1

$\therefore$ Slope of line l_2 (m_2) = m_1 = 1

Line l_3 is perpendicular to line l_1

$\therefore$ Slope of line l_3 $(m_3) = -\frac{1}{m_1} = -1$

Thus, slopes of lines l_2 and l_3 respectively are 1 and –1.

17. (*a*) Consider the equation given in option (*a*) that is

$$3x + 3y = 6$$

Dividing both sides by 6, we get

$$\frac{x}{2} + \frac{y}{2} = 1$$

The above line cuts positive x-axis and positive y-axis at equal distance (2 units) from the origin.

Thus, $3x + 3y = 6$ is the required line.

It can be verified that intercepts made by the lines given in other options are not the same.

18. (*b*) Assertion (A) is true:

The given polynomial is $x^3 + 2x^2 - x - 2$.

Clearly, the degree of given polynomial is 3.

Therefore, Assertion (A) is true.

Reason (R) is true:

If $x + 2$ is a factor of the polynomial $p(x) = x^3 + 2x^2 - x - 2$, then $p(-2) = 0$

$\therefore p(-2) = (-2)^3 + 2(-2)^2 - (-2) - 2$

$= -8 + 8 + 2 - 2 = 0$

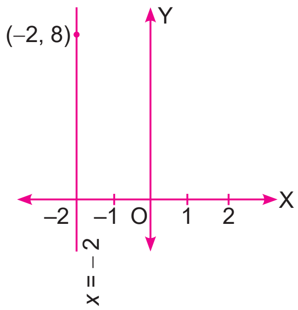
Therefore, Reason (R) is true.

Thus, both Assertion (A) and Reason (R) are true statements but. Reason (R) is not the correct explanation for Assertion (A).

Hence, option (*b*) is the correct answer.

19. (*c*) Assertion (A) is true:

Clearly, the point (–2, 8) lies itself on the line $x = -2$, therefore (–2, 8) is invariant under reflection in line $x = -2$.



Alternatively:

Reflection of a point (x, y) in the line $x = a$ is given by

$$M_{x=a}(x, y) = (2a - x, y)$$

$$\therefore \quad M_{x=-2}(-2, 8) = (2(-2) + 2, 8) \qquad \text{[Putting values of } a, x \text{ and } y.]$$

$$= (-2, 8)$$

Therefore, Assertion (A) is true.

Reason (R) is false:

If a point has its x-coordinate 0, *i.e.*, the point lies on y-axis, then the point is invariant under reflection in y-axis only.

Therefore, the statement given in Reason (R) is false.

Hence, option (*c*) is the correct answer.

20. (*b*) Here, matrix A is 2 × 2 matrix, so number of rows in matrix M should be equal to number of columns in matrix A, *i.e.*, 2 and number columns in matrix M should be equal to number of columns in matrix B, *i.e.*, 1.

Therefore, the order of matrix M is 2 × 1.

21. (*d*) Given $a_1, a_2, a_3 \ldots$ and $b_1, b_2, b_3 \ldots$ are real numbers such that $a_1 - b_1 = a_2 - b_2 = a_3 - b_3 = \ldots$ are all equal. Then, $a_1 - b_1, a_2 - b_2, a_3 - b_3, \ldots$ forms an arithmetic progression. Since all terms of the A.P. are equal, in this case common difference of the A.P. will be 0, *i.e.*, $d = 0$.

II: Short Answer Questions-1 (3 marks each)

22. The given inequation $\frac{1}{2}(2x-1) \le 2x + \frac{1}{2} \le 5\frac{1}{2} + x$ can be written as

$$\frac{1}{2}(2x-1) \le 2x + \frac{1}{2} \quad \text{and} \quad 2x + \frac{1}{2} \le 5\frac{1}{2} + x$$

$$\Rightarrow \quad x - \frac{1}{2} \le 2x + \frac{1}{2} \quad \text{and} \quad 2x + \frac{1}{2} \le \frac{11}{2} + x$$

$$\Rightarrow \quad x - 2x \le \frac{1}{2} + \frac{1}{2} \quad \text{and} \quad 2x - x \le \frac{11}{2} - \frac{1}{2}$$

$$\Rightarrow \quad -x \le 1 \quad \text{and} \quad x \le 5$$

$$\text{or} \quad x \ge -1 \quad \text{and} \quad x \le 5$$

Thus, the solution of the given inequation is $-1 \le x \le 5$.

(*a*) When $x \in$ R, x can take the maximum value 5 and minimum value as −1.

(*b*) When $x \in$ W, x can take the maximum value 5 and minimum value as 0.

Therefore, there is no change in maximum value but minimum value changes to 0 from −1.

23. The given equation is $\frac{5}{x} + 4\sqrt{3} = \frac{2\sqrt{3}}{x^2}, x \ne 0$

By cross multiplication, we have

$$x^2\left(\frac{5}{x} + 4\sqrt{3}\right) = 2\sqrt{3}$$

$$\Rightarrow \quad 5x + 4\sqrt{3}\,x^2 = 2\sqrt{3}$$

$$\Rightarrow \quad 4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$$

$$\Rightarrow \quad 4\sqrt{3}\,x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$\Rightarrow \quad 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$$

$$\Rightarrow \quad (\sqrt{3}x + 2)(4x - \sqrt{3}) = 0$$

$$\Rightarrow \quad \sqrt{3}x + 2 = 0 \quad \text{or} \quad 4x - \sqrt{3} = 0$$

$$\Rightarrow \quad x = \frac{-2}{\sqrt{3}} \quad \text{or} \quad x = \frac{\sqrt{3}}{4}$$

Thus, $x = \frac{-2}{\sqrt{3}}$ and $x = \frac{\sqrt{3}}{4}$ are the solutions of the given equation.

24. Given that price of the toy including GST is ₹ 24.

Taking marked price as ₹ x, we have

$$x + x\% \text{ of } x = 24 \qquad [\because \text{ Percentage of GST is same as marked price.}]$$

$\Rightarrow \quad x + \frac{x}{100} \times x = 24$

$\Rightarrow \quad x + \frac{x^2}{100} = 24$

$\Rightarrow \quad x^2 + 100x - 2400 = 0$

$\Rightarrow \quad x^2 + 120x - 20x - 2400 = 0$

$\Rightarrow \quad x(x + 120) - 20(x + 120) = 0$

$\Rightarrow \quad (x + 120)(x - 20) = 0$

$\Rightarrow \quad x = 20 \quad \text{or} \quad x = -120$

$\Rightarrow \quad x = 20$ [$\because$ Marked price cannot be negative, $x \neq -120$.]

Thus, marked price of the toy is ₹ 20 and GST charged on it is 20%.

25. Let a and b be the required numbers.

Given, mean proportion between two numbers is 6.

$\therefore \quad a : 6 :: 6 : b$

$\Rightarrow \quad \frac{a}{6} = \frac{6}{b}$

$\Rightarrow \quad ab = 36$ ...(1)

Also, third proportion between two numbers is 48.

$\therefore \quad a : b :: b : 48$

$\Rightarrow \quad \frac{a}{b} = \frac{b}{48}$

$\Rightarrow \quad b^2 = 48a$

$\Rightarrow \quad b^2 = 48\left(\frac{36}{b}\right)$ $\left[\because \text{From (1), } a = \frac{36}{b}\right]$

$\Rightarrow \quad b^3 = 48 \times 36$

$= 16 \times 3 \times 9 \times 4$

$= 64 \times 27$

$\therefore \quad b = \sqrt[3]{64 \times 27} = 4 \times 3 = 12$

Putting the value of b in (1), we have

$12a = 36 \quad \text{or} \quad a = 3$

Thus, 3 and 12 are the required two numbers

26. The given polynomial is $p(x) = 2x^3 + 3x^2 - 3x - 2$.

If $(x + 2)$, $(x - 1)$ and $(x - 2)$ are the factors of $p(x)$, then by Factor Theorem, we should have $p(-2) = 0$, $p(1) = 0$ and $p(2) = 0$.

Now, $\quad p(-2) = 2(-2)^3 + 3(-2)^2 - 3(-2) - 2$

$= -16 + 12 + 6 - 2 = 18 - 18 = 0$

$\therefore \quad (x + 2)$ is a factor of $p(x)$.

$p(1) = 2(1)^3 + 3(1)^2 - 3(1) - 2$

$= 2 + 3 - 3 - 2 = 5 - 5 = 0$

$\therefore \quad (x - 1)$ is also a factor of $p(x)$

$p(2) = 2(2)^3 + 3(2)^2 - 3(2) - 2$

$= 16 + 12 - 6 - 2 = 28 - 8 = 20 \neq 0$

Since $p(2) \neq 0$, $(x - 2)$ is not a factor of $p(x)$.

We have verified that $(x + 2)$ and $(x - 1)$ are the two factors of $p(x)$, therefore $p(x)$ must be divisible by $(x + 2)$ and $(x - 1)$ both or $p(x)$ must be divisible by $(x + 2)(x - 1)$, *i.e.*, $x^2 + x - 2$.

By division algorithm,

$$\begin{array}{r|l} & 2x + 1 \\ x^2 + x - 2 & 2x^3 + 3x^2 - 3x - 2 \\ & \underline{\underset{-}{}2x^3 \underset{-}{+} 2x^2 \underset{+}{-} 4x} \\ & \quad x^2 + x - 2 \\ & \quad \underline{\underset{-}{}x^2 \underset{-}{+} x \underset{+}{-} 2} \\ & \qquad 0 \end{array}$$

$\therefore \quad 2x^3 + 3x^2 - 3x - 2 = (2x + 1)(x^2 + x - 2)$

Thus, the third factor is $(2x + 1)$.

Hence, $(x + 2)$, $(x - 1)$ and $(2x + 1)$ are the correct factors of the given polynomial.

Alternatively:

Third factor can also be determined by breaking the given polynomial in such a way so that $x^2 + x - 2$ be one of the factors of $p(x)$.

The factorisation process is as under:

$$\begin{aligned} p(x) &= 2x^3 + 3x^2 - 3x - 2 \\ &= 2x^3 + x^2 + 2x^2 + x - 4x - 2 \\ &= x^2(2x + 1) + x(2x + 1) - 2(2x + 1) \\ &= (2x + 1)(x^2 + x - 2). \end{aligned}$$

27. Given, $A = \begin{bmatrix} -6 & 0 \\ 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix}$

$\therefore$ Matrix $M = \frac{1}{2}A - 2B + 5I$, where I is the identity matrix

$$= \frac{1}{2}\begin{bmatrix} -6 & 0 \\ 4 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 2 & 6 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -3-2+5 & 0+0+0 \\ 2-2+0 & 1-6+5 \end{bmatrix}$$

$$= \begin{bmatrix} -5+5 & 0 \\ 2-2 & 6-6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$, which is the required matrix.

28. (*a*) The whole numbers which are divisible are 3 and 7 both are the numbers divisible by their product, *i.e.*, 21.

Therefore the required A.P. is 21, 42, 63, ...

For this A.P., first term, $a = 21$, common difference, $d = 42 - 21 = 21$

$\therefore$ nth term, $T_n = a + (n - 1)d$

$$= 21 + (n - 1)(21) = 21 + 21n - 21 = 21n$$

So, nth term of the A.P. is $21n$.

(*b*) Clearly, 4th term of this A.P. is 84 and 5th term is 105, which is a three-digit number.

So, first four terms of the A.P. are two-digit numbers.

Thus, there are *four* two-digit numbers in the A.P. and these numbers are 21, 42, 63 and 84.

(*c*) For the A.P.: 21, 42, 63, ...

Sum of first 10 terms, $S_{10} = \frac{10}{2}[2 \times 21 + (10 - 1)\, 21]$ $\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$$= 5[42 + 9 \times 21]$$

$$= 5[42 + 189] = 5 \times 231 = 1155$$

Thus, the sum of first 10 terms of the A.P. is 1155.

29. The given sequence is $(\sqrt{3})^n$, $n \in N$

Putting $n = 1, 2, \ldots 5$, we get the first five terms as:

$$t_1 = (\sqrt{3})^1 = \sqrt{3}$$
$$t_2 = (\sqrt{3})^2 = 3$$
$$t_3 = (\sqrt{3})^3 = 3\sqrt{3}$$
$$t_4 = (\sqrt{3})^4 = 9$$
$$t_5 = (\sqrt{3})^5 = 9\sqrt{3}$$

Thus, required terms are $\sqrt{3}$, 3, $3\sqrt{3}$, 9, $9\sqrt{3}$.

(*a*) Here, common ratio for the above sequence is same, *i.e.*, $\frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \ldots$

The given sequence is a G.P.

(*b*) For the G.P.: $\sqrt{3}$, 3, $3\sqrt{3}$, ...

We have, first term, $a = \sqrt{3}$, common ratio, $r = \frac{3}{\sqrt{3}} = \sqrt{3}$

Sum of first *n* terms of a G.P. is given by

$$S_n = \frac{a(r^n - 1)}{r - 1}, \; r > 1$$

$\therefore$ Sum of first 10 terms, $S_{10} = \frac{\sqrt{3}[(\sqrt{3})^{10} - 1)]}{\sqrt{3} - 1}$

$$= \frac{\sqrt{3}(3^5 - 1)}{\sqrt{3} - 1}$$

$$= \frac{\sqrt{3}(243 - 1)}{\sqrt{3} - 1} = \frac{242\sqrt{3}}{\sqrt{3} - 1}$$

$$= \frac{242\sqrt{3}(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{242\sqrt{3}(\sqrt{3} + 1)}{2}$$

$$= 121\sqrt{3}(\sqrt{3} + 1) = 121(3 + \sqrt{3})$$

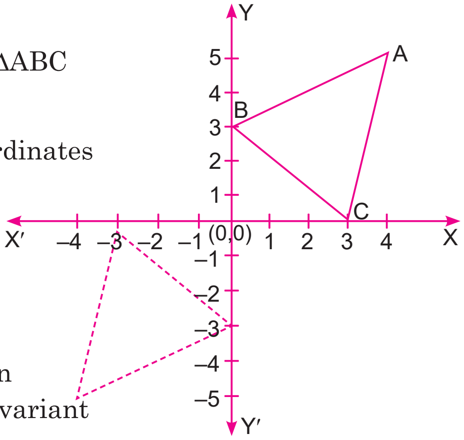
Given that sun of first ten terms is $p(3 + \sqrt{3})$. Therefore,

$$p(3 + \sqrt{3}) = 121(3 + \sqrt{3})$$

$$\Rightarrow \quad p = 121$$

Hence, the value of *p* is 121.

30. (*a*) In the figure, coordinates of the vertices A, B and C of ΔABC respectively are (4, 5), (0, 3) and (3, 0).



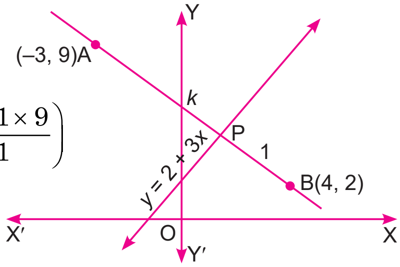
We know that on reflection through the origin, the coordinates of a point (*x*, *y*) are (–*x*, –*y*).

$\therefore$ On reflection through the origin, the coordinates of A, B and C respectively are (–4, –5), (0, –3) and (–3, 0).

(*b*) We know that a point on *x*-axis remains invariant when reflected on the *x*-axis and a point on *y*-axis remains invariant when reflected on the *y*-axis.

Therefore, when ΔABC is reflected on the *x*-axis, point C(3, 0) remains invariant and point B(0, 3) remains invariant when ΔABC is reflected on the *y*-axis.

31. Let the line $y = 2 + 3x$ divides the line segment AB joining the points A(–3, 9) and B(4, 2) at point P in the ratio $k : 1$. Then,



By section formula, coordinates of P are $\left(\frac{k\times 4+1\times(-3)}{k+1}, \frac{k\times 2+1\times 9}{k+1}\right)$

$$= \left(\frac{4k-3}{k+1}, \frac{2k+9}{k+1}\right)$$

Since point P also lies on the line $y = 2 + 3x$, coordinates of P satisfy the given line. Therefore,

$$\frac{2k+9}{k+1} = 2+3\left(\frac{4k-3}{k+1}\right)$$

$$\Rightarrow \quad \frac{2k+9}{k+1} = 2+\frac{12k-9}{k+1}$$

$$\Rightarrow \quad \frac{2k+9}{k+1} = \frac{2k+2+12k-9}{k+1}$$

$$\Rightarrow \quad \frac{2k+9}{k+1} = \frac{14k-7}{k+1}$$

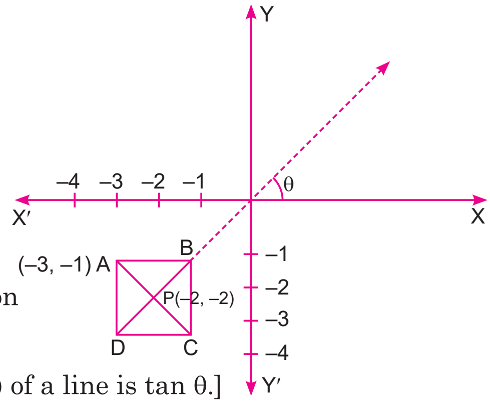
$$\Rightarrow \quad 2k + 9 = 14k - 7$$

$$\Rightarrow \quad 9 + 7 = 14k - 2k$$

$$\Rightarrow \quad 16 = 12k \quad \Rightarrow \quad k = \frac{16}{12} \quad \text{or} \quad k = \frac{4}{3}$$

Thus, the required ratio is $\frac{4}{3} : 1$, *i.e.*, 4 : 3.

32. Given that square ABCD lies in third quadrant in XY plane and coordinates of A are (–3, –1). Diagonal DB is equally inclined to both the axis when produced and diagonals AC and BD meet at P(–2, –2).



(*a*) On producing, DB equally inclined to both the axes. Therefore, DB makes angle 45° with positive direction of x-axis.

∴ Slope (m) of line BD = tan 45° = 1 [∵ Slope (m) of a line is tan θ.]

Thus, slope of the line BD is 1.

(*b*) Given that AC and BD meet at P(–2, –2).

Therefore, line AC is passing through A(–3, –1) and P(–2, –2).

Using two points form, $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Equation of AC is:

$$y - (-1) = \frac{-2-(-1)}{-2-(-3)}(x - (-3))$$

$$\Rightarrow \quad y + 1 = \frac{-2+1}{-2+3}(x + 3)$$

$$\Rightarrow \quad y + 1 = -(x + 3)$$

$$\Rightarrow \quad y + 1 = -x - 3$$

$\Rightarrow \quad x + y + 4 = 0$, which is the required equation.

III: Short Answer Questions-2 (4 marks each)

33. (*a*) The given inequation $\frac{11+3x}{5} \geq 3 - x > -\frac{-3}{2}$, $x \in$ R can be rewritten as

$$\frac{11+3x}{5} \geq 3 - x \quad \text{and} \quad 3 - x > \frac{-3}{2}$$

$$\Rightarrow \quad 11 + 3x \geq 5(3 - x) \quad \text{and} \quad 2(3 - x) > -3$$

$$\Rightarrow \quad 11 + 3x \geq 15 - 5x \quad \text{and} \quad 6 - 2x > -3$$

$$\Rightarrow \quad 3x + 5x \geq 15 - 11 \quad \text{and} \quad 6 + 3 > 2x$$

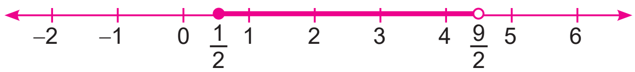
$$\Rightarrow \quad 8x \geq 4 \quad \text{and} \quad 9 > 2x$$

$$\Rightarrow \quad x \geq \frac{1}{2} \quad \text{and} \quad x < \frac{9}{2}$$

The above solution is equivalent to $\frac{1}{2} \leq x < \frac{9}{2}$.

Thus, the required solution set is $\left\{x \mid x \in \text{R and } \frac{1}{2} \leq x < \frac{9}{2}\right\}$.

(*b*) On the number line above solution can be represented as:



34. (*a*) The given quadratic equation is

$$5x^2 - 9x + 4 = 0$$

The above equation will have real roots if discriminant, D = $b^2 - 4ac \geq 0$.

Therefore, $\quad$ D = $b^2 - 4ac$

$$= (-9)^2 - 4 \times 5 \times 4 = 81 - 80 = 1$$

Since D $\geq$ 0, the roots of the given equation are real.

(*b*) The roots of the given equation can be identified by the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x = \frac{-(-9) \pm \sqrt{1}}{2 \times 5} \qquad [\because \text{ D} = 1]$$

$$= \frac{9 \pm 1}{10}$$

Taking +ve sign, we have $x = \frac{9+1}{10} = 1$

Taking –ve sign, we have $x = \frac{9-1}{10} = \frac{4}{5}$

Thus, $x = 1$ and $x = \frac{4}{5}$ are the two roots of the given equation.

35. The table showing number of customers and profit per week is:

Week Number	**Week 1**	**Week 2**	**Week 3**	**Week 4**
Number of Customers	1400	5600	x	3212
Profit (in ₹)	28000	112000	32140	y

(*a*) We know that if four numbers *a*, *b*, *c* and *d* are in continued proportion, then

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

Here, $\quad \frac{a}{b} = \frac{1400}{28000} = \frac{1}{20}$

and $\frac{b}{c} = \frac{28000}{5600} = 5$

Since $\frac{a}{b} \neq \frac{b}{c}$, the number of customers and profit per week are not in continued proportion.

(*b*) Here, ratio of number of customers and profit for Week 1

$$= \frac{1400}{28000} = \frac{1}{20}$$

and ratio of number of customers and profit for Week 2

$$= \frac{5600}{112000} = \frac{1}{20}$$

Since the two ratios are equal, the number of customers and profit per week are in proportion.

$$\therefore \quad \frac{1400}{28000} = \frac{x}{32140}$$

$$\Rightarrow \quad \frac{1}{20} = \frac{x}{32140} \quad \Rightarrow \quad x = \frac{32140}{20} = 1607$$

Also, $\frac{1400}{28000} = \frac{3212}{y}$

$$\Rightarrow \quad \frac{1}{20} = \frac{3212}{y} \quad \Rightarrow \quad y = 3212 \times 20 = 64240$$

Thus, values of x and y are 1607 and 64240 respectively.

36. (*a*) Let $p(x) = 9x^3 - mx^2 - nx + 8$

Given that $9x^2 - 4$ is a factor $p(x) = 9x^3 - mx^2 - nx + 8$.

We have, $9x^2 - 4 = (3x - 2)(3x + 2)$

Therefore, by Factor Theorem, $(3x - 2)$ and $(3x + 2)$ both divide $p(x)$ completely,

i.e., $p\left(\frac{2}{3}\right) = 0$ and $p\left(\frac{-2}{3}\right) = 0$.

$$\therefore \quad p\left(\frac{2}{3}\right) = 0 \quad \Rightarrow \quad 9\left(\frac{2}{3}\right)^3 - m\left(\frac{2}{3}\right)^2 - n\left(\frac{2}{3}\right) + 8 = 0$$

$$\Rightarrow \quad \frac{8}{3} - \frac{4m}{9} - \frac{2n}{3} + 8 = 0$$

$$\Rightarrow \quad \frac{24 - 4m - 6n + 72}{9} = 0$$

$$\Rightarrow \quad -4m - 6n + 96 = 0$$

$$\Rightarrow \quad 2m + 3n = 48 \qquad \ldots(1)$$

Also, $p\left(\frac{-2}{3}\right) = 0 \quad \Rightarrow \quad 9\left(\frac{-2}{3}\right)^3 - m\left(\frac{-2}{3}\right)^2 - n\left(\frac{-2}{3}\right) + 8 = 0$

$$\Rightarrow \quad -\frac{8}{3} - \frac{4}{9}m + \frac{2n}{3} + 8 = 0$$

$$\Rightarrow \quad -\frac{-24 - 4m + 6n + 72}{9} = 0$$

$$\Rightarrow \quad -4m + 6n + 48 = 0$$

$$\Rightarrow \quad 2m - 3n = 24 \qquad \ldots(2)$$

On adding (1) and (2), we get

$$4m = 72 \quad \Rightarrow \quad m = 18$$

Putting the value of m in (1), we get

$$2 \times 18 + 3n = 48$$

$$\Rightarrow \quad 3n + 36 = 48 \quad \Rightarrow \quad 3n = 12$$

$$\Rightarrow \quad n = 4$$

Thus, $m = 18$ and $n = 4$ are the required values.

(*b*) Putting $m = 18$ and $n = 4$ in the given polynomial, we have

$$p(x) = 9x^3 - 18x^2 - 4x + 8$$

Since $9x^2 - 4$, *i.e.*, $(3x - 2)$ and $(3x + 2)$ are the factors of the given polynomial, the third factor can be determined by dividing the polynomial by $9x^2 - 4$.

$$\begin{array}{r|l} & x - 2 \\ \hline 9x^2 - 4 & 9x^3 - 18x^2 - 4x + 8 \\ & \underline{-9x^3 \mp 4x} \\ & -18x^2 + 8 \\ & \underline{\mp 18x^2 \pm 8} \\ & 0 \end{array}$$

Thus, $9x^3 - 18x^2 - 4x + 8 = (9x^2 - 4)(x - 2)$, *i.e.*, $(3x - 2)(3x + 2)(x - 2)$.

37. (*a*) Here, $A = \begin{bmatrix} x & 1 \\ y & 2 \end{bmatrix}$ and $B = \begin{bmatrix} x \\ x-2 \end{bmatrix}$

We know that if A is a $m \times n$ matrix and B is $n \times p$ matrix, then the product matrix AB will be $m \times p$ matrix.

Since A is a 2×2 matrix and B is a 2×1 matrix, the product matrix AB will be 2×1 matrix.

(*b*) Given that AB is a null matrix, therefore

$$\begin{bmatrix} x & 1 \\ y & 2 \end{bmatrix}\begin{bmatrix} x \\ x-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x^2 + 1(x-2) \\ xy + 2(x-2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x^2 + x - 2 \\ xy + 2x - 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad x^2 + x - 2 = 0 \quad \ldots(1)$$

and $\quad xy + 2x - 4 = 0 \quad \ldots(2)$

From (1), we have $\quad x^2 + x - 2 = 0$

$$\Rightarrow \quad x^2 + 2x - x - 2 = 0$$

$$\Rightarrow \quad x(x + 2) - 1(x + 2) = 0$$

$$\Rightarrow \quad (x + 2)(x - 1) = 0$$

$$\Rightarrow \quad x = 1 \quad \text{or} \quad x = -2$$

Putting $x = 1$ in (2), we have

$$y + 2 - 4 = 0 \quad \Rightarrow \quad y - 2 = 0 \quad \text{or} \quad y = 2$$

Putting $x = -2$ in (2), we have

$$-2y - 4 - 4 = 0 \quad \Rightarrow \quad -2y = 8 \quad \text{or} \quad y = -4$$

Thus, $x = 1$ and $y = 2$ or $x = -2$ and $y = -4$ are the possible values of x and y.

38. (*a*) The given A.P. is: 20, 17, 14, …

For this A.P., first term, $a = 20$ and common difference, $d = -3$

Since sum of first n terms of an A.P. is $S_n = \frac{n}{2}[2a + (n - 1)d]$

Therefore, $\quad 65 = \frac{n}{2}[2 \times 20 + (n - 1)(-3)] \quad [\because S_n = 65]$

$$\Rightarrow \quad 65 = \frac{n}{2}[40 + (-3n + 3)]$$

$$\Rightarrow \quad 65 = \frac{n}{2}[43 - 3n]$$
$$\Rightarrow \quad 130 = 43n - 3n^2$$
$$\Rightarrow \quad 3n^2 - 43n + 130 = 0$$
$$\Rightarrow \quad 3n^2 - 30n - 13n + 130 = 0$$
$$\Rightarrow \quad 3n(n - 10) - 13(n - 10) = 0$$
$$\Rightarrow \quad (n - 10)(3n - 13) = 0$$
$$\Rightarrow \quad n = 10 \quad \text{or} \quad n = \frac{13}{3}$$
$$\Rightarrow \quad n = 10 \quad \left[\because \text{Number of terms cannot be in fractions, } n \neq \frac{13}{3}\right]$$

Thus, there are 10 terms in the given A.P.

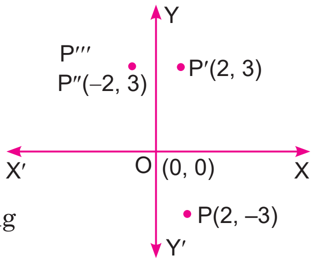
(*b*) Let l be the last term of the A.P. Then,

Using $l = a + (n - 1)d$, we have

$$l = 20 + (10 - 1)(-3)$$
$$= 20 + 9 \times (-3) = 20 - 27 = -7$$

Thus, last term of the given A.P. is –7.

39. (*a*) Given, point P(2, –3) on reflection becomes P′(2, 3)



Symbolically, $P(2, -3) \xrightarrow{\text{On reflection}} P'(2, 3)$.

Here, after reflection only the y-coordinate of the point changes in sign. Therefore, line of reflection, *i.e.*, line L_1 is x-axis (See figure).

(*b*) Point P′ is reflected to P″ along the line L_2 which is perpendicular to the line L_1 and passes through the point, which is invariant along both the axes.

Since origin remains invariant along both the axes, line L_2 passes through origin. So line L_2 passes through origin and perpendicular to line L_1, *i.e.*, x-axis. It means line L_2 is y-axis (See figure).

$$P'(2, 3) \xrightarrow{y\text{-axis}} P''(-2, 3)$$

Thus, coordinates of P″ are (–2. 3).

(*c*) Point of intersection of line L_1 (x-axis) and line L_2 (y-axis) is origin O(0, 0).

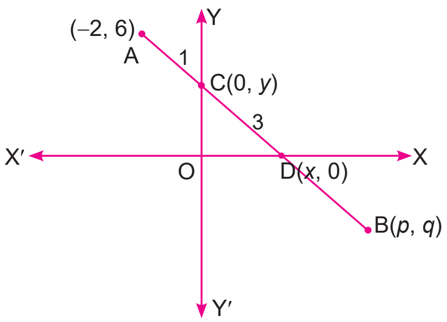
(*d*) Point P is reflected to P‴ on reflection through the point named in the answer of part (*c*), *i.e.*, origin. Therefore,

$$P(2, -3) \xrightarrow{\text{origin}} P'''(-2, 3)$$

Thus, coordinates of P‴ are (–2, 3)

Since P″ and P‴ have same coordinates, P″ and P‴ are coincident points (See figure).

40. Given, line segment AB intersects y-axis at C and D respectively such that AC : AD = 1 : 4 and D is the mid-point of CB.



Since AC : AD = 1 : 4 $\Rightarrow$ AC : CD = 1 : 3.

Let coordinates of C and D be $(0, y)$ and $(x, 0)$ respectively.

Then, by section formula

$$(0, y) = \left(\frac{1 \times x + 3 \times (-2)}{1 + 3}, \frac{1 \times 0 + 3 \times 6}{1 + 3}\right) \quad [\because \text{ C divides AD in the ratio } 1 : 3.]$$

$$(0, y) = \left(\frac{x-6}{4}, \frac{18}{4}\right)$$

$\Rightarrow$ $$(0, y) = \left(\frac{x-6}{4}, \frac{9}{2}\right)$$

$\Rightarrow$ $$\frac{x-6}{4} = 0 \text{ and } y = \frac{9}{2}$$

$\Rightarrow$ $$x = 6 \text{ and } y = \frac{9}{2}$$

Thus, coordinates of C are $\left(0, \frac{9}{2}\right)$ and that of D are (6, 0).

Given that D is the mid-point of CB, therefore

By mid-point formula, $(6, 0) = \left(\frac{p+0}{2}, \frac{q+\frac{9}{2}}{2}\right)$

$\Rightarrow$ $$(6, 0) = \left(\frac{p}{2}, \frac{2q+9}{4}\right)$$

$\Rightarrow$ $$6 = \frac{p}{2} \quad \text{and} \quad 0 = \frac{2q+9}{4}$$

$\Rightarrow$ $$p = 12 \quad \text{and} \quad q = \frac{-9}{2}$$

Hence, coordinates of B are $\left(12, \frac{-9}{2}\right)$.

41. Equation of given line is $x + 2y = 4$ ...(1)

$\therefore$ In slope-intercept form, eq (1) can be written as

$$y = -\frac{x}{2} + 2 \quad \text{...(2)}$$

Comparing eq (2) with $y = mx + c$, we have

$$m = \frac{-1}{2} \text{ and } c = 2$$

Since required line is perpendicular to the given line, therefore

Slope of required line $= \frac{-1}{m} = 2$

Given that the required line cuts an intercept of 2 units from the positive y-axis, therefore $c = 2$

Thus, equation of the required line is

$$y = 2x + 2$$

or $$2x - y = -2 \quad \text{...(3)}$$

Solving (1) and (3) simultaneously, we get the intersection point of the two lines.

From eq (3), putting $y = 2x + 2$ in eq (1), we have

$$x + 2(2x + 2) = 4$$

$\Rightarrow$ $$x + 4x + 4 = 4$$

$\Rightarrow$ $$5x + 4 = 4 \quad \Rightarrow \quad 5x = 0 \quad \text{or} \quad x = 0$$

Substituting the value of x in (1), we have

$$0 + 2y = 4 \quad \Rightarrow \quad 2y = 4 \quad \text{or} \quad y = 2$$

Hence, (0, 2) is the point of intersection of the two lines.

42. The given linear inequation is:

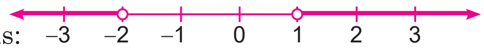
$$5(2 - 4x) > 18 - 16x > 22 - 20x, \ x \in R$$

The above inequation can be rewritten as:

$$\begin{aligned} & 5(2-4x) > 18-16x \quad \text{and} \quad 18-16x > 22-20x \\ \Rightarrow\ & 10-20x > 18-16x \quad \text{and} \quad 20x-16x > 22-18 \\ \Rightarrow\ & -20x+16x > 18-10 \quad \text{and} \quad 4x > 4 \\ \Rightarrow\ & -4x > 8 \quad \text{and} \quad x > 1 \\ \Rightarrow\ & x < -2 \quad \text{and} \quad x > 1 \end{aligned}$$

Thus, the solution set is $\{x : x < -2 \text{ or } x > 1, x \in R\}$

On the number line the solution set can be represented as:



43. Let the given polynomial be $p(x) = x^3 + 2x^2 - ax + b$. ...(1)

Given that $p(x)$ leaves a remainder –6 when divided by $x + 1$.

Therefore, by Remainder Theorem,

$$\begin{aligned} p(-1) &= -6 \\ \Rightarrow\ (-1)^3 + 2(-1)^2 - a(-1) + b &= -6 \\ \Rightarrow\ -1 + 2 + a + b &= -6 \\ \Rightarrow\ 1 + a + b &= -6 \\ \Rightarrow\ a + b &= -7 \qquad ...(1) \end{aligned}$$

Also, $(x - 2)$ is a factor of $p(x)$. Therefore, by Factor Theorem,

$$\begin{aligned} p(2) &= 0 \\ \Rightarrow\ (2)^3 + 2(2)^2 - a(2) + b &= 0 \\ \Rightarrow\ 8 + 8 - 2a + b &= 0 \\ \Rightarrow\ 16 - 2a + b &= 0 \\ \text{or}\ 2a - b &= 16 \qquad ...(2) \end{aligned}$$

On adding (1) and (2), we get

$$3a = 9 \quad \text{or} \quad a = 3$$

Putting $a = 3$ in (1), we get

$$3 + b = -7 \quad \text{or} \quad b = -10$$

Thus, $a = 3$ and $b = -10$ are the required values.

44. We have, $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$, $C = [1 \quad -4]$ and $D = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

(*a*) Here, matrix A is 2 × 2 matrix and C is 1 × 2 matrix. Since number of columns in matrix A is not same as the number of rows in matrix C, the product matrix AC is not possible.

(*b*) We have,

$$X = AB + B^2 - DC$$

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 1 \end{bmatrix}[1 \quad -4]$$

$$= \begin{bmatrix} -1\times1+3\times0 & -1\times(-2)+3\times3 \\ 2\times1+0\times0 & 2\times(-2)+0\times3 \end{bmatrix} + \begin{bmatrix} 1\times1+(-2)\times0 & 1\times(-2)+(-2)\times3 \\ 0\times1+3\times0 & 0\times(-2)+3\times3 \end{bmatrix} - \begin{bmatrix} 4\times1 & 4\times(-4) \\ 1\times1 & 1\times(-4) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 11 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 4 & -16 \\ 1 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1-4 & 11-8+16 \\ 2+0-1 & -4+9+4 \end{bmatrix} = \begin{bmatrix} -4 & 19 \\ 1 & 9 \end{bmatrix}$$

Hence, matrix X is $\begin{bmatrix} -4 & 19 \\ 1 & 9 \end{bmatrix}$.

IV: Long Answer Questions (5 marks each)

45. The points A(2, 2) and B(6, –2) are plotted in the graph given below.

(*a*) A point P(x, y) when reflected in the origin changes to P′($-x$, $-y$)

i.e., $P(x, y) \xrightarrow{\text{Origin}} P'(-x, -y)$

$\therefore$ $A(2, 2) \xrightarrow{\text{Origin}} D(-2, -2)$

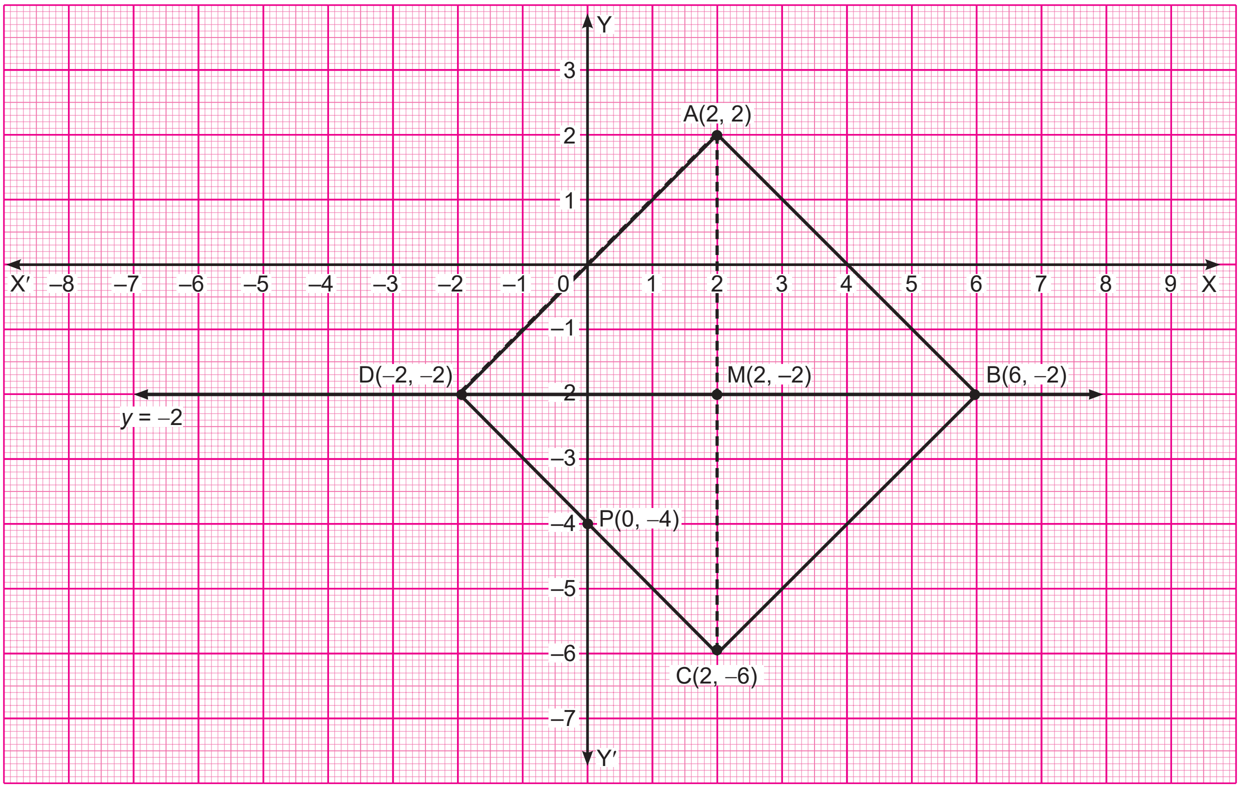
Thus, coordinates of point D are (–2, –2).

(*b*) Reflection of a point P(x, y) in the line $y = b$ is P′(x, $2b - y$)

i.e., $P(x, y) \xrightarrow{y=b} P'(x, 2b - y)$

$\therefore$ $A(2, 2) \xrightarrow{y=-2} C(2, -6)$

Thus, coordinates of point C are (2, –6).



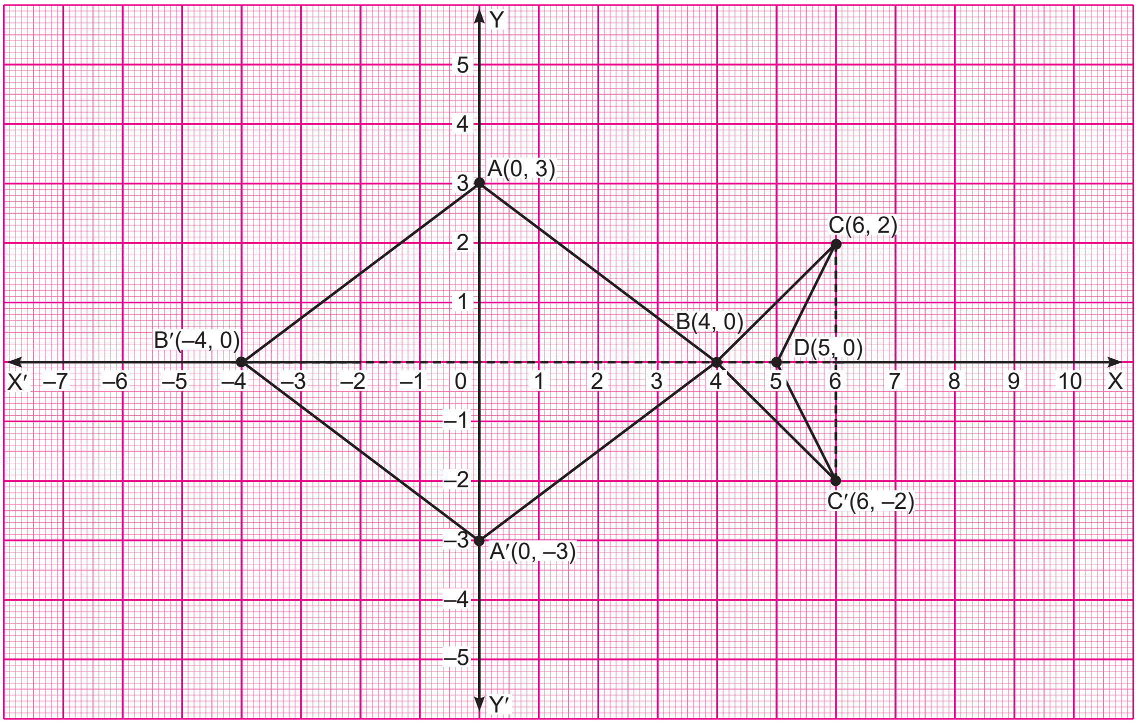
(*c*) A point which is invariant under reflection in $x = 0$, *i.e.*, y-axis lies itself on y-axis. Given, P lies on the line CD, therefore P must be the point of intersection of line CD and y-axis.

Since CD cuts y-axis at (0, –4), the coordinates of point P are (0, –4).

(*d*) By joining the four points A, B, C and D, we get a quadrilateral whose diagonals AC and BD are equal (each 8 units) and perpendicular to each other. Therefore, closed figure ABCD is a square.

(*e*) From the graph, point M is the point of intersection of the diagonals of ABCD and coordinates of point M are (2, –2).

46. The points A(0, 3), B(4, 0), C(6, 2) and D(5, 0) are plotted in the following graph.



(*a*) When a point is reflected on *x*-axis its abscissa remains the same but the sign of its ordinate changes.

$\therefore \quad A(0, 3) \xrightarrow{x\text{-axis}} A'(0, -3)$

So, the coordinates of A are (0, –3).

(*b*) When a point is reflected on *y*-axis, its ordinate remains the same but the sign of its abscissa changes.

$\therefore \quad B(4, 0) \xrightarrow{y\text{-axis}} B'(-4, 0).$

So, the coordinates of B′ are (–4, 0).

(*c*) As explained in part (*a*), we have

$$C(6, 2) \xrightarrow{x\text{-axis}} C'(6, -2).$$

(*d*) In the graph, point D is lying on *x*-axis. It will remain invariant when reflected in the *x*-axis. Since equation of *x*-axis is $y = 0$, the point D will remain invariant on the line whose equation is $y = 0$.

(*e*) On joining the points A, B, C, D, C′, A′, B′ and A, we get a closed figure as shown in the graph. The close figure BCDC′ is a concave quadrilateral.

47. The given polynomial is $p(x) = 2x^2 + 11x + 12$.

By Factor theorem, $(2x + 3)$ will a factor of $p(x)$, if $p\left(\frac{-3}{2}\right) = 0$.

$$\therefore \quad p\left(\frac{-3}{2}\right) = 2\left(\frac{-3}{2}\right)^2 + 11\left(\frac{-3}{2}\right) + 12$$

$$= 2\left(\frac{9}{4}\right) - \frac{33}{2} + 12$$

$$= \frac{9}{4} - \frac{33}{2} + 12$$

$$= \frac{9 - 33 + 24}{2} = \frac{33 - 33}{2} = \frac{0}{2} = 0$$

Thus, $(2x + 3)$ is a factor of the given polynomial.

By division algorithm,

$$\begin{array}{r|l} & x + 4 \\ 2x+3 & 2x^2 + 11x + 12 \\ & \underline{-2x^2 \pm 3x} \\ & \quad 8x + 12 \\ & \quad \underline{-8x \pm 12} \\ & \qquad 0 \end{array}$$

Thus, $2x^2 + 11x + 12 = (2x + 3)(x + 4)$.

Alternatively:

The given polynomial can also be factorised by splitting the middle term.

$$\begin{aligned} 2x^2 + 11x + 12 &= 2x^2 + 8x + 3x + 12 \\ &= 2x(x + 4) + 3(x + 4) \\ &= (2x + 3)(x + 4) \end{aligned}$$

The other polynomial is $q(x) = x^2 + 3x - 4$

On factorising, $x^2 + 3x - 4 = x^2 + 4x - x - 4$

$$\begin{aligned} &= x(x + 4) - 1(x + 4) \\ &= (x - 1)(x + 4) \end{aligned}$$

Now, the polynomial $q(x)$ will be a factor of the polynomial $p(x)$, if $p(x)$ is divisible by both the factors of $q(x)$, *i.e.*, $(x - 1)$ and $(x + 4)$.

Since $(x + 4)$ is already a factor of $p(x)$, the polynomial $q(x)$ will be a factor of $p(x)$, if $p(x)$ is multiplied by $(x - 1)$, *i.e.*, the other factor of $q(x)$.

So, $p(x)$ must be multiplied by $(x - 1)$.

Let $r(x)$ be the resulting polynomial. Then,

$$\begin{aligned} r(x) &= (2x^2 + 11x + 12)(x - 1) \\ &= 2x^3 + 11x^2 + 12x - 2x^2 - 11x - 12 \\ &= 2x^3 + 9x^2 + x - 12 \end{aligned}$$

Thus, $2x^3 + 9x^2 + x - 12$ is the resulting polynomial.

48. The given sequence is 2, 9, 16, …

(*a*) We have, $9 - 2 = 7$ and $16 - 9 = 7$

Since the difference between two successive terms is same, *i.e.*, 7, the given sequence is an A.P.

(*b*) For the given A.P., first term, $a = 2$, common difference, $d = 7$

$\therefore$ 20th term, $a_{20} = 2 + (20 - 1)7$ $\quad [\because a_n = a + (n - 1)d]$

$$\begin{aligned} &= 2 + 19 \times 7 \\ &= 2 + 133 = 135 \end{aligned}$$

Thus, 20th term of the given A.P. is 135.

(*c*) Sum of first 22 terms, $S_{22} = \frac{22}{2}[2 \times 2 + (22 - 1)7]$ $\quad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$$\begin{aligned} &= 11[4 + 21 \times 7] \\ &= 11[4 + 147] = 11 \times 151 = 1661 \end{aligned}$$

Sum of first 25 terms, $S_{25} = \frac{25}{2}[2 \times 2 + (25 - 1)7]$

$$\begin{aligned} &= \frac{25}{2}[4 + 24 \times 7] \\ &= \frac{25}{2}[4 + 168] = \frac{25}{2} \times 172 = 2150 \end{aligned}$$

∴ Difference between the sum of first 22 and 25 terms

$= S_{25} - S_{22}$

$= 2150 - 1661 = 489.$

Thus, the difference between the sum of first 22 and 25 terms is 489.

(*d*) Let 102 be the nth term of the A.P. Then,

$102 = 2 + (n - 1)7$ [Using $a_n = a + (n - 1)d$]

$\Rightarrow \quad 102 = 2 + 7n - 7$

$\Rightarrow \quad 102 = 7n - 5$

$\Rightarrow \quad 7n = 102 + 5 = 107$

or $\quad n = \frac{107}{7}$, which is not a whole number

Since a term of a sequence cannot be in fraction, 102 is not a term of the A.P.

(*e*) When k is added to each term, the sequence is

$2 + k, 9 + k, 16 + k, \ldots$

In this case, $(9 + k) - (2 + k) = 7$ and $(16 + k) - (9 + k) = 7$

Since the difference between the two successive terms of the new sequence remains the same *i.e.*, 7, the new sequence is still an A.P.

49. We have,

Equation of line L_1 is: $x - y = 1$ ...(1)

Equation of line L_2 is: $x + y = 5$ ...(2)

Given, the lines L_1 and L_2 intersect at Q(3, 2).

(*a*) Since line L_3 is parallel to line L_1, the slopes of the two lines must be equal.

From eq (1), we have $y = x - 1$

Comparing it with $y = mx + c$, we have $m = 1$

So, the slope of line L_1 is 1.

∴ Slope of line $L_3 = 1$ [∵ Slopes of L_1 and L_3 are same.]

Given, the line L_3 has y-intercept 3, *i.e.*, $c = 3$

Putting the values of m and c in $y = mx + c$, we get

$y = x + 3$, which is the equation of line L_3

(*b*) Given that the line L_3 meets the line L_2 at a point P(k, 4).

Therefore, P(k, 4) is also a point on the line L_3.

∴ Putting $x = k$ and $y = 4$ in the equation of line L_3, we get

$4 = k + 3 \qquad k = 4 - 3 = 1$

Thus, the value of k is 1.

(*c*) We know that ordinate of any point lying on x-axis is 0.

Let coordinates of point R be (x, 0).

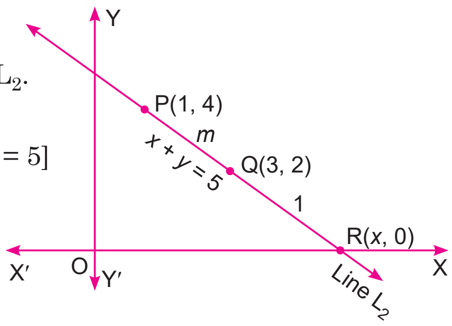
Since point R also lies on line L_2, it will satisfy the line L_2.

∴ Putting $y = 0$ in the equation of line L_2, we have

$x + 0 = 5$ [∵ Equation of line L_2 is $x + y = 5$]

$\Rightarrow \quad x = 5$

Thus, coordinates of point R are (5, 0).



Now, let the line L_2 divide the points P, Q and R in the ratio $m : 1$, *i.e.*, PQ : QR = $m : 1$. Therefore, By section formula, coordinates of Q are

$$\left(\frac{m\times x+1\times 1}{m+1}, \frac{m\times 0+1\times 4}{m+1}\right)$$

$$=\left(\frac{mx+1}{m+1}, \frac{4}{m+1}\right)$$

Since coordinates of Q are given as (3, 2), therefore

$$\left(\frac{mx+1}{m+1}, \frac{4}{m+1}\right) = (3, 2)$$

Equating ordinates on both sides, we have

$$\frac{4}{m+1} = 2 \quad \Rightarrow \quad 4 = 2m + 2$$

$$\Rightarrow \quad 2m = 4 - 2 = 2$$

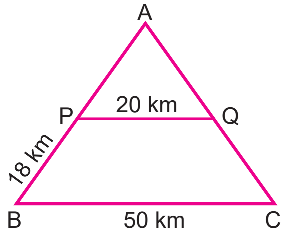
$$\text{or} \quad m = 1$$

Thus, the required ratio is 1 : 1, *i.e.*, Q is the mid-point of PR.

Unit III: Geometry (Chapters 11–15)

I: Multiple Choice Questions (1 mark each)

1. (*d*) In ΔABC, given that PQ || BC.



Therefore, by AA Similarity Criterion, ΔAPQ ~ ΔABC

$$\Rightarrow \quad \frac{AP}{AB} = \frac{PQ}{BC}$$

$$\Rightarrow \quad \frac{AP}{18+AP} = \frac{20}{50} \quad \Rightarrow \quad \frac{AP}{18+AP} = \frac{2}{5}$$

$$\Rightarrow \quad 5AP = 36 + 2AP$$

$$\Rightarrow \quad 5AP - 2AP = 36 \quad \Rightarrow \quad 3AP = 36 \text{ or } AP = 12 \text{ km}$$

Thus, AB = AP + PB = 12 km + 18 km = 30 km

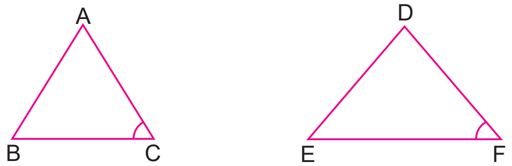
Obviously, AB = AP + PB is the shortest route.

$\therefore$ Time taken by the train to reach B from A $= \frac{30}{90}$ hour $\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$

$= \frac{1}{3}$ hour or 20 minutes

Thus, option (*d*) is the correct answer.

2. (*d*) In the given diagram, we have



∠C = ∠F; and

$$\frac{AB}{DE} = \frac{BC}{EF} \quad \text{or} \quad \frac{AB}{BC} = \frac{DE}{EF}$$

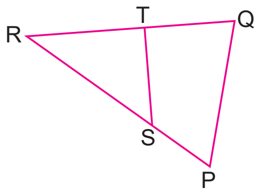
Clearly, ∠C is not the included angle between the sides AB and BC of ΔABC.

Similarly, ∠F is not the included angle between the sides DE and EF of ΔDEF.

Therefore, SAS is not a Similarly Criterion in Δs ABC and DEF.

Hence, the similarity of given triangles cannot be determined.

3. (*c*) In the given diagram, ST is not parallel to PQ.



Here, ∠R is common to both the triangles, therefore

∠PRQ = ∠SRT

By AA Similarity Criterion, triangles PQR and TSR will be similar if ∠PQR = ∠TSR or ∠QPR = ∠STR.

Thus, the necessary and sufficient condition for ΔPQR ~ ΔTSR is either ∠PQR = ∠TSR or ∠QPR = ∠STR.

Clearly, option (*c*) is the correct answer.

4. (*d*) Given, $\frac{\text{Height of Soina in picture}}{\text{Actual height of Sonia}} = \frac{20 \text{ cm}}{1.6 \text{ m}} = \frac{20 \text{ cm}}{160 \text{ cm}} = \frac{1}{8}$

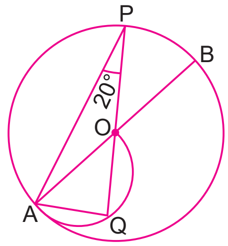
$\therefore$ Scale factor, $k = \frac{1}{8}$

Now,

Height of Sonia in picture = k. Actual height of Sonia

$$\Rightarrow \quad 18 \text{ cm} = \frac{1}{8} \times \text{Actual height of Sonia}$$

$\Rightarrow$ Actual height of Sonia = 8 × 18 cm = 1.44 cm or 1.44 m.



5. (*c*) In given figure, O is the centre of bigger circle, therefore

OP = OA [Radii of the circle]

⇒ ∠OAP = ∠OPA

⇒ ∠OAP = 20° ...(1)

Also, ∠OQA is the angle drawn in semicircle taken OA as diameter, therefore

∠OQA = 90° ⇒ ∠PQA = 90° [∵ ∠OQA = ∠PQA]

Now, in ΔPQA, we have

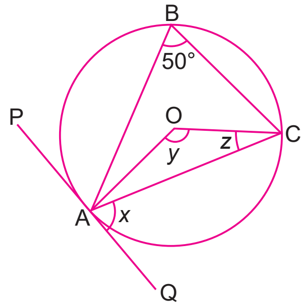
∠PAQ = 180° – (∠QPA + ∠PQA)

= 180° – (20° + 90°) [∵ ∠QPA = ∠OPA = 20°]

= 180° – 110° = 70°

∴ ∠OAQ = ∠PAQ – ∠OAP

= 70° – 20° = 50°.



6. (*a*) In the given diagram, chord AC makes ∠CAQ with the tangent PQ at A and ∠ABC in the alternate segment of the circle.

∴ ∠CAQ = ∠ABC [∵ Angles in alternate segments are equal.]

⇒ $x = 50°$ [∠ABC = 50°]

Now, ∠AOC is subtended at the centre and ∠ABC at the circumference of the circle on the same arc. Therefore,

∠AOC = 2 ∠ABC [∵ Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.]

⇒ $y = 2 \times 50°$

⇒ $y = 100°$

In, ΔAOC, OC = OA [Radii of the same circle]

⇒ ∠OAC = ∠OCA [Angles opposite to equal sides are also equal.]

By angle sum property of a triangle, we have

∴ ∠OAC + ∠AOC + ∠OCA = 180°

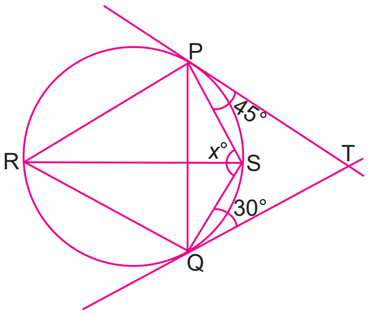
⇒ ∠OCA + y + ∠OCA = 180° [∵ ∠OAC = ∠OCA]

⇒ 2∠OCA + 100° = 180° [∵ y = 100°]

⇒ $2z = 180° – 100°$ [∵ ∠OCA = z]

⇒ $2z = 80°$ or $z = 40°$

Thus, $x = 50°$, $y = 100°$ and $z = 40°$.



7. (*d*) In the figure, chord PS makes ∠SPT with the tangent PT and chord QS makes ∠SQT with the tangent QT.

Since ∠SPT and ∠PRS are in alternate segments,

⇒ ∠PRS = ∠SPT [∵ Angles in the alternate segments are equal.]

⇒ ∠PRS = 45° ...(1)

Similarly, ∠SQT and ∠QRS are in the alternate segments,

∠QRS = ∠SQT

⇒ ∠QRS = 30° ...(2)

On adding (1) and (2), we get

∠PRS + ∠QRS = 45° + 30°

⇒ ∠PRQ = 75°

Now, PRQS is a cyclic quadrilateral, therefore

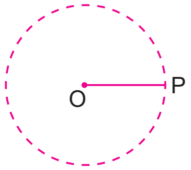
$$\angle PRQ + \angle PSQ = 180°$$ [∵ Opposite angles of a cyclic quadrilateral are supplementary.]

$\Rightarrow \quad 75° + x = 180°$

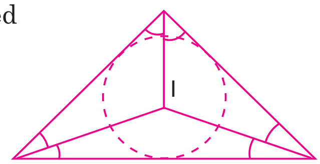
$\Rightarrow \quad x = 180° - 75° = 105°$

Thus, the value of x is 105°.

8. (*a*) Locus of the moving point is a circle if it moves such that it keeps a fixed distance from a fixed point. Here, fixed point is the centre and fixed distance is the radius of the circle.

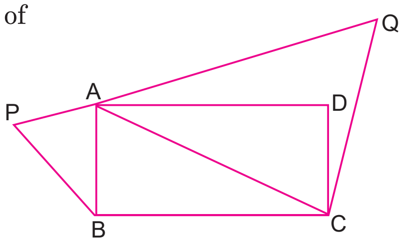


9. (*b*) The point of concurrence of the angle bisectors of a triangle is called the incentre of the triangle.



II: Short Answer Questions-1 (3 marks each)

10. Given a rectangle ABCD in which BC = 2AB and ΔACQ ~ ΔBAP.



Since ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides, we have

$$\frac{ar(\Delta BAP)}{ar(\Delta ACQ)} = \frac{AB^2}{AC^2} \quad \ldots(1)$$

In rectangle ABCD, triangle ABC is right triangle. Therefore,

By Pythagora's Theorem, $AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC^2 = AB^2 + (2AB)^2 \quad$ [∵ BC = 2AB]

$\Rightarrow \quad AC^2 = AB^2 + 4AB^2$

$\Rightarrow \quad AC^2 = 5AB^2$

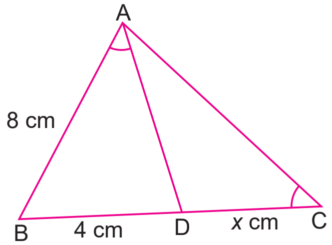
or $\quad \dfrac{AB^2}{AC^2} = \dfrac{1}{5}$

Using this result in eq (1), we have

$$\frac{ar(\Delta BAP)}{ar(\Delta ACQ)} = \frac{1}{5}$$

Thus, $ar(\Delta BAP) : ar(\Delta ACQ) = 1 : 5$.

11. Given a ΔABC in which D is a point on BC such that BD = 4 cm and DC = x cm.



(*a*) In Δs ABD and CBA, we have

$\angle BAD = \angle ACB$ [Given]

and $\angle ABC = \angle ABC$ [Common]

∴ By AA Similarity Criterion, ΔABD ~ ΔCBA.

(*b*) Since Δs ABD and CBA are similar triangles, we have

$$\frac{AB}{BC} = \frac{BD}{AB} = \frac{AD}{AC}$$

$\Rightarrow \quad \dfrac{8}{4+x} = \dfrac{4}{8}$ [Using ratios of first two corresponding sides]

$\Rightarrow \quad 4(4 + x) = 64$

$\Rightarrow \quad 16 + 4x = 64$

$\Rightarrow \quad 4x = 64 - 16 \quad \Rightarrow \quad 4x = 48 \quad$ or $\quad x = 12$

Thus, the value of x is 12 cm.

12. Here, scale factor (k) $= \dfrac{2 \text{ cm}}{1 \text{ km}}$

$$= \frac{2}{100000} \qquad [\because 1 \text{ km} = 100000 \text{ cm}]$$

$$= \frac{1}{50000}$$

(a) The length of the cart track between two settlements is 7.6 cm.

$\therefore$ Length of cart track on the map = k. Actual length of the cart track on the ground

$$\Rightarrow \quad 7.6 \text{ cm} = \frac{1}{50000} \text{ (Actual length of the cart track on the ground)}$$

$\Rightarrow$ Actual length of the cart track on the ground

$$= 7.6 \text{ cm} \times 50000$$

$$= 380000 \text{ cm} = 3.8 \text{ km}$$

(b) Area of the square grid on the map = k^2. Actual area of the square grid

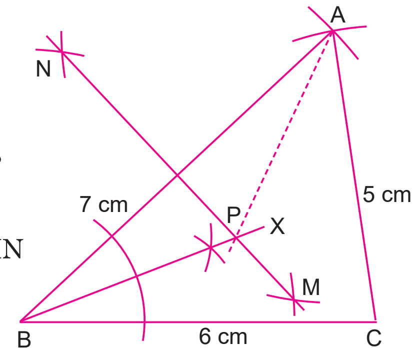
$$\Rightarrow \quad 4 \text{ cm}^2 = \left(\frac{1}{50000}\right)^2 \text{. Actual area of the square grid}$$

$\Rightarrow$ Actual area of the square grid = $4 \text{ cm}^2 \times (50000)^2$

$$= \frac{4 \times 50000 \times 50000}{100000 \times 100000} \text{ km}^2$$

$$= 1 \text{ km}^2$$

13. With the given measurements, ΔABC is constructed alongside.



(a) (i) The locus of the point equidistant from BC and BA is the bisector BX of the angle ABC.

(ii) The locus of the point equidistant from points A and B is the perpendicular bisector MN of sides AB

(b) The two loci, *i.e.*, bisector BX and perpendicular bisector MN meet at P as marked in the figure.

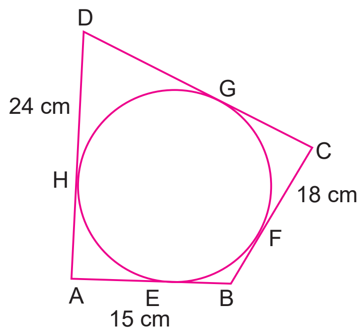
On measuring, length PA = 3.8 cm.

14. Given a quadrilateral ABCD where sides AB, BC, CD and DA touch the circle at E, F, G and H respectively.

In the figure, we have

AE = AH ...(1) [Length of tangents drawn from an external point to a circle are equal.]

and EB = BF ...(2)



On adding (1) and (2), we get

$$AE + EB = AH + BF$$

$$\Rightarrow \quad AB = AH + BF \qquad ...(3)$$

Similarly, we have

$$CD = CF + DH \qquad ...(4)$$

On adding (3) and (4), we get

$$AB + CD = AH + BF + CF + DH$$

$$= (AH + HD) + (BF + FC)$$

$$= AD + BC$$

∴ AB + CD = AD + BC

Putting the values of AB, BC and AD, we have

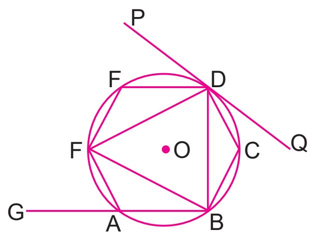
15 cm + CD = 24 cm + 18 cm [∵ AB = 15 cm, BC = 18 cm and AD = 24 cm]

⇒ 15 cm + CD = 42 cm

⇒ CD = 42 cm – 15 cm = 27 cm

Thus, the length of CD is 27 cm.

15. In the figure, a regular hexagon ABCDEF is inscribed in a circle with centre O. PQ is a tangent to the circle at D.



(*a*) We know that each interior angle of a regular hexagon is 120°.

Therefore, ∠FAB = 120°.

Since GAB is a line,

∠FAG = ∠GAB – ∠FAB

= 180° – 120° = 60°.

(*b*) ∠BCD is an interior angle of regular hexagon ABCDEF.

Therefore, ∠BCD = 120°.

(*c*) In ΔDEF, we have

DE = EF [Sides of a regular hexagon]

⇒ ∠DFE = ∠EDF [∵ Angles opposite to equal sides are also equal.]

Also, ∠FED = 120°.

∴ ∠DFE = ∠EDF = 30°. [By angle sum property of a triangle]

Now, ∠PDE and ∠DFE are the angles in alternate segments

Therefore, ∠PDE = ∠DFE [∵ Angles in alternate segments are equal.]

⇒ ∠PDE = 30°. [∵ ∠DFE = 30°]

16. Given, AB and CD intersect at the centre O of the circle and ∠EBA = 33° and ∠EAC = 82°.

(*a*) Clearly, AB is the diameter of the circle and ∠AEB is an angle in semicircle. Therefore,

∠AEB = 90° [∵ Angle in a semicircle is a right angle.]

Now, in ΔAEB

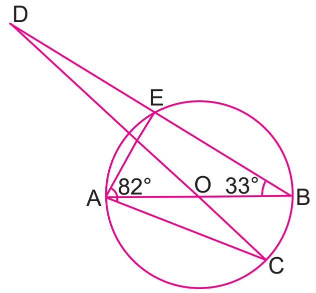
∠BAE + ∠ABE + ∠AEB = 180°

⇒ ∠BAE + 33° + 90° = 180°

⇒ ∠BAE + 123° = 180°

⇒ ∠BAE = 180° – 123° = 57°.

Thus, measure of ∠BAE is 57°.



(*b*) We have, ∠EAC = 82° [Given]

⇒ ∠BAE + ∠BAC = 82°

⇒ 57° + ∠BAC = 82°

⇒ ∠BAC = 82° – 57° = 25°

∴ ∠BOC = 2 ∠BAC [∵ Angle subtended by an arc at the centre is double the angle subtended by it in the remaining part of the circle.]

= 2 × 25° = 50°

Thus, measure of ∠BOC is 50°.

(*c*) In ΔODB, $\angle BOC$ is an exterior angle.

Therefore, $\angle BOC = \angle OBD + \angle ODB$ [$\because$ Exterior angle of a triangle is equal to sum of its two interior opposite angles.]

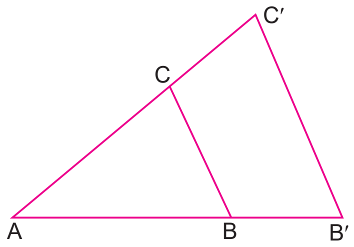
$\Rightarrow \quad 50° = 33° + \angle OBD$

$\Rightarrow \quad \angle OBD = 50° - 33° = 17°$

Thus, measure of $\angle ODB$ is 17°.

III: Short Answer Questions-2 (4 marks each)

17. Given, ΔABC is enlarged along the side BC to $\Delta AB'C'$ such that BC : B′C′ is 3 : 5.



(*a*) We know that on enlargement the figure obtained is similar to its original figure. Therefore,

$$\Delta ABC \sim \Delta AB'C',$$

Since ratios of the corresponding sides of two similar triangles are proportional, we have

$$\frac{AB}{AB'} = \frac{BC}{B'C'}$$

$$\Rightarrow \quad \frac{AB}{AB'} = \frac{3}{5} \qquad \left[\because \frac{BC}{B'C'} = \frac{3}{5}\right]$$

Let $AB = 3x$, and $AB' = 5x$. Then,

$$AB' = AB + BB'$$

$$5x = 3x + BB' \quad \Rightarrow \quad BB' = 5x - 3x = 2x$$

$$\therefore \quad \frac{AB}{BB'} = \frac{3x}{2x} \quad \text{or} \quad AB : BB' = 3 : 2.$$

(*b*) From part (*a*), we have

$$\frac{AB}{BB'} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{AB}{4} = \frac{3}{2} \quad \Rightarrow \quad AB = \frac{3 \times 4}{2} = 6$$

Therefore, length of AB is 6 cm.

(*c*) Yes, $\Delta ABC \sim \Delta AB'C'$.

Justification:

In Δs ABC and AB′C′, we have

$$\frac{AB}{AB'} = \frac{AC}{AC'} \quad \text{or} \quad \frac{AB}{AC} = \frac{AB'}{AC'}$$

and $\angle BAC = \angle B'AC'$ [Common]

$\therefore$ By SAS Similarity Criterion, $\Delta ABC \sim \Delta AB'C'$.

(*d*) Since the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides, we have

$$\frac{ar(\Delta ABC)}{ar(\Delta AB'C')} = \left(\frac{AB}{AB'}\right)^2$$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta AB'C')} = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

Let $ar(\Delta ABC) = 9a$ and $ar(\Delta AB'C') = 25a$. Then,

$ar(\text{quad. } BB'C'C) = ar(\Delta AB'C') - ar(\Delta ABC)$

$= 25a - 9a = 16a$

Thus, $\dfrac{ar(\Delta ABC)}{ar(\text{quad. } BB'C'C)} = \dfrac{9a}{16a}$

or $ar(\Delta ABC) : ar(\text{quad. } BB'C'C) = 9 : 16$.

18. Given, volume of human eye = 6.5 cm^3 (approx)

and volume of model eye = 1403 cm^3

(*a*) As the volume of model eye is greater than the volume of human eye, scale factor (k) is greater than 1 ($k > 1$).

(*b*) (*i*) By volume of scale factor (k^3), we have

$$k^3 = \frac{\text{Volume of model eye}}{\text{Volume of human eye}}$$

$$= \frac{1403}{6.5} = 216 \text{ (approx)}$$

$\therefore$ $k^3 = 6^3 \Rightarrow k = 6$

Thus, the value of k is 6.

(*ii*) Given that radius of model eye is 7.2 cm. Then,

By scale factor (k), we have

$$k = \frac{\text{Radius of model eye}}{\text{Radius of human eye}}$$

$\Rightarrow$ $6 = \dfrac{7.2}{\text{Radius of human eye}}$

$\Rightarrow$ Radius of human eye $= \dfrac{7.2}{6} = 1.2$

$\therefore$ Diameter of human eye = 2 × 1.2 cm = 2.4 cm.

(*iii*) Given that surface area of the model eye is 651.6 cm^2. Then,

By area of scale factor (k^2), we have

$$k^2 = \frac{\text{Surface area of model eye}}{\text{Surface area of human eye}}$$

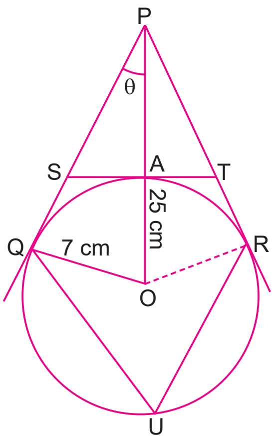
$\Rightarrow$ $(6)^2 = \dfrac{651.6}{\text{Surface area of human eye}}$

$\Rightarrow$ Surface area of human eye $= \dfrac{651.6}{36} = 18.1$

Thus, the external surface area of the human eye is 18.1 cm^2.

19. In the diagram, PQ, PR and ST are the tangents to the circle with centre O.

Given, radius of the circle = 7 cm and OP = 25 cm.



(*a*) In ΔPQO, we have

$\angle PQO = 90°$ [$\because$ Tangent is $\perp$ to the radius through the point of contact.]

$\therefore$ In right ΔPQO, $PQ^2 = OP^2 - OQ^2$

$= (25)^2 - (7)^2 = 625 - 49 = 576$

$\Rightarrow$ $PQ = \sqrt{576} = 24$ cm

Therefore, $\tan\theta = \dfrac{OQ}{PQ}$

$\Rightarrow$ $\tan\theta = \dfrac{7}{24}$ ...(1)

Similarly, ST is tangent to the circle at A and OA is the radius of the circle. Therefore,

$$PA = OP - OA = 25 - 7 = 18 \text{ cm}$$

Now, in right ΔPAS, we have

$$\tan\theta = \frac{AS}{PA}$$

$$\Rightarrow \quad \tan\theta = \frac{AS}{18} \quad \quad ...(2)$$

From (1) and (2), we get

$$\frac{7}{24} = \frac{AS}{18}$$

$$\Rightarrow \quad AS = \frac{18\times7}{24} = \frac{21}{4} \text{ cm}$$

$$\therefore \quad ST = 2AS \quad \quad [\because \text{ A is the mid-point of ST.}]$$

$$= 2 \times \frac{21}{4} \text{ cm} = 10.5 \text{ cm}$$

Hence, length of ST is 10.5 cm.

(*b*) From (1), we have

$$\tan\theta = \frac{7}{24}$$

$$\Rightarrow \quad \tan\theta = 0.292 \quad \text{or} \quad \theta = 16° \, 16' \quad \quad [\text{Using mathematical tables.}]$$

(*c*) Join OR.

In right ΔPQO, we have

$$\angle POQ + \angle PQO + \angle QPO = 180°$$

$$\Rightarrow \quad \angle POQ + 90° + 16° \, 16' = 180°$$

$$\Rightarrow \quad \angle POQ = 180° - 90° - 16° \, 16' = 73° \, 44' \approx 74° \text{ (To the nearest degree)}$$

Also, by SSS Congruence Criterion, ΔPOQ ≅ ΔPOR

Therefore, $\angle QOR = \angle POQ + \angle POR = 2\angle POQ$ [∵ $\angle POR = \angle POQ$]

$$= 2 \times 74° = 148°$$

Now, $\angle QOR = 2\angle QUR$ [∵ Angle subtended by an arc at the centre is double the angle subtended by the same arc in the remaining part of the circle.]

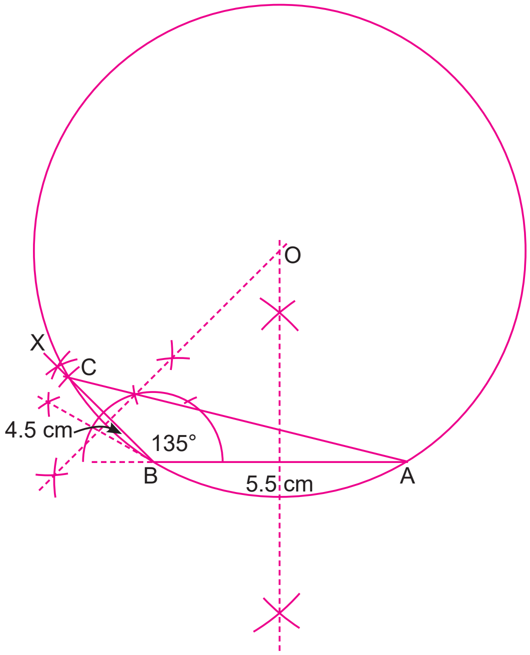
$$\Rightarrow \quad \angle QUR = \frac{1}{2}\angle QOR = \frac{1}{2} \times 148° = 74°.$$

20. Steps of Construction:

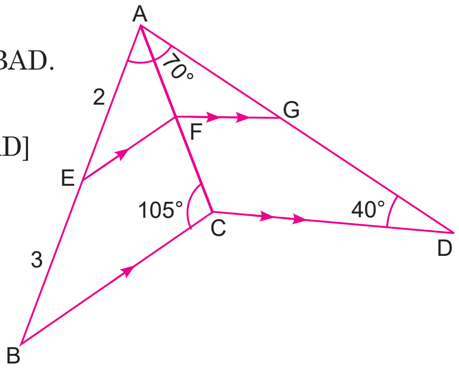
1. Draw line segment AB = 5.5 cm.
2. At B, draw ABX = 135°
3. From Ray BX, cut off BC = 4.5 cm.
4. Join AC and complete ΔABC.
5. Draw perpendicular bisectors of AB and BC to intersect each other at O.
6. Taking OA, OB or OC as radius, draw a circle passing through A, B and C.

The circle thus drawn is the required circumcircle to ΔABC.

On measuring, AC = 9.1 cm and radius of the circumcircle = 6.5 cm.



21. In the given figure, BC || EF, CD || FG, AE : EB = 2 : 3, ∠BAD = 70°, ∠ACB = 105°, ∠ADC = 40° and AC bisects ∠BAD.



(*a*) We have,

∠BAC = ∠DAC = 35° [∵ AC bisects ∠BAD]

In ΔABC, BC || EF, therefore

∠AFE = ∠ACB [Corresponding angles]

⇒ ∠AFE = 105° [∵ ∠ACB = 105°]

In ΔADC, CD || FG, therefore

∠AGF = ∠ADC [Corresponding angles]

⇒ ∠AGF = 40° [∵ ∠ADC = 40°]

In ΔAEF, ∠AEF = 180° – (∠AFE + ∠EAF)

= 180° – (105° + 35°) [∵ ∠EAF = ∠BAC = 35°]

= 180° – 140° = 40°

Also, in ΔAGF,

∠AFG = 180° – (∠AGF + ∠GAF)

= 180° – (40° + 35°) [∵ ∠GAF = ∠DAC = 35°]

= 180° – 75° = 105°

Now, in Δs AEF and AGF

∠EAF = ∠GAF [Each 35°]

∠AEF = ∠AGF [Each 40°]

∠AFE = ∠AFG [Each 105°]

∴ By AAA Similarity Criterion, ΔAEF ~ ΔAGF.

(*b*) (*i*) In ΔABC, EF || BC

∴ By Basic Proportionality Theorem, $\frac{AE}{EB} = \frac{AF}{FC}$ ...(1)

Similarly, in ΔADC, FG || CD

∴ By Basic Proportionality Theorem, $\frac{AG}{GD} = \frac{AF}{FC}$ ...(2)

From (1) and (2), we have

$$\frac{AE}{EB} = \frac{AG}{GD}$$

or $$\frac{EB}{AE} = \frac{GD}{AG}$$

⇒ $$1 + \frac{EB}{AE} = 1 + \frac{GD}{AG}$$

⇒ $$\frac{AE + EB}{AE} = \frac{AG + GD}{AG}$$

⇒ $$\frac{5}{2} = \frac{AD}{AG}$$ [∵ AE + EB = 2 + 3 = 5]

or $$\frac{AG}{AD} = \frac{2}{5}$$

Thus, AG : AD = 2 : 5.

(*ii*) In ΔADC, we have

∠ACD = AFG [Corresponding angles]

⇒ ∠ACD = 105° [∵ ∠AFG = 105°, proved in part (*a*)]

Now, in Δs ABC and ADC,

$\angle BAC = \angle DAC$ [Each 35°]

$AC = AC$ [Common]

$\angle ACB = \angle ACD$ [Each 105°]

∴ By ASA Congruence Criterion, $\Delta ABC \cong \Delta ADC$

We know that areas of two congruent triangles are always equal.

Therefore, $ar(\Delta ACB) = ar(\Delta ACD)$

or $$\frac{ar(\Delta ACB)}{ar(\Delta ACD)} = 1$$

$\Rightarrow ar(\Delta ACB) : ar(\Delta ACD) = 1 : 1.$

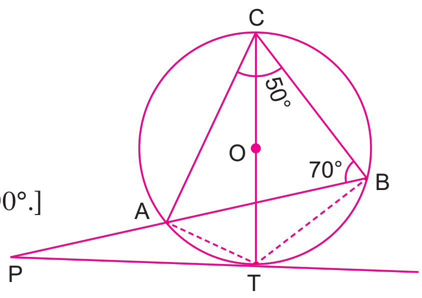
(*iii*) $ar(\text{quad. } ABCD) = ar(\Delta ACB) + ar(\Delta ACD)$

$= 2ar(\Delta ACB)$ [$\because ar(\Delta ACB) = ar(\Delta ACD)$]

$$\Rightarrow \frac{ar(\text{quad. } ABCD)}{ar(\Delta ACB)} = 2$$

Hence, $ar(\text{quad. } ABCD) : ar(\Delta ACB) = 2 : 1.$

22. In the given figure, we have $\angle ABC = 70°$ and $\angle ACB = 50°$. PT is the tangent to the circle at point T.



(*a*) Join BT.

Clearly, $\angle CBT$ is the angle in semicircle.

∴ $\angle CBT = 90°$. [∵ Angle in a semicircle is 90°.]

(*b*) Join AT.

Since $\angle CAT$ is also in semicircle,

∴ $\angle CAT = 90°$

In ΔABC, we have

$\angle CAB = 180° - (\angle ABC + \angle ACB)$ [By angle sum property of a triangle]

$= 180° - (70° + 50°)$

$= 180° - 120° = 60°$

∴ $\angle BAT = \angle CAT - \angle CAB$

$= 90° - 60° = 30°$

Thus, $\angle BAT = 30°$.

(*c*) $\angle PBT = \angle CBT - \angle ABC$

$= 90° - 70° = 20°$

Thus, $\angle PBT = 20°$.

(*d*) We have, $\angle ATP = \angle ABT$ [∵ Angles in the alternate segments are equal.]

or $\angle ATP = \angle PBT$ [$\because \angle ABT = \angle PBT$]

or $\angle ATP = 20°$

Also, $\angle PAT = 180° - \angle BAT$ [∵ PAB is a line.]

$= 180° - 30° = 150°$

Now, in ΔPAT, we have

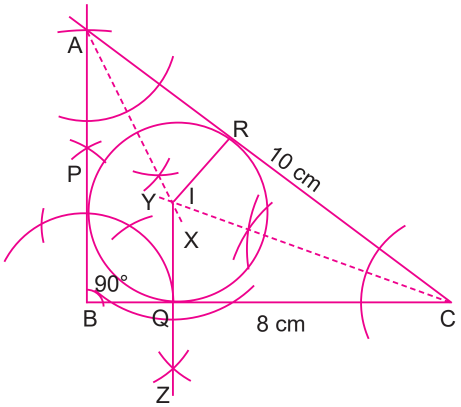
$\angle APT = 180° - (\angle PAT + \angle ATP)$

$= 180° - (150° + 20°)$

$= 180° - 170° = 10°$

Thus, $\angle APT = 10°$.

23. (*a*) The required triangle ABC is constructed in the figure shown above.



(*b*) The bisectors AX and CY of ∠BAC and ∠ACB respectively intersect each other at I.
From I, draw perpendicular IZ on BC to meet BC at Q.
Taking IQ as radius, draw a circle touching the sides of ΔABC.

The circle thus drawn is the required incircle.

(*c*) On measuring, in-radius IQ = 2 cm.

(*d*) The length of tangents CR and CQ each is 5.8 cm.

(*e*) In quad. CQIR, we have

$\angle CQI = \angle CRI = 90°$ [∵ Tangent is ⊥ to radius through the point of contact.]

$\therefore \angle RIQ + \angle QCR = 360° - (\angle CQI + \angle CRI)$

$= 360° - (90° + 90°) = 360° - 180° = 180°$

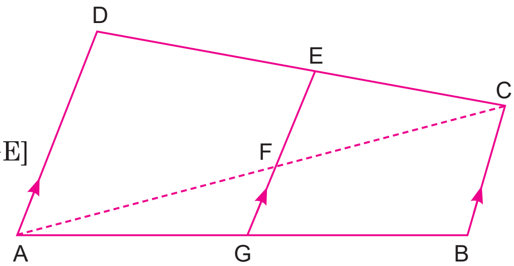
Hence, ∠RIQ + ∠QCR = 180°, which is the required relation between the given angles.

IV: Long Answer Questions (5 marks each)

24. In the given figure, we have

AD || GE || BC, DE = 18 cm,

EC = 3 cm and AD = 35 cm,



(*a*) In ΔADC, AD || EF   [∵ AD || GE]

∴ By Basic Proportionality Theorem,

$$\frac{DE}{EC} = \frac{AF}{FC}$$

$$\Rightarrow \quad \frac{18}{3} = \frac{AF}{FC}$$

$$\Rightarrow \quad \frac{AF}{FC} = \frac{6}{1} \quad \text{or} \quad AF : FC = 6 : 1$$

Thus, AF : FC = 6 : 1.

(*b*) In Δs ADC and FEC, we have

$\angle ACD = \angle FCE$ [Common]

and $\angle ADC = \angle FEC$ [Corresponding angles]

∴ By AA Similarity Criterion, ΔADC ~ ΔFEC

Therefore,

$$\frac{AD}{FE} = \frac{DC}{EC}$$ [∵ Corresponding sides of similar triangles are in proportion.]

$$\Rightarrow \quad \frac{35}{EF} = \frac{DE + EC}{EC}$$ [∵ FE = EF]

$$\Rightarrow \quad \frac{35}{EF} = \frac{18 + 3}{3}$$

$$\Rightarrow \quad \frac{35}{EF} = \frac{21}{3}$$

$$\Rightarrow \quad \frac{35}{EF} = 7 \quad \text{or} \quad EF = \frac{35}{7} = 5$$

Thus, the length of EF is 5 cm.

(*c*) We know that the ratio of the area of two similar triangles is equal to the square of the ratio of the corresponding sides of the two triangles. Therefore,

$$\frac{ar(\Delta ADC)}{ar(\Delta FEC)} = \left(\frac{DC}{EC}\right)^2$$

$$\Rightarrow \quad \frac{ar(\text{trap. ADEF}) + ar(\Delta FEC)}{ar(\Delta FEC)} = \left(\frac{21}{3}\right)^2$$

$$\Rightarrow \quad \frac{ar(\text{trap. ADEF})}{ar(\Delta FEC)} + \frac{ar(\Delta FEC)}{ar(\Delta FEC)} = (7)^2$$

$$\Rightarrow \quad \frac{ar(\text{trap. ADEF})}{ar(\Delta FEC)} + 1 = 49$$

$$\Rightarrow \quad \frac{ar(\text{trap. ADEF})}{ar(\Delta FEC)} = 49 - 1$$

$$\Rightarrow \quad \frac{ar(\text{trap. ADEF})}{ar(\Delta FEC)} = 48$$

or $\quad ar(\text{trap. ADEF}) : ar(\Delta FEC) = 48 : 1$

Thus, area (trapezium ADEF) : area (ΔEFC) is 48 : 1.

(*d*) In ΔABC, given GF || BC [$\because$ GE || BC]

$\therefore$ By Basic Proportionality Theorem,

$$\Rightarrow \quad \frac{BC}{GF} = \frac{AC}{AF}$$

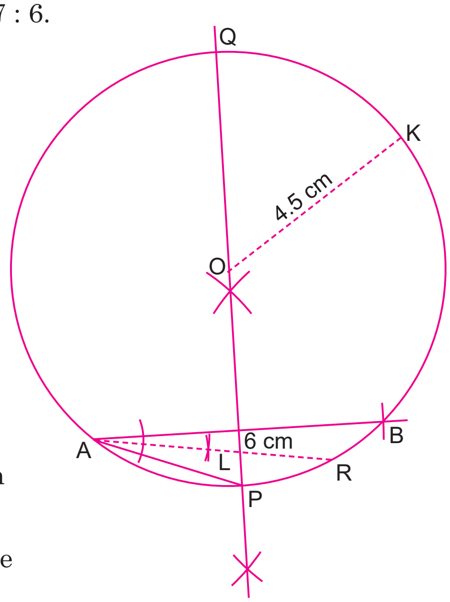
$$\Rightarrow \quad \frac{BC}{GF} = \frac{AF + FC}{AF}$$

$$\Rightarrow \quad \frac{BC}{GF} = 1 + \frac{FC}{AF}$$

$$\Rightarrow \quad \frac{BC}{GF} = 1 + \frac{1}{6} \qquad \left[\because \text{From part } (a), \frac{AF}{FC} = \frac{6}{1}\right]$$

$$\Rightarrow \quad \frac{BC}{GF} = \frac{7}{6} \quad \text{or} \quad BC : GF = 7 : 6.$$

25. (*a*) The locus of a moving point which moves such that it keeps a fixed distance of 4.5 cm from a fixed-point O is a circle whose centre is O and radius is 4.5 cm. In the figure, the point K moves such that it remains at a fixed distance of 4.5 cm from the fixed point O.



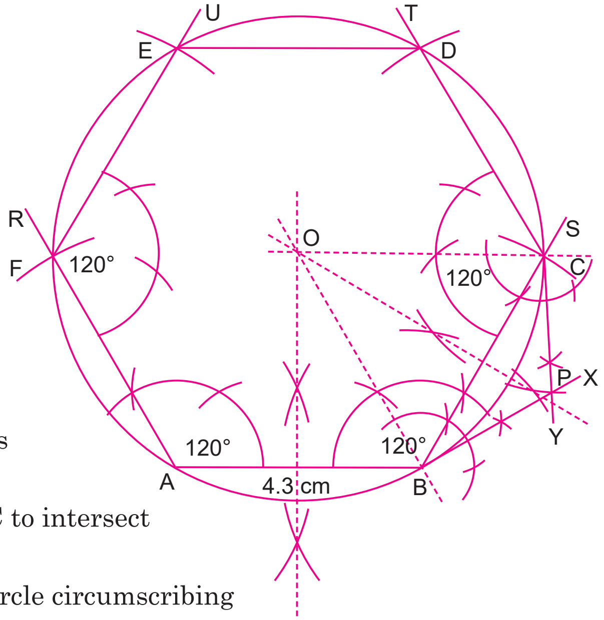
(*b*) Take a point A on the circle and with radius as 6 cm, draw an arc to intersect the circle at B. Join AB. Thus, A and B are two points on the locus drawn in part (*a*).

(*c*) The locus of all points equidistant from A and B is the perpendicular bisector of AB which cuts the locus drawn in part (*a*) at P and Q respectively.

(*d*) The locus of all points equidistant from AP and AB is the bisector AL of $\angle$BAP which intersects the circle at R.

On measuring, the length AR = 4.8 cm.

26. Steps of Construction:



1. Draw a line segment AB = 4.3 cm.
2. At A and B, draw ∠BAR = ∠ABS = 120° respectively.
3. From AR and BS cut off AF = BC each of length 4.3 cm.
4. At C and F, draw ∠BCT = ∠AFU = 120° respectively.
5. From CT and FU, cut off CD = FE each of length 4.3 cm.
6. Join DE.

Thus, the required regular hexagon ABCDEF is constructed.

7. Draw perpendicular bisectors of AB and BC to intersect each other at O.
8. Taking radius equal to OA or OB, draw a circle circumscribing the regular hexagon ABCDEF.
9. Join OB and OC and draw perpendiculars BX and CY at B and C respectively. BX and CY are the required tangents meeting each other at point P.

On measuring, ∠BPC = ∠120°.

Unit IV: Mensuration (Chapter 16)

I: Multiple Choice Questions (1 mark each)

1. (*b*) Let '*d*' be the diameter and '*h*' be the height of the Borosil cylindrical glass. Then,

$$\frac{d}{h} = \frac{3}{5} \quad [\because \text{Ratio of diameter to height is } 3 : 5.]$$

$$\Rightarrow \quad h = \frac{5d}{3}$$

$$\Rightarrow \quad h = \frac{5 \times 6}{3} \quad [\because \text{Diameter, } d = 6 \text{ cm, given}]$$

$$\Rightarrow \quad h = 10 \text{ cm}$$

So, curved surface area of the glass = $2\pi rh = \pi dh$ $[\because 2r = d]$

$= \pi\,(6)\,(10) = 60\pi \text{ cm}^2$.

2. (*c*) We know that volume of a solid is the quantity of material with which the solid is made of. Since quantity of material in the metallic wire remains the same when it is stretched to double its length, the volume of the wire does not change with the change in its length or shape.

3. (*b*) Volume of the cone = $\frac{1}{3}\pi r^2 h$

$$\Rightarrow \quad 9702 = \frac{1}{3} \times \frac{22}{7} \times r^2 \times r$$ [∵ Radius of the base and height of the cone are equal.]

$$\Rightarrow \quad 9702 = \frac{22}{21} r^3$$

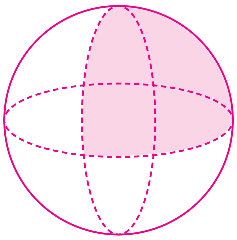
$$\Rightarrow \quad r^3 = \frac{9702 \times 21}{22}$$

$$\Rightarrow \quad r^3 = 441 \times 21 \quad \Rightarrow \quad r^3 = 21 \times 21 \times 21$$

$$\Rightarrow \quad r = 21 \text{ cm}$$

∴ Diameter of the base of the cone = $2r$ = 42 cm.

4. (*b*) When a solid sphere is cut into 4 identical pieces there will be one curved and two flat surfaces in one quarter piece.

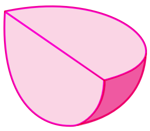


We know that total surface area of a solid sphere of radius '*r*' is $4\pi r^2$, therefore

Area of one curved surface of quarter piece = $\frac{1}{4} \times 4\pi r^2 = \pi r^2$

Also, area of one circular flat surface of radius '*r*' is πr^2, therefore

Area of two flat surfaces of quarter piece = $2 \times \frac{\pi r^2}{2} = \pi r^2$



∴ Total surface area of one quarter piece = $\pi r^2 + \pi r^2$

$= 2\pi r^2$

Given that radius of the sphere is 4 cm.

∴ Total surface area of one quarter piece = $2\pi\,(4)^2$ sq cm

$= 32\pi$ sq cm.

5. (*b*) Total surface area of a hemisphere of radius 'r' is $3\pi r^2$.

$\therefore$ Total surface area of two identical hemispheres $= 3\pi r^2 + 3\pi r^2 = 6\pi r^2$

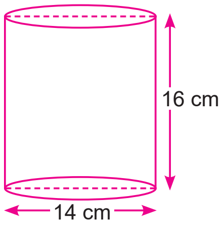
Total surface area of the sphere formed with the two hemispheres $= 4\pi r^2$

$\therefore$ Required ratio $= 6\pi r^2 : 4\pi r^2 = 6 : 4$ or $3 : 2$.

II: Short Answer Questions-1 (3 marks each)

6. Given,

Radius of the cylindrical can, $R = \frac{14}{2}$ cm $= 7$ cm



Height of the cylindrical can, H = 16 cm

$\therefore$ Volume of the cylindrical can $= \pi R^2 H$

$= \pi(7)^2 \times 16$ cu cm

$= 784\pi$ cu cm

Radius of each spherical rasgulla, $r = \frac{6}{2}$ cm $= 3$ cm

$\therefore$ Volume of each spherical rasgulla $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (3)^3$ cu cm $= 36\pi$ cu cm

So, volume of 20 spherical rasgullas $= 20 \times 36\pi$ cu cm $= 720\pi$ cu cm

Now,

Volume of the cylindrical can left for sweetened liquid

= Volume of the cylindrical can – Volume of 20 spherical rasgullas

$= 784\pi$ cu cm $- 720\pi$ cu cm

$= 64\pi$ cu cm $= 64 \times 3.14$ cu cm

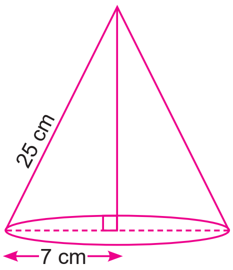
$= 200.96$ cu cm

Thus, the cylindrical can contains 200.96 cu cm sweetened liquid.

7. (*a*) Given that a solid metallic cylinder is melted and made into a cone whose diameter and slant height are given.

Let 'H' cm be the height of the cone. Then,

Radius of the cone, $R = \frac{14}{2}$ cm $= 7$ cm, and



Slant height of the cone, L = 25 cm

$\therefore$ Height of the cone, $H = \sqrt{L^2 - R^2}$

$= \sqrt{(25)^2 - (7)^2}$

$= \sqrt{625 - 49} = \sqrt{576} = 24$ cm

Thus, height of the cone is 24 cm.

(*b*) Since the metallic cylinder is melted and made into a cone, the volume of two solids would be the same.

Given, ratio for the radius and the height of the cylinder = 7 : 27.

Let $7x$ and $27x$ be the radius and height of the cylinder. Then,

Volume of the cylinder $= \pi r^2 h$

$= \pi(7x)^2\ (27x)$ cu cm [Taking $r = 7x$ and $h = 27x$]

$= \pi(49x^2)\ (27x)$ cu cm

Also,

Volume of the cone $= \frac{1}{3}\pi R^2 H$

$= \frac{1}{3}\pi \times (7)^2 \times 24$ cu cm [Taking R = 7 cm and H = 24 cm]

$\because$ Volume of the cylinder = Volume of the cone, we have

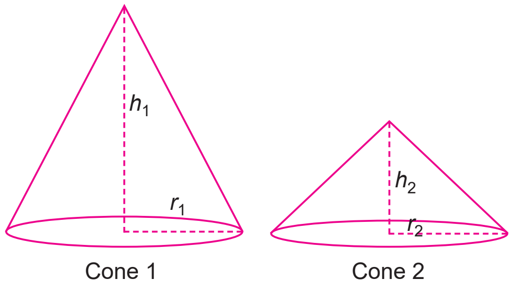
$$\pi(49x^2)(27x) = \frac{1}{3}\pi(49) \times 24$$

$$\Rightarrow \quad x^3 = \frac{8}{27} \quad \text{or} \quad x = \frac{2}{3}$$

Thus, height of the cylinder, $27x = 27 \times \frac{2}{3}$ cm, *i.e.*, 18 cm.

III: Short Answer Questions-2 (4 marks each)

8. (*a*) Let C_1 and C_2 be the curved surface areas of the two cones and V_1 and V_2 be their respective volumes. Given that curved surface area of first cone is half of the other cone. Therefore,



$$C_1 = \frac{1}{2}C_2 \quad \text{or} \quad \frac{C_1}{C_2} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{r_1}{r_2}\left(\frac{l_1}{l_2}\right) = \frac{1}{2} \quad \ldots(1)$$

Given that ratio of their slant heights is 2 : 1.

$$\therefore \quad \frac{l_1}{l_2} = \frac{2}{1} \quad \text{or} \quad \frac{l_1}{l_2} = 2$$

Putting the value of $\frac{l_1}{l_2}$ in (1), we have

$$\frac{r_1}{r_2}\ (2) = \frac{1}{2}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{1}{4} \quad \text{or} \quad r_1 : r_2 = 1 : 4.$$

Thus, the ratio of their radii is 1 : 4.

(*b*) Given, ratio of the volumes of the two cones is 3 : 1.

$$\frac{V_1}{V_2} = \frac{3}{1} \quad \Rightarrow \quad \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{3}{1}$$

$$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^2\left(\frac{h_1}{h_2}\right) = \frac{3}{1}$$

$$\Rightarrow \quad \left(\frac{1}{4}\right)^2\left(\frac{h_1}{h_2}\right) = \frac{3}{1} \quad \left[\because \frac{r_1}{r_2} = \frac{1}{4}\right]$$

$$\Rightarrow \quad \frac{1}{16}\left(\frac{h_1}{h_2}\right) = \frac{3}{1}$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{48}{1}$$

$$\text{or} \quad h_1 : h_2 = 48 : 1$$

Thus, the ratio of their heights is 48 : 1.

IV: Long Answer Questions (5 marks each)

9. Here, volume of the three solids made from the terracotta clay will be the same.

Given radius of the sphere, $r_1 = 7$ cm

$$\therefore \text{ Volume of the sphere} = \frac{4}{3}\pi r_1^3$$

$$= \frac{4}{3}\pi(7)^3 \text{ cu cm}$$

Radius of the cone, $r_2 = 14$ cm

Let h_2 be the height of the cone. Then,

$$\text{Volume of the cone} = \frac{1}{3}\pi r_2^2 h_2$$

$$= \frac{1}{3}\pi(14)^2 h_2 \text{ cu cm}$$

$\because$ Volume of the cone = Volume of the sphere

$$\Rightarrow \quad \frac{1}{3}\pi(14)^2 h_2 = \frac{4}{3}\pi(7)^3$$

$$\Rightarrow \quad (14)^2 \times h_2 = 4 \times (7)^3$$

$$\Rightarrow \quad h_2 = \frac{4 \times 7 \times 7 \times 7}{14 \times 14} = 7 \text{ cm}$$

Thus, height of the cone is 7 cm.

Height of the cylinder, $h_3 = \frac{7}{3}$ cm

Let radius of the cylinder formed be r_3 cm. Then,

$$\text{Volume of the cylinder} = \pi r_3^2 h_3$$

$$= \pi r_3^2 \left(\frac{7}{3}\right) \text{ cu cm}$$

$\because$ Volume of the cylinder = Volume of the sphere

$$\Rightarrow \quad \pi r_3^2 \left(\frac{7}{3}\right) = \frac{4}{3}\pi(7)^3$$

$$\Rightarrow \quad r_3^2 \left(\frac{7}{3}\right) = \frac{4}{3}(7)^3$$

$$\Rightarrow \quad {r_3}^2 = 4 \times (7)^2$$

$$\Rightarrow \quad r_3 = 2 \times 7 \quad \text{or} \quad r_3 = 14 \text{ cm}$$

Thus, radius of the cylinder is 14 cm.

Now,

$$\text{Total surface area of the sphere} = 4\pi r_1^2$$

$$= 4\pi(7)^2 \text{ sq cm}$$

$$\text{Total surface area of the cylinder} = 2\pi r_3 h_3 + 2\pi r_3^2$$
$$= 2\pi r_3(h_3 + r_3)$$
$$= 2\pi \times 14\left(\frac{7}{3}+14\right) \text{ sq cm} = 2\pi \times 14\left(\frac{49}{3}\right) \text{ sq cm}$$

$$\therefore \frac{\text{Total surface area of the sphere}}{\text{Total surface area of the cylinder}} = \frac{4\pi(7)^2}{2\pi\times 14\left(\frac{49}{3}\right)}$$
$$= \frac{4\times 49\times 3}{2\times 14\times 49} = \frac{3}{7}$$

Thus, total surface area of the sphere is $\frac{3}{7}$th of total surface area of the cylinder.

Unit V: Trigonometry (Chapters 17–19)

I: Multiple Choice Questions (1 mark each)

1. (*c*) We have,

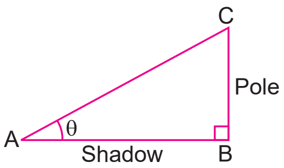
$\text{cosec}^2\theta + \sec^2\theta$

$$= \cot^2\theta + 1 + \tan^2\theta + 1$$
$$= \cot^2\theta + \tan^2\theta + 2$$
$$= \cot^2\theta + \tan^2\theta + 2\cot\theta\cdot\tan\theta \qquad [\because \cot\theta\cdot\tan\theta = 1]$$
$$= (\cot\theta + \tan\theta)^2.$$

2. (*d*) Given, $a = 3\sec^2\theta$ and $b = 3\tan^2\theta - 2$

$$\therefore \quad a - b = 3\sec^2\theta - (3\tan^2\theta - 2)$$
$$= 3\sec^2\theta - 3\tan^2\theta + 2$$
$$= 3(\sec^2\theta - \tan^2\theta) + 2$$
$$= 3(1 + \tan^2\theta - \tan^2\theta) + 2 \qquad [\because \sec^2\theta = 1 + \tan^2\theta]$$
$$= 3 + 2 = 5.$$

3. (*a*) Let θ be the angle of elevation of the sun. Then,

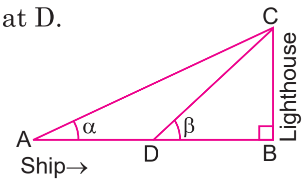


$$\tan\theta = \frac{BC}{AB}$$

Given, $$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \theta = 30°.$$

4. (*b*) Let A be the initial position of the ship and in 10 minutes it reaches at D.



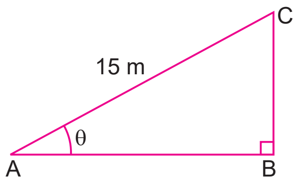
Then, $\angle BAC = \alpha$ and $\angle BDC = \beta$

In ΔADC, $\angle DAC < \angle BDC$ [∵ Exterior angle of a triangle is greater than each of its interior opposite angles.]

i.e., $\alpha < \beta$.

II: Short Answer Questions-1 (3 marks each)

5. (*a*) For the inclined plane, we have



$$AB = \sqrt{3}\,BC$$

$$\Rightarrow \quad \frac{AB}{BC} = \sqrt{3}$$

Also, in right ΔABC,

$$\frac{AB}{BC} = \cot\theta$$

$$\Rightarrow \quad \cot\theta = \sqrt{3} \quad \text{or} \quad \theta = 30° \qquad [\because \cot 30° = \sqrt{3}]$$

Thus, value of θ is 30°.

(*b*) Length of inclined plane AC = 15 m

In right ΔABC,

$$\cos\theta = \frac{AB}{AC}$$

$$\Rightarrow \quad \cos 30° = \frac{AB}{15}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AB}{15} \qquad \left[\because \cos 30° = \frac{\sqrt{3}}{2}\right]$$

$$\Rightarrow \qquad AB = \frac{15\sqrt{3}}{2}$$

$$= \frac{15 \times 1.732}{2} = \frac{25.98}{2} = 12.99 \text{ m} \approx 13 \text{ m}$$

Thus, length of the base AB (in nearest metre) is 13 m.

6. We have,

L.H.S. $= \tan^2\theta + \cos^2\theta - 1$

$$= \tan^2\theta - (1 - \cos^2\theta)$$

$$= \tan^2\theta - \sin^2\theta \qquad [\because 1 - \cos^2\theta = \sin^2\theta]$$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

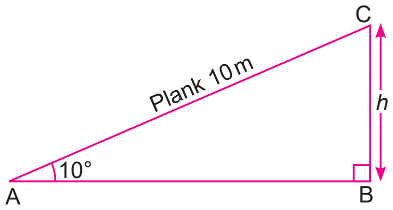
$$= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta\,(1 - \cos^2\theta)}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} \cdot \sin^2\theta$$

$$= \tan^2\theta \cdot \sin^2\theta = \text{R.H.S.}$$

III: Short Answer Questions-2 (4 marks each)

7. Let ABC be the plank making an angle of 10° with the horizontal ground and BC = h m be its height from the ground. Then,



$$\sin 10° = \frac{BC}{AC}$$

$$\Rightarrow \quad \sin 10° = \frac{h}{10}$$

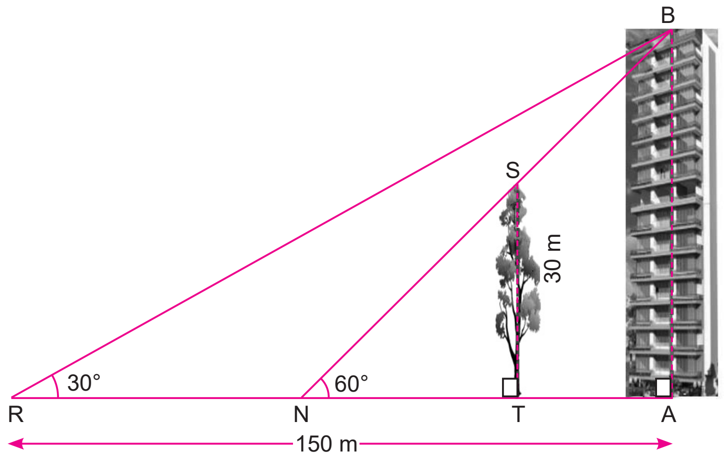
$$\Rightarrow \quad 0.1736 = \frac{h}{10} \qquad [\because \sin 10° = 0.1736, \text{ using mathematical tables}]$$

$\Rightarrow \quad h = 0.1736 \times 10 = 1.736 \approx 1.74$ (correct to 3 significant figures)

Thus, the height from which the cylindrical drum was rolled down is 1.74 m.

IV: Long Answer Questions (5 marks each)

8. In the figure, TS is 30 m high tree and AB is a tall building. R and N are the positions of Rahul and Neha. Distance RA = 150 m, angle of elevation of the building from R is 30° and angle of elevation of the tree and the building from N is 60°.



(*a*) In right triangle RAB, we have

$$\tan 30° = \frac{AB}{RA}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{150} \quad \Rightarrow \quad AB = \frac{150}{\sqrt{3}} = \frac{150\sqrt{3}}{3} = 50\sqrt{3}\text{ m} \quad \text{or} \quad 86.6 \text{ m}$$

Thus, height of the building is 86.6 m.

(*b*) (*i*) In right triangle NAB, we have

$$\tan 60° = \frac{AB}{NA}$$

$$\Rightarrow \quad \sqrt{3} = \frac{50\sqrt{3}}{NA}$$ [From part (*a*), AB = 50 $\sqrt{3}$]

$$\Rightarrow \quad NA = \frac{50\sqrt{3}}{\sqrt{3}} = 50 \text{ m}$$

Thus, distance between Neha and the foot of the building is 50 m.

(*ii*) From the figure, distance between Rahul and Neha is RN.

∴ RN = RA – NA = 150 m – 50 m = 100 m.

(*iii*) In right triangle NTS, we have

$$\tan 60° = \frac{TS}{NT}$$

$$\Rightarrow \quad \sqrt{3} = \frac{30}{NT} \quad \Rightarrow \quad NT = \frac{30}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3} \text{ m}$$

or 17.32 m

Thus, distance between Neha and the tree is 17.32 m.

(*iv*) From the figure, distance between building and the tree is TA.

∴ TA = NA – NT

= 50 m – 17.32 m = 32.68 m

Thus, distance between building and the tree is 32.68 m.

Unit VI: Statistics (Chapters 20–21)

I: Multiple Choice Questions (1 mark each)

1. (*c*) Assertion (A) is true:

The given frequency distribution is:

Class Interval	20–30	30–40	40–50	50–60	60–70
Frequency	1	3	2	6	4

Here, modal class and median class both lie in the class interval 50–60.

Class marks of 50–60 = $\frac{50+60}{2}$ = 55

Since class marks of both modal and median classes is 55, the difference of the two class marks is 0.

Therefore, Assertion (A) is true.

Reason (R) is false:

The statement given in Reason (R) may be true for some particular frequency distribution but in general the two classes are always not same.

Therefore, Reason (R) is false.

Hence, option (*c*) is the correct answer.

2. (*c*) Assertion (A) is true:

When the number of observations in a collection of arrayed data is odd, the median is the middle number.

Therefore, Assertion (A) is true.

Reason (R) is false:

The given data in ascending order is:

5, 7, 9, 10, 10, 11, 13

Here, number of observations, $n = 7$ is odd, therefore

$$\text{Median} = \left(\frac{n+1}{2}\right)\text{th observation}$$

$$= \left(\frac{7+1}{2}\right)\text{th observation} = 4\text{th observation} = 10$$

Thus, median of the given data is 10.

Therefore, Reason (R) is false.

Hence, option (*c*) is the correct answer.

II: Short Answer Questions-1 (3 marks each)

3. (*a*) According to the graph given in the question, the class marks of the data arc : 13, 15, 17, 19, 21 and 23.

Since class size is the difference between two successive class marks, so the class size is 15 – 13, *i.e.*, 2

$\therefore$ Lower limit of first class interval = $13 - \left(\frac{2}{2}\right)$ = 12; and

Upper limit of first class interval = $13 + \left(\frac{2}{2}\right)$ = 14.

Therefore, first class interval is 12–14 and from the graph its frequency is 8.

In this way other class intervals can be obtained and their respective frequencies can be read from the graph.

Thus, following is the required table showing the class intervals and their frequencies.

Class Interval	Frequency
12–14	8
14–16	2
16–18	3
18–20	4
20–22	5
22–24	6

(*b*) To find the mean we use the formula

Mean $(\overline{x}) = \dfrac{f_1x_1 + f_2x_2 + \ldots + f_nx_n}{f_1 + f_2 + \ldots + f_n}$, where $x_1\ x_2 \ldots x_n$ are class marks and $f_1, f_2 \ldots f_n$ are their respective frequencies

$$\therefore \text{ Mean } (\overline{x}) = \frac{8 \times 13 + 2 \times 15 + 3 \times 17 + 4 \times 19 + 5 \times 21 + 6 \times 23}{8 + 2 + 3 + 4 + 5 + 6}$$

$$= \frac{104 + 30 + 51 + 76 + 105 + 138}{28} = \frac{504}{28} = 18$$

Thus, mean of the given data is 18.

4. Given, mean of 5, 7, 8, 4 and m is n

$$\therefore \quad \frac{5 + 7 + 8 + 4 + m}{5} = n$$

$$\Rightarrow \quad 24 + m = 5n \quad \Rightarrow \quad 5n - m = 24 \quad \ldots(1)$$

and mean of 5, 7, 8, 4 and n is m

$$\therefore \quad \frac{5 + 7 + 8 + 4 + n}{5} = m$$

$$\Rightarrow \quad 24 + n = 5m \quad \Rightarrow \quad 5m - n = 24 \quad \ldots(2)$$

From (1) and (2),

$$5n - m = 5m - n$$

$$\Rightarrow \quad 6m = 6n \quad \Rightarrow \quad m = n$$

Again from (1), $\quad 5m - m = 24 \qquad [\because m = n]$

$$\Rightarrow \quad 4m = 24 \quad \text{or} \quad m = 6$$

Hence, $m = n = 6$.

III: Short Answer Questions-2 (4 marks each)

5. The marks of 12 students arranged in ascending order are:

2, 3, 3, 3, 4, x, $x + 2$, 8, p, q, 8, 9

Here, $n = 12$, which is even, therefore

$$\text{Median} = \text{Mean of } \left(\frac{n}{2}\right)\text{th and } \left(\frac{n}{2} + 1\right)\text{th observations}$$

$$= \text{Mean of 6th and 7th observations}$$

$$\therefore \quad 6 = \frac{x + x + 2}{2} \qquad [\because \text{Median of the data is 6.}]$$

$$\Rightarrow \quad 2x + 2 = 12$$

$$\Rightarrow \quad 2x = 12 - 2 = 10 \quad \text{or} \quad x = 5$$

With this value of x, the given marks of 12 students are:

2, 3, 3, 3, 4, 5, 7, 8, p, q, 8, 9

Now, it is given that mode of the data is 8 and mode is the most frequent occurring value in a data.

For the given data, if mode is 8 its frequency must be the highest which will be only possible if p and q both have same value 8.

So, if we take $p = q = 8$, the frequency of 8 will be 4 which is the highest.

Hence, both p and q have the same value 8.

6. The given data is:

Class Marks (Wages)	425	475	525	575	625	675
Number of Workers	6	12	15	17	7	13

From the above table,

Difference between two consecutive class marks = 475 – 425 = 50

So, class widths of class intervals = 50

$\therefore$ Adjustment factor $= \frac{50}{2} = 25$

Thus, lower limit of first class interval = 425 – 25 = 400

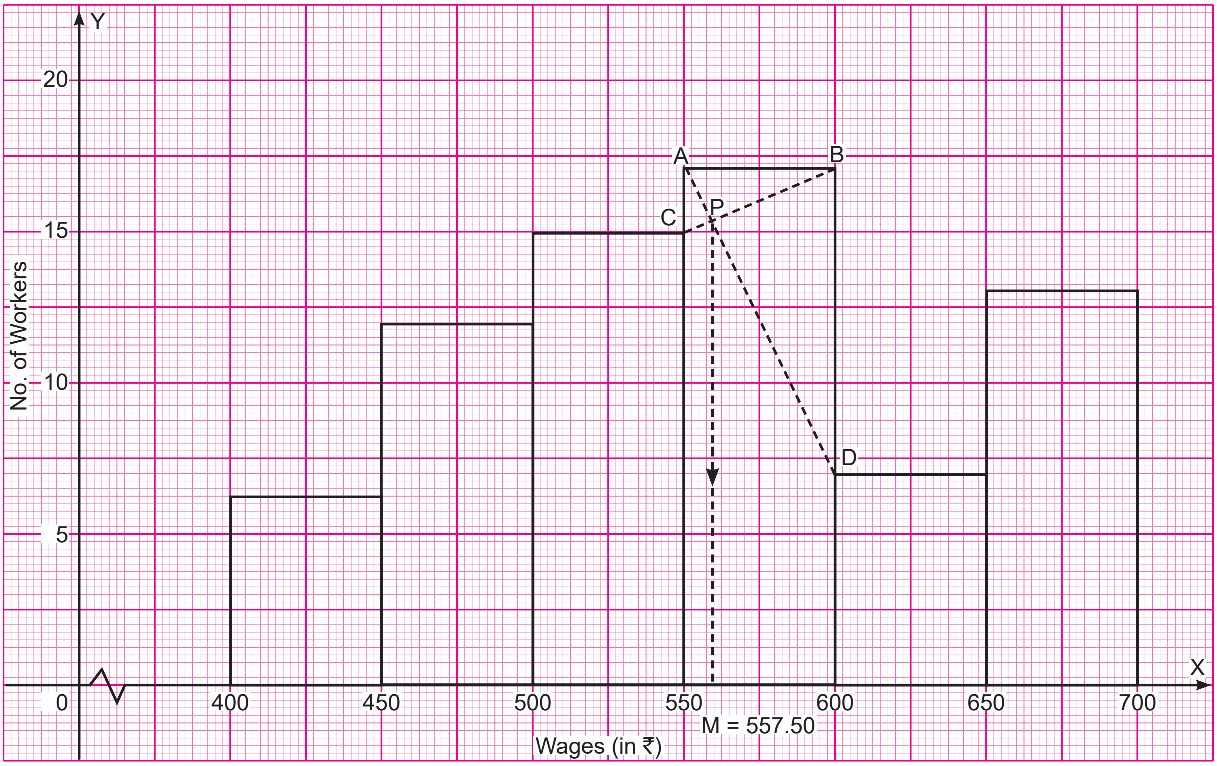
and uppar limit of first class interval = 425 + 25 = 450

Therefore, 400–450 is the first class interval.

Similarly, other class intervals can be formed and on this basis, we have the required frequency distribution table as under:

Class Intervals	**Frequency**
400–450	6
450–500	12
500–550	15
550–600	17
600–650	7
650–700	13

Now, taking class intervals on x-axis and their respective frequencies on y-axis, we plot the histogram as depicted below.



To find the modal wage, we consider the highest rectangle and join its two upper ends A and B with the ends of the rectangles lying on its either sides. In the above histogram AD and BC meet each other at P.

Through P, we draw a perpendicular to meet x-axis at M.

The abscissa of point M, *i.e.*, 557.50 represents the mode of the data.

Thus, modal wage of the workers is ₹ 557.50.

IV: Long Answer Questions (5 marks each)

7. (*a*) From the given data, we have the following cumulative frequency table.

Daily Wages (in ₹)	Number of Employees (f)	Cumulative Frequency (cf)
30–40	8	8
40–50	14	22
50–60	12	34
60–70	17	51
70–80	20	71
80–90	26	97
90–100	13	110
100–110	10	120

Taking upper limits of class intervals on x-axis and their respective cumulative frequencies on y-axis, we plot (40, 8), (50, 22), ... on a graph paper and join these points with a freehand curve. The curve so drawn is the required ogive as represented below:

(*b*) (*i*) Here, total number of employees, $n = 120$

$\therefore \frac{n}{2} = 60$

From A = 60 on y-axis, we draw a line parallel to x-axis to meet the ogive at P. From P, we draw perpendicular which meet x-axis at M = 74. The value of M represents the median.

$\therefore$ Median wage of the employees is ₹ 74.

(*ii*) Through L = 84 on x-axis, we draw a perpendicular to meet the ogive at Q. From Q, we draw a line parallel to x-axis to meet the y-axis at B = 81.

∴ Number of employees earning more than ₹ 84 per day = 120 – 81 = 39

Therefore, required percentage of employees = $\frac{39}{120} \times 100\% = 32.5\%$

Thus, 32.5% employees earn more than ₹ 84 per day.

(*iii*) Through N = 56 on x-axis, we draw a perpendicular to meet the ogive at R. From R, we draw a line parallel to x-axis to meet the y-axis at C = 30.

Thus, number of employees who earn ₹ 56 and below is 30.

8. (*a*) From the given histogram, we have the following frequency table:

Height (in cm)	**Number of Students**
120–130	6
130–140	29
140–150	34
150–160	22
160–170	12

(*b*) Number of students whose height is less than 150 cm

$$= 6 + 29 + 34 = 69$$

(*c*) Total number of students = 6 + 29 + 34 + 22 + 12 = 103

(*d*) Perpendicular drawn from the point of intersection of two lines in the highest rectangle meet the x-axis at 143, which represents the modal height.

Thus, modal height of the students is 143 cm.

(*e*) Given, mean or average height = 145.5 cm

and modal height = 143 cm

∴ Required difference = 145.5 – 143 = 2.5

Thus, difference in the modal height and the mean height is 2.5 cm.

9. (*a*) The data of age distribution of 100 policy holders is:

Age in Years	**Number of Policy Holders (f)**
15–20	7
20–25	12
25–30	15
30–35	22
35–40	f
40–45	14
45–50	8
50–55	4
Total	$N = \Sigma f = 82 + f$

Given, total number of policy holders, N = 100

∴ $82 + f = 100 \Rightarrow f = 100 - 82 = 18$

Thus, the unknown frequency is 18.

(*b*) We substitute the value of f in the given table and form the following cumulative frequency table:

Age in Years	Number of Policy Holders (f)	Cumulative Frequency (cf)
15–20	7	7
20–25	12	19
25–30	15	34
30–35	22	56
35–40	18	74
40–45	14	88
45–50	8	96
50–55	4	100

Here, $\frac{N}{2} = \frac{100}{2} = 50$

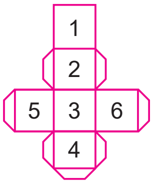
Cumulative frequency just greater than 50 is 56 and the class corresponding to this frequency is 30–35.

Therefore, median class of the distribution is 30–35.

Unit VII: Probability (Chapter 22)

I: Multiple Choice Questions (1 mark each)

1. (*a*) As shown in the picture, out of six numbers, the numbers 4 and 6 are composite numbers whereas 2, 3 and 5 are prime numbers.



$\therefore$ Probability of getting a composite number $= \frac{2}{6}$, *i.e.*, $\frac{1}{3}$

Probability of getting a prime number $= \frac{3}{6}$, *i.e.*, $\frac{1}{2}$

$\therefore$ Ratio of the two probabilities $= \frac{1}{3} : \frac{1}{2}$, *i.e.*, 2 : 3.

II: Short Answer Questions-1 (3 marks each)

2. Given, probability of selecting a blue marble $= \frac{1}{3}$

and probability of selecting a red marble $= \frac{1}{5}$

$\therefore$ Probability of selecting a green marble $= 1 - \left(\frac{1}{3} + \frac{1}{5}\right)$ [Sum of all probabilities containing all possible outcomes is 1.]

$$= 1 - \left(\frac{8}{15}\right) = \frac{7}{15}$$

Also,

Probability of selecting a green marble $= \frac{\text{Number of green marbles in the bag}}{\text{Total number of marbles in the bag}}$

$\Rightarrow \quad \frac{7}{15} = \frac{14}{\text{Total number of marbles in the bag}}$

$\Rightarrow$ Total number of marbles in the bag $= \frac{14 \times 15}{7} = 30$

Now,

Probability of selecting a red marble $= \frac{\text{Number of red marbles in the bag}}{\text{Total number of marbles in the bag}}$

$\Rightarrow \quad \frac{1}{5} = \frac{\text{Number of red marbles in the bag}}{30}$

$\Rightarrow$ Number of red marbles in the bag $= \frac{30}{5} = 6$

Thus, on the basis of the above solution, we have

(*a*) Number of red marbles = 6

(*b*) Total number of marbles in the bag = 30.

III: Short Answer Questions-2 (4 marks each)

3. From the given marks distribution, we prepare the following cumulative frequency table:

Marks Scored	Number of Students	Cumulative Frequency
0–10	4	4
10–20	5	9
20–30	9	18
30–40	7	25
40–50	13	38
50–60	12	50
60–70	15	65
70–80	11	76
80–90	14	90
90–100	10	100

Here, total number of students = 100

(*a*) Number of students who have scored less than 20 = 9

$$\therefore \text{ Required probability} = \frac{\text{Number of students who have scored less than 20}}{\text{Total number of students}} = \frac{9}{100}.$$

(*b*) Number of students who have scored below 60 = 50

Number of students who have scored below 30 = 18

∴ Number of students who have scored below 60 but 30 or more = 50 – 18 = 32

$$\therefore \text{ Required probability} = \frac{\text{Number of students who have scored below 60 but 30 or more}}{\text{Total number of students}} = \frac{32}{100} = \frac{8}{25}.$$

(*c*) Number of students who have scored below 70 = 65

∴ Number of students who have scored more than or equal to 70 = 100 – 65 = 35

$$\therefore \text{ Required probability} = \frac{\text{Number of students who have scored more than or equal to 70}}{\text{Total number of students}} = \frac{35}{100} = \frac{7}{20}.$$

(*d*) Number of students who have scored above 89, *i.e.*, 90 and more = 10

$$\therefore \text{ Required probability} = \frac{\text{Number of students who have scored above 89, } i.e., \text{ 90 and more}}{\text{Total number of students}} = \frac{10}{100} = \frac{1}{10}.$$

4. Given, a bag contains 13 red cards, 13 black cards and 13 green cards. Each set of cards are numbered 1 to 13. So, total number of cards in the bag = 39

(*a*) Number of green cards in the bag = 13

Total number of cards in the bag = 39

$$\therefore \text{ P(a green card)} = \frac{\text{Number of green cards in the bag}}{\text{Total number of cards in the bag}} = \frac{13}{39} = \frac{1}{3}.$$

(*b*) In each set of cards there are six cards with even numbers, namely—2, 4, 6, 8, 10 and 12. So, in all there are 18 such cards in three sets. Therefore,

Number of cards with an even number = 18

Total number of cards in the bag = 39

$$\therefore \text{ P (a card with an even number)} = \frac{\text{Number of cards with an evern number}}{\text{Total number of cards in the bag}}$$

$$= \frac{18}{39} = \frac{6}{13}.$$

(*c*) There are four red and four black cards with a number which is a multiple of three, *i.e.*, 3, 6, 9 and 12. So, in all there are 8 such cards in three sets. Therefore,

Number of red or black cards with a number which is a multiple of three = 8

Total number of cards in the bag = 39

∴ P (a red or black card with a number which is a multiple of three)

$$= \frac{\text{Number of red or black cards with a number which is a multiple of three}}{\text{Total number of cards in the bag}} = \frac{8}{39}.$$