Specimen Questions (with Solutions) 2025 for Practice

Question 1

Choose the correct answers to the questions from the given options: (15)

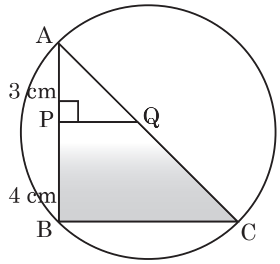
(*i*) A polynomial in 'x' is divided by $(x - a)$ and for $(x - a)$ to be a factor of this polynomial, the remainder should be

(*a*) $-a$. (*b*) 0. (*c*) a. (*d*) $2a$.

(*ii*) Radha deposited ₹ 400 per month in a recurring deposit account for 18 months. The qualifying sum of money for the calculation of interest is

(*a*) ₹ 3,600. (*b*) ₹ 7,200. (*c*) ₹ 68,400. (*d*) ₹ 1,36,800.

(*iii*) In the adjoining figure, AC is a diameter of the circle. AP = 3 cm and PB = 4 cm and QP ⊥ AB. If the area of ΔAPQ is 18 cm^2, then the area of shaded portion QPBC is



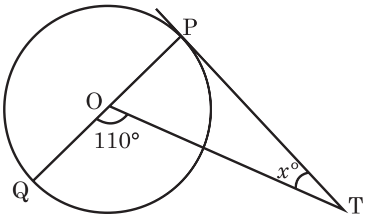
(*a*) 32 cm^2.

(*b*) 49 cm^2.

(*c*) 80 cm^2.

(*d*) 98 cm^2.

(*iv*) In the adjoining diagram, O is the centre of the circle and PT is a tangent. The value of x is



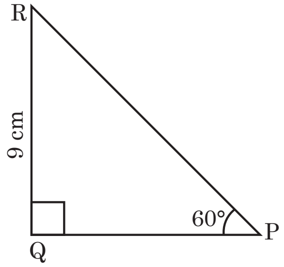
(*a*) 20°.

(*b*) 40°.

(*c*) 55°.

(*d*) 70°.

(*v*) In the adjoining diagram, the length of PR is



(*a*) $3\sqrt{3}$ cm.

(*b*) $6\sqrt{3}$ cm.

(*c*) $9\sqrt{3}$ cm.

(*d*) 18 cm.

(*vi*) A solid sphere is cut into two identical hemispheres.

Statement 1: The total volume of two hemispheres is equal to the volume of the original sphere.

Statement 2: The total surface area of two hemispheres together is equal to the surface area of the original sphere.

Which of the following is valid?

(*a*) Both the statements are true.

(*b*) Both the statements are false.

(*c*) Statement 1 is true, and Statement 2 is false.

(*d*) Statement 1 is false, and Statement 2 is true.

(*vii*) Given that the sum of the squares of the first seven natural numbers is 140, then their mean is

(*a*) 20. (*b*) 70. (*c*) 280. (*d*) 980.

(*viii*) A bag contains 3 red and 2 blue marbles. A marble is drawn at random. The probability of drawing a black marble is

(*a*) 0. (*b*) $\frac{1}{5}$. (*c*) $\frac{2}{5}$. (*d*) $\frac{3}{5}$.

(*ix*) If $A = \begin{bmatrix} 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 4 \\ 2 & 0 \end{bmatrix}$

Assertion (A): Product AB of the two matrices A and B is possible.

Reason (R): Number of columns of matrix A is equal to number of rows in matrix B.

(*a*) A is true, R is false.
(*b*) A is false, R is true.
(*c*) Both A and R are true, and R is the correct reason for A.
(*d*) Both A and R are true, and R is incorrect reason for A.

(*x*) A mixture of paint is prepared by mixing 2 parts of red pigments with 5 parts of the base. Using the given information in the following table, find the values of *a*, *b* and *c* to get the required mixture of paint.

Parts of red pigment	2	4	*b*	6
Parts of base	5	*a*	12.5	*c*

(*a*) $a = 10, b = 10, c = 10$ (*b*) $a = 5, b = 2, c = 5$
(*c*) $a = 10, b = 5, c = 10$ (*d*) $a = 10, b = 5, c = 15$

(*xi*) An article which is marked at ₹ 1,200 is available at a discount of 20% and the rate of GST is 18%. The amount of SGST is

(*a*) ₹ 216.00. (*b*) ₹ 172.80. (*c*) ₹ 108.00. (*d*) ₹ 86.40.

(*xii*) The sum of money required to buy 50, ₹ 40 shares at ₹ 38.50 is

(*a*) ₹ 1,920. (*b*) ₹ 1,924. (*c*) ₹ 1,925. (*d*) ₹ 1,952.

(*xiii*) The roots of quadratic equation $x^2 - 1 = 0$ are

(*a*) 0. (*b*) 1. (*c*) −1. (*d*) ± 1.

(*xiv*) Which of the following equation represents a line equally inclined to the axes?

(*a*) $2x - 3y + 7 = 0$ (*b*) $x - y = 7$ (*c*) $x = 7$ (*d*) $y = -7$

(*xv*) Given, $x + 2 \leq \frac{x}{3} + 3$ and x is a prime number. The solution set for x is

(*a*) ϕ. (*b*) {0}. (*c*) {1}. (*d*) {0, 1}.

Solution:

(*i*) (*b*) When a polynomial $p(x)$ is divided by $(x - a)$ the remainder is given by $p(a)$ and for $(x - a)$ to be a factor of this polynomial, $p(a) = 0$, *i.e.*, remainder is zero.

(*ii*) (*c*) If ₹ P is deposited every month for n months in a R.D., the qualifying sum of money for calculating the interest is ₹ $P \times \frac{n(n+1)}{2}$ = ₹ $400 \times \frac{18 \times 19}{2}$ = ₹ 68,400.

(*iii*) (*c*) Here, $\Delta APQ \sim \Delta ABC \Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{(3)^2}{(7)^2}$

$$\Rightarrow \frac{18}{ar(\Delta ABC)} = \frac{9}{49} \Rightarrow ar(\Delta ABC) = 98 \text{ cm}^2$$

$\therefore ar(\text{quad. QPBC}) = ar(\Delta ABC) - ar(\Delta APQ) = 98 \text{ cm}^2 - 18 \text{ cm}^2 = 80 \text{ cm}^2$.

(*iv*) (*a*) Here, $\angle OPT = 90°$ and $\angle POT = 70°$

$\therefore \angle OTP$ or $x° = 180° - \angle OPT - \angle POT = 180° - 90° - 70° = 20°$.

(*v*) (*b*) In ΔPQR, $\sin 60° = \frac{9}{\text{PR}} \Rightarrow \frac{\sqrt{3}}{2} = \frac{9}{\text{PR}}$ or $\text{PR} = \frac{18}{\sqrt{3}} = 6\sqrt{3}$ cm.

(*vi*) (*c*) Let '*r*' be the radius of the original solid sphere. Then,

Volume of the sphere $= \frac{4}{3}\pi r^3$ and volume of two hemispheres $= 2 \times \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3$

∴ Total volume of two hemispheres is equal to the volume of the original sphere.

Thus, Statement 1 is true.

Total surface area of the sphere $= 4\pi r^2$ and total surface area of two hemispheres $= 6\pi r^2$

∴ Total surface area of two hemispheres together is not equal to the surface area of the original sphere.

Thus, Statement 2 is false.

Therefore, Statement 1 is true, and Statement 2 is false.

(*vii*) (*a*) Given that $1^2 + 2^2 + \ldots + 7^2 = 140$ or $\sum_{i=1}^{7} x_i^2 = 140$ and $n = 7$.

$$\therefore \text{Mean } (\bar{x}) = \frac{\sum_{i=1}^{7} x_i^2}{7} = \frac{140}{7} = 20.$$

(*viii*) (*a*) Since there is no black marble in the bag, P(*a* black marble) = 0.

(*ix*) (*c*) Here, matrix A is a 1 × 2 matrix and matrix B is a 2 × 2 matrix.

∴ Number of columns of matrix A is equal to number of rows in matrix B and it is the necessary condition for the product AB of the two matrices A and B.

So, both A and R are true and R is the correct reason for A.

(*x*) (*d*) Here, ratio of parts of red pigments to the parts of base is 2 : 5.

$$\therefore \quad \frac{2}{5} = \frac{4}{a} \quad \Rightarrow \quad 2a = 20 \text{ or } a = 10$$

$$\text{Similarly, } \frac{2}{5} = \frac{b}{12.5} \quad \Rightarrow \quad 5b = 25 \text{ or } b = 5$$

$$\text{and} \quad \frac{2}{5} = \frac{b}{c} \quad \Rightarrow \quad 2c = 30 \text{ or } c = 15.$$

(*xi*) (*d*) Price of the article after 20% discount = 80% of ₹ 1200 = ₹ 960

and GST charged at 18% = 18% of ₹ 960 = ₹ 172.80

∴ Amount of SGST $= \frac{1}{2} \times$ ₹ 172.80 = ₹ 86.40.

(*xii*) (*c*) Given that market price of ₹ 40 share is ₹ 38.50.

∴ Money required to buy 50 shares = 50 × ₹ 38.50 = ₹ 1,925

(*xiii*) (*d*) Given, $x^2 - 1 = 0 \Rightarrow (x - 1)(x + 1) = 0$

$$\Rightarrow x = 1 \text{ or } x = -1, \ i.e., x = \pm 1.$$

(*xiv*) (*b*) Clearly, the slope (m) of the line $x - y = 7$ is 1, *i.e.*, $m = \tan 45°$.

Therefore, the line $x - y = 7$ is equally inclinded to the axes.

(*xv*) (*a*) Here, $x + 2 \le \frac{x}{3} + 3 \Rightarrow x - \frac{x}{3} \le 3 - 2$

$$\Rightarrow \frac{2x}{3} \le 1 \text{ or } x \le \frac{3}{2}$$

Since there is no prime number less than or equal to $\frac{3}{2}$, the given solution set of the inequation is an empty set, *i.e.*, ϕ.

Question 2

(*i*) While factorising a given polynomial, using remainder and factor theorem, a student finds that $(2x + 1)$ is a factor of $2x^3 + 7x^2 + 2x - 3$.

(*a*) Is the student's solution correct stating that $(2x + 1)$ is a factor of the given polynomial?
(*b*) Give a valid reason for your answer.

Also, factorise the given polynomial completely. (4)

(*ii*) A line segment joining P(2, –3) and Q(0, –1) is cut by the *x*-axis at the point R. A line AB cuts the *y*-axis at T(0, 6) and is perpendicular to PQ at S.

Find the:

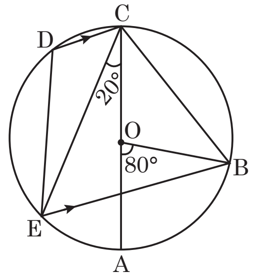
(*a*) equation of line PQ. (*b*) equation of line AB.

(*c*) coordinates of points R and S. (4)

(*iii*) In the given figure, AC is the diameter of the circle with centre O. CD is parallel to BE. $\angle AOB = 80°$ and $\angle ACE = 20°$. Calculate

(*a*) $\angle BEC$ (*b*) $\angle BCD$

(*c*) $\angle CED$. (4)



Solution:

(*i*) (*a*) Let the given polynomial be $f(x) = 2x^3 + 7x^2 + 2x - 3$.

If $(2x + 1)$ is a factor of $f(x) = 2x^3 + 7x^2 + 2x - 3$, then by Factor Theorem $f\left(\frac{-1}{2}\right) = 0$.

Now,
$$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + 7\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) - 3$$
$$= 2\left(-\frac{1}{8}\right) + 7\left(\frac{1}{4}\right) + 2\left(-\frac{1}{2}\right) - 3$$
$$= -\frac{1}{4} + \frac{7}{4} - 1 - 3 = \frac{7}{4} - \frac{17}{4} = \frac{-5}{2} \neq 0$$

Since $f\left(-\frac{1}{2}\right) \neq 0$, $(2x + 1)$ is not a factor of $f(x)$.

Hence, the student's solution is not correct.

(*b*) By Factor Theorem, $f\left(-\frac{1}{2}\right) \neq 0$.

Factorising the given polynomial:

Here,
$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + 7\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 3 = 0$$

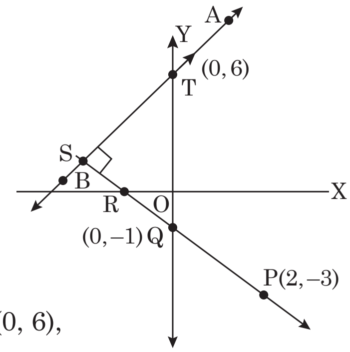
$\therefore$ $(2x - 1)$ is a factor of $f(x)$.

By long division,
$$\begin{array}{r|l} & x^2 + 4x + 3 \\ 2x - 1 & 2x^3 + 7x^2 + 2x - 3 \\ & \underline{\,_-2x^3 \mp x^2} \\ & \quad 8x^2 + 2x \\ & \quad \underline{_-8x^2 \mp 4x} \\ & \qquad 6x - 3 \\ & \qquad \underline{_-6x \mp 3} \\ & \qquad\quad 0 \end{array}$$

Therefore, $f(x) = (2x - 1)(x^2 + 4x + 3)$

or $f(x) = (2x - 1)(x + 3)(x + 1)$.

(*ii*) (*a*) Since line PQ passes through P(2, –3) and Q (0, –1),



its slope $(m_1) = \frac{-1-(-3)}{0-2} = \frac{2}{-2} = -1$

∴ Equation of line PQ is: $y - (-3) = -1(x - 2)$

⇒ $y + 3 = -x + 2$

or $x + y + 1 = 0$ ...(1)

(*b*) Since line AB is perpendicular to line PQ and passes through T(0, 6),

therefore slope (m_2) of line AB = 1 [$\because m_1 \times m_2 = -1$]

∴ Equation of line AB is: $y - 6 = 1(x - 0)$

⇒ $y - 6 = x$

or $x - y + 6 = 0$ ...(2)

(*c*) Point R lies on line PQ and cuts *x*-axis, therefore its ordinate is 0.

∴ Putting $y = 0$ in (1), we have

$$x + 1 = 0 \Rightarrow x = -1$$

So, the coordinates of R are (–1, 0).

Point S is the point of intersection of the lines PQ and AB therefore, to get the coordinates of S, we solve equation (1) and (2).

On adding (1) and (2), we get $2x + 7 = 0 \Rightarrow x = \frac{-7}{2}$

Putting $x = \frac{-7}{2}$ in (1), we get $\frac{-7}{2} + y + 1 = 0$

⇒ $y = \frac{7}{2} - 1 = \frac{5}{2}$

So, the coordinates of S are $\left(-\frac{7}{2}, \frac{5}{2}\right)$.

(*iii*) (*a*) We have, ∠AOB = 80° and ∠BOC = 180° – ∠AOB

⇒ ∠BOC = 180° – 80° = 100°

Now, ∠BOC = 2∠BEC [Angle at the centre is twice the angle in the remaining part of the circle.]

⇒ $\angle BEC = \frac{1}{2} \angle BOC$

⇒ $\angle BEC = \frac{1}{2} \times 100° = 50°$.

(*b*) Given that CD || BE.

∴ ∠ECD = ∠BEC [Alternate angles]

⇒ ∠ECD = 50° [∵ ∠BEC = 50°]

Now, ∠BCD = ∠BCA + ∠ACE + ∠ECD

= 40° + 20° + 50° = 110°. [$\because \angle BCA = \frac{1}{2} \angle AOB = 40°$]

(*c*) In ΔBCE, ∠CBE = 180° – ∠BCE – ∠BEC

= 180° – 60° – 50° = 180° – 110° = 70°

Now, BCDE is a cyclic quad., therefore

∠CBE + ∠CDE = 180°

⇒ ∠CDE = 180° – 70° = 110°

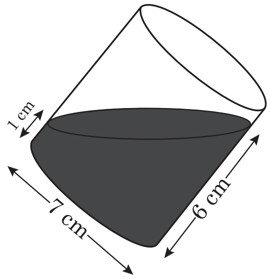
In ΔCED, ∠CED = 180° – ∠CDE – ∠ECD

= 180° – 110° – 50° = 20°.

Question 3

(*i*) In a Geometric Progression (G.P.) the first term is 24 and the fifth term is 8. Find the ninth term of the G.P. . (4)

(*ii*) In the adjoining diagram, a tilted right circular cylindrical vessel with base diameter 7 cm contains a liquid. When placed vertically, the height of the liquid in the vessel is the mean of two heights shown in the diagram. Find the area of wet surface, when the cylinder is placed vertically on a horizontal surface. $\left(\text{Use } \pi = \frac{22}{7}\right)$ (4)



(*iii*) Study the graph and answer each of the following:

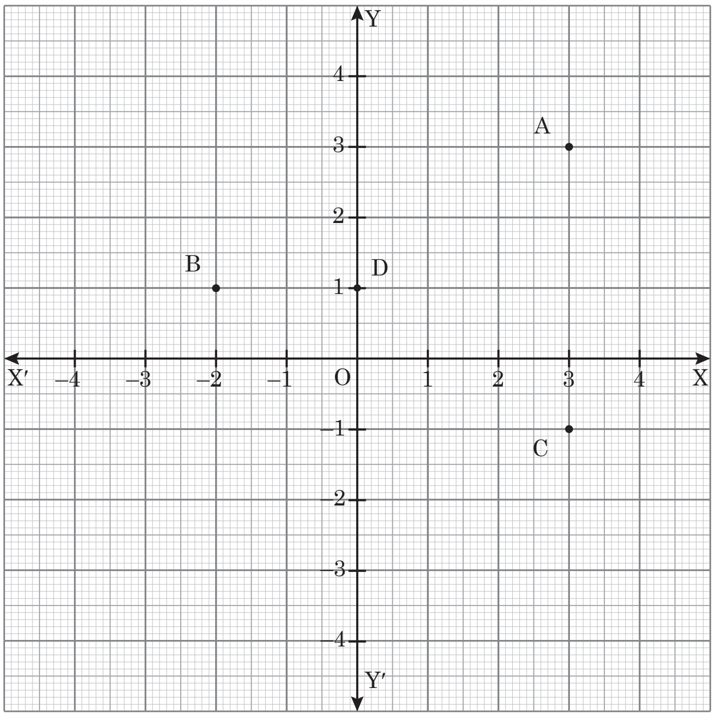
(*a*) Write the coordinates of points A, B, C and D.

(*b*) Given that, point C is the image of point A. Name and write the equation of the line of reflection.

(*c*) Write the coordinates of the image of the point D under reflection in *y*-axis.

(*d*) What is the name given to a point whose image is the point itself?

(*e*) On joining the points A, B, C, D and A in order, a figure is formed. Name the closed figure. (5)



Solution:

(*i*) Given, first term, $a = 24$ and fifth term, $T_5 = 8$.

$$T_5 = 8 \Rightarrow ar^4 = 8 \Rightarrow r^4 = \frac{8}{24} = \frac{1}{3}$$

$\therefore$ Ninth term, $T_9 = ar^8 = a(r^4)^2 = 24 \times \left(\frac{1}{3}\right)^2 = \frac{8}{3}$.

(*ii*) When placed vertically, height of the liquid in the vessel, $h = \frac{1}{2}(1 + 6)$ cm $\Rightarrow h = \frac{7}{2}$ cm

$\therefore$ Area of wet surface $= \pi r^2 + 2\pi rh = \pi r(r + 2h)$

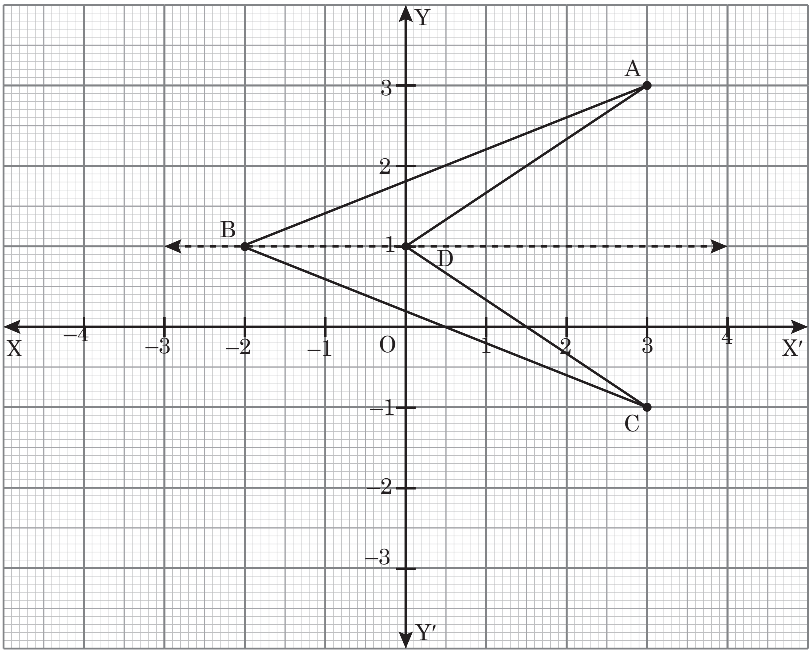
$$= \frac{22}{7} \times \frac{7}{2}\left(\frac{7}{2} + 2 \times \frac{7}{2}\right) = 115.5 \text{ cm}^2.$$

(*iii*) (*a*) Coordinates of points A, B, C and D are A(3, 3), B(−2, 1), C(3, −1) and D(0, 1).

(*b*) Here, line of reflection is parallel to *x*-axis and passes through mid-point of AC, *i.e.*,

$$\left(\frac{3+3}{2}, \frac{3-1}{2}\right) = (3, 1).$$

Since the point (3, 1) lies on the line BD, therefore line BD or $y = 1$ is the line of reflection.



(*c*) Since point D lies on the *y*-axis, its image under reflection in *y*-axis is the point D(0, 1) itself.

(*d*) A point whose image w.r.t. a line is the point itself is known as invariant point.

(*e*) Concave quadrilateral or an arrowhead.

Question 4

(*i*) A man buys 250, ten-rupee shares each at ₹ 12.50. If the rate of dividend is 7%, find the:

(*a*) dividend he receives annually.

(*b*) percentage return on his investment. (3)

(*ii*) Solve the following inequation, write the solution set and represent it on the real number line.

$$5x - 21 < \frac{5x}{7} - 6 \le -3\frac{3}{7} + x, \; x \in R. \quad (3)$$

(*iii*) Prove the following trigonometry identity:

$(\sin + \cos\theta)(\operatorname{cosec}\theta - \sec\theta) = \operatorname{cosec}\theta . \sec\theta - 2\tan\theta$ (4)

Solution:

(*i*) (*a*) Annual dividend = Number of shares × Rate of dividend × Face value of a share

$$= ₹\ 250 \times \frac{7}{100} \times 10 = ₹\ 175.$$

(*b*) Income from one share = 7% of ₹ 10 = ₹ 0.7

$$\text{Return \%} = \frac{\text{Income from 1 share}}{\text{M.V. of 1 share}} \times 100\%$$

$$= \frac{0.7}{12.50} \times 100\% = \frac{7 \times 10}{12.50}\% = 5.6\%.$$

(*ii*) The given inequation $5x - 21 < \frac{5x}{7} - 6 \leq -3\frac{3}{7} + x$, $x \in R$ can be rewritten as:

$$5x - 21 < \frac{5x}{7} - 6 \quad \text{and} \quad \frac{5x}{7} - 6 \leq -3\frac{3}{7} + x$$

$$\Rightarrow \quad 5x - \frac{5x}{7} < -6 + 21 \quad \text{and} \quad \frac{5x}{7} - x \leq -\frac{24}{7} + 6$$

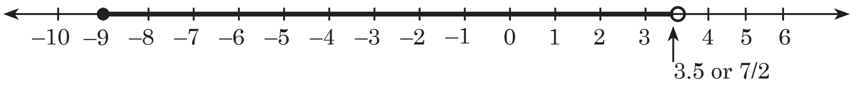
$$\Rightarrow \quad \frac{35x - 5x}{7} < 15 \quad \text{and} \quad \frac{5x - 7x}{7} \leq \frac{-24 + 42}{7}$$

$$\Rightarrow \quad 30x < 105 \quad \text{and} \quad -2x \leq 18$$

$$\Rightarrow \quad x < \frac{7}{2} \quad \text{and} \quad x \geq -9$$

Thus, the solution set is: $\left\{x : -9 \leq x < \frac{7}{2}, x \in R\right\}$.

The above solution can be represented on the real number line as:



(*iii*) We have, L.H.S. = (sin θ + cos θ) (cosec θ – sec θ)

$$= (\sin\theta + \cos\theta)\left(\frac{1}{\sin\theta} - \frac{1}{\cos\theta}\right)$$

$$= (\sin\theta + \cos\theta)\left(\frac{\cos\theta - \sin\theta}{\sin\theta.\cos\theta}\right)$$

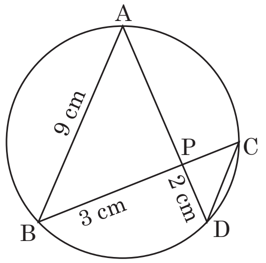
$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta.\cos\theta} = \frac{1 - 2\sin^2\theta}{\sin\theta.\cos\theta}$$

$$= \frac{1}{\sin\theta.\cos\theta} - \frac{2\sin^2\theta}{\sin\theta.\cos\theta}$$

$$= \operatorname{cosec}\theta.\sec\theta - 2\tan\theta = \text{R.H.S.}$$

Question 5

(*i*) In the given figure (drawn not to scale) chords AD and BC intersect at P, where AB = 9 cm, PB = 3 cm and PD = 2 cm.



(*a*) Prove that ΔAPB ~ ΔCPD.

(*b*) Find the length of CD.

(*c*) Find area ΔAPB : area ΔCPD. (3)

(*ii*) Mr. Sameer has a recurring deposit account and deposits ₹ 600 per month for 2 years. If he gets ₹ 15,600 at the time of maturity, find the rate of interest earned by him. (3)

(*iii*) Using step-deviation method, find mean for the following frequency distribution.

Class	0–15	15–30	30–45	45–60	60–75	75–90
Frequency	3	4	7	6	8	2

(4)

Solution:

(*i*) (*a*) In ΔAPB and ΔCPD, we have

∠BAP = ∠DCP [Angles in the same segment]

∠ABP = ∠CDP [Angles in the same segment]

∴ By AA Similarity Criterion, ΔAPB ~ ΔCPD.

(*b*) In ΔAPB and ΔCPD,

$$\frac{AB}{CD} = \frac{PB}{PD} \Rightarrow \frac{9}{CD} = \frac{3}{2} \Rightarrow CD = 6 \text{ cm}$$

(*c*) Since ΔAPB ~ ΔCPD, we have

$$\frac{ar(\Delta APB)}{ar(\Delta CPD)} = \frac{PB^2}{PD^2} = \frac{(3)^2}{(2)^2} = \frac{9}{4}$$

∴ area of ΔAPB : area of ΔCPD = 9 : 4.

(*ii*) Here, monthly deposit (P) = ₹ 600, time (n) = 2 years, *i.e.*, 24 months and maturity value (M.V.) = ₹ 15,600.

Let r% p.a. be the rate of interest.

Interest (I) on a R. D. is calculated by the formula, $I = \frac{P \times n \times (n+1)}{2} \times \frac{r}{100} \times \frac{1}{12}$

∴ Interest (I) = ₹ $\frac{600 \times 24 \times 25}{2} \times \frac{r}{100} \times \frac{1}{12}$ = ₹ $150r$

Maturity value (M.V.) = ₹ 15600

⇒ $600 \times 24 + 150r = 15600$ [∵ M.V. = P × n + I]

⇒ $150r = 15600 - 14400 \Rightarrow r = \frac{1200}{150} = 8\%.$

Thus, rate of interest earned by Sameer on his R.D. is 8% p.a.

(*iii*) To compute the mean from step-deviation method, we prepare the following table:

Here, assumed mean (A) = 52.5 and class size (h) = 15.

Class Interval	*Class Mark* (x_i)	*Step Deviation* $\left(u_i = \frac{x_i - A}{h}\right)$	*Frequency* (f_i)	*Product* ($f_i u_i$)
0–15	7.5	–3	3	–9
15–30	22.5	–2	4	–8
30–45	37.5	–1	7	–7
45–60	52.5	0	6	0
60–75	67.5	1	8	8
75–90	82.5	2	2	4
Total			$\Sigma f_i = 30$	$\Sigma f_i u_i = -12$

Therefore, mean $(\bar{x}) = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 52.5 + \frac{-12}{30} \times 15$

$= 52.5 - 6 = 46.5$

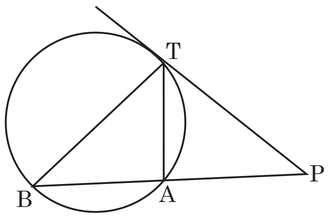
Question 6

(*i*) Find the coordinates of the centroid P of the ΔABC, whose vertices are A(–1, 3), B(3, –1) and C(0, 0). Hence, find the equation of a line passing through P and parallel to AB. (3)

(*ii*) In the given figure, PT is a tangent to the circle. Chord BA produced meets the tangent PT at P. Given PT = 20 cm and PA = 16 cm.

(*a*) Prove ΔPTB – ΔPAT.

(*b*) Find the length of AB. (3)



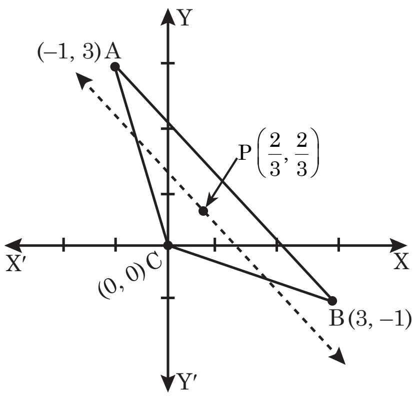
(*iii*) The following bill shows the GST rate and the marked price of articles:

Rajdhani Departmental Store				
S.No.	*Item*	*Marked Price*	*Discount*	*Rate of GST*
(*a*)	Dry fruits (1 kg)	₹ 1200	₹ 100	12%
(*b*)	Packed wheat flour (5 kg)	₹ 286	Nil	5%
(*c*)	Bakery products	₹ 500	10%	12%

Find the total amount to be paid (including GST) for the above bill. (4)

Solution:

(*i*) Coordinates of centroid = $P\left(\frac{-1+3+0}{3}, \frac{3+(-1)+0}{3}\right) = P\left(\frac{2}{3}, \frac{2}{3}\right)$



Slope (m) of line AB $= \frac{-1-3}{3-(-1)} = \frac{-4}{4} = -1$

Since the required line is parallel to AB, therefore
Slope of required line = Slope of line AB = –1

Also, the required line passes though $P\left(\frac{2}{3}, \frac{2}{3}\right)$.

$\therefore$ Equation of the required line is $y - \frac{2}{3} = -1\left(x - \frac{2}{3}\right)$

$\Rightarrow$ $3x + 3y = 4$.

(*ii*) (*a*) In ΔPTB and ΔPAT,

$\angle PTA = \angle PBT$ [By Alternate Segment Theorem]

$\angle TPA = \angle BPT$ [Common angle]

$\therefore$ By AA Similarity Criterion, $\Delta PTB \sim \Delta PAT$.

(*b*) Using the Theorem, $PA \times PB = PT^2$, we have

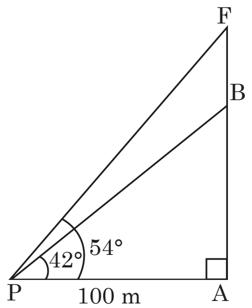
$16(16 + AB) = 400 \Rightarrow 16 + AB = 25 \Rightarrow AB = 9$ cm.

(*iii*) Here, we first calculate discounted prices and then GST collected on given items as shown in the table below:

Rajdhani Departmental Store					
S.No.	*Item*	*Marked Price*	*Discounted Price*	*Rate of GST*	*GST Collected*
(*a*)	Dry Fruits (1 kg)	₹ 1200	₹ 1100	12%	$\frac{12 \times 1100}{100}$ = ₹ 132.00
(*b*)	Wheat Flour (5 kg)	₹ 286	₹ 286	5%	$\frac{5 \times 286}{100}$ = ₹ 14.30
(*c*)	Bakery Products	₹ 500	₹ 450	12%	$\frac{12 \times 450}{100}$ = ₹ 54.00
	Total		₹ 1836		₹ 200.30

$\therefore$ Total amount to be paid (including GST) = ₹ 1836 + ₹ 200.30 = ₹ 2036.30

Question 7



(*i*) A vertical tower standing on a horizontal plane is surmounted by a vertical flagstaff. At a point 100 m away from the foot of the tower, the angle of elevation of the top and bottom of the flagstaff are 54° and 42° respectively. Find the height of the flagstaff. Give your answer correct to nearest metre. (5)

(*ii*) The marks of 200 students in a test were recorded as follows:

Marks %	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
No. of Students	5	7	11	20	40	52	36	15	9	5

Using graph sheet draw ogive for the given data and use it to find the,

(*a*) median.
(*b*) number of students who obtained more than 65% marks.
(*c*) number of students who did not pass, if the pass percentage was 35. (5)

Solution:

(*i*) (*a*) In the given figure, AB is a vertical tower and FB is a flagstaff on it. P is a point 100 m away from the foot of the tower. Also, ∠APF and ∠APB are the angles of elevation of the top and bottom of the flagstaff.

Now, in right ΔPAB, we have

$$\frac{AB}{PA} = \tan 42°$$

$$\Rightarrow \quad \frac{AB}{100} = 0.9004 \qquad [\because \tan 42° = 0.9004]$$

$$\Rightarrow \quad AB = 100 \times 0.9004 = 90.04 \text{ m}$$

Also, in right ΔPAF, we have

$$\frac{AF}{PA} = \tan 54°$$

$$\Rightarrow \quad \frac{AF}{100} = 1.3764 \qquad [\because \tan 54° = 1.3764]$$

$$\Rightarrow \quad AF = 100 \times 1.3764 = 137.64 \text{ m}$$

$$\therefore \quad FB = AF - AB = 137.64 \text{ m} - 90.04 \text{ m} = 47.60 \text{ m} \approx 48 \text{ m}$$

Thus, height of the flagstaff (correct to the nearest metre) is 48 m.

(*ii*) To draw an ogive for the given data, we prepare the following cumulative frequency table:

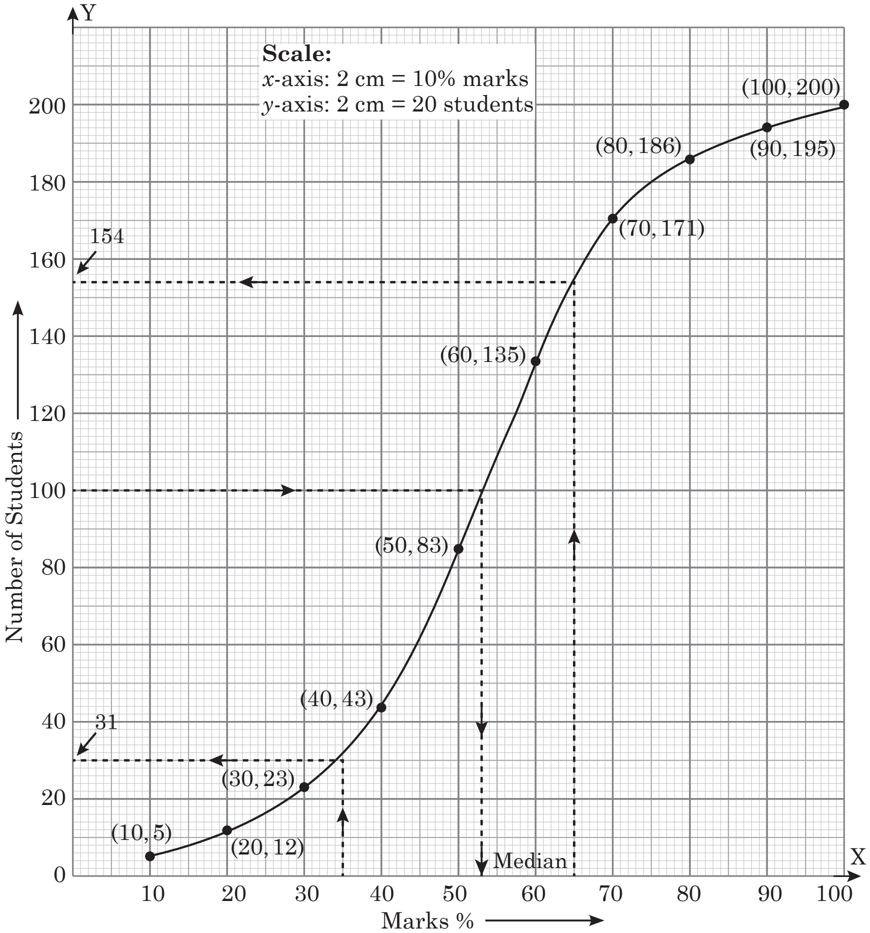
Marks (%)	*Frequency (f)*	*Cumulative Frequency (cf)*
0–10	5	5
10–20	7	12
20–30	11	23
30–40	20	43
40–50	40	83
50–60	52	135
60–70	36	171
70–80	15	186
80–90	9	195
90–100	5	200

On the basis of the adjoining ogive, we have

(*a*) Median marks (in %) = 53.

(*b*) Number of students who obtained more than 65% marks = 200 – 154 = 46.

(*c*) Number of students who didn't pass = 31.



Question 8

(*i*) In a TV show, a contestant opts for video call a friend life line to get an answer from three of his friends—named Amar, Akbar and Anthony. The question which he asks from one of his friends has four options. Find the probability that:

(*a*) Akbar is chosen for the call.

(*b*) Akbar couldn't give the correct answer. (3)

(*ii*) If x, y and z are in continued proportion, prove that:

$$\frac{x}{y^2 \cdot z^2} + \frac{y}{x^2 \cdot z^2} + \frac{z}{x^2 \cdot y^2} = \frac{1}{x^3} + \frac{1}{y^3} + \frac{1}{z^3}.$$

(3)

(*iii*) A manufacturing company prepares spherical ball bearings, each of radius 7 mm and mass 4 gm. These ball bearings are packed into boxes. Each box can have maximum of 2156 cm^3 of ball bearings. Find the:

(*a*) maximum number of ball bearings that each box can have.

(*b*) mass of each box of ball bearings in kg. $\left(\text{Use } \pi = \frac{22}{7}\right)$ (4)

Solution:

(*i*) From the three friends, there is only one way of chosing Akbar for video call.

(*a*) So, P(Akbar) $= \frac{1}{3}$.

(*b*) Out of 4 options, only 1 option is correct. Therefore,

P(correct answer) = $\frac{1}{4}$

So, P(not correct answer) = $1 - \frac{1}{4} = \frac{3}{4}$.

(*ii*) If x, y, z are in continued proportion, we have

$$\frac{x}{y} = \frac{y}{z} \Rightarrow y^2 = xz$$

Now,

$$\text{L.H.S.} = \frac{x}{y^2 \cdot z^2} + \frac{y}{z^2 \cdot x^2} + \frac{z}{x^2 \cdot y^2} = \frac{x^3 + y^3 + z^3}{x^2 y^2 z^2}$$

$$= \frac{x^3 + y^3 + z^3}{x^3 z^3} \qquad [\because y^2 = xz]$$

$$= \frac{x^3}{x^3 z^3} + \frac{y^3}{x^3 z^3} + \frac{z^3}{x^3 z^3}$$

$$= \frac{1}{z^3} + \frac{y^3}{y^6} + \frac{1}{x^3} \qquad [\because x^3 z^3 = y^6]$$

$$= \frac{1}{z^3} + \frac{1}{y^3} + \frac{1}{x^3} = \text{R.H.S.}$$

(*iii*) (*a*) Number of ball bearings in each box = $\frac{\text{Max. volume a box can have}}{\text{Volume of 1 ball bearing}}$

$$= \frac{2156}{\frac{4}{3} \times \pi \times r^3} = \frac{2156}{\frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{10}\right)^3}$$

$$= \frac{2156 \times 3 \times 7 \times 10 \times 10 \times 10}{4 \times 22 \times 7 \times 7 \times 7} = 1500.$$

(*b*) Mass of each box = Number of ball bearings in the box × Mass of each ball bearing

= 1500 × 4 gm = 6000 gm or 6 kg.

Question 9

(*i*) The table given below shows the runs scored by a cricket team during the overs of a match.

Overs	*Runs Scored*
20–30	37
30–40	45
40–50	40
50–60	60
60–70	51
70–80	35

Use graph sheet for this question.

Take 2 cm = 10 overs along one axis and 2 cm = 10 runs along the other axis.

(*a*) Draw a histogram representing the above distribution.

(*b*) Estimate the modal runs scored. (3)

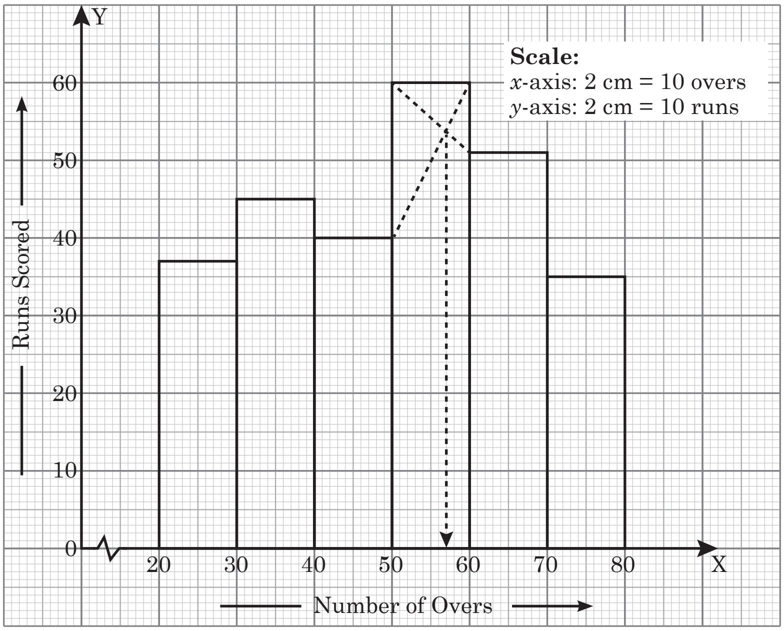
(*ii*) An Arithmetic Progression (A.P.) has 3 as its first term. The sum of the first 8 terms is twice the sum of the first 5 terms. Find the common difference of the A.P. (3)

(*iii*) The roots of equation $(q - r)\, x^2 + (r - p)\, x + (p - q) = 0$ are equal.

Prove that: $2q = p + r$, that is, p, q and r are in A.P. (4)

Solution:

(*i*) (*a*) For the given distribution, the required histogram is drawn below.



(*b*) From the above histogram the model runs scored is 57.

(*ii*) Given, first term, $a = 3$, $S_8 = 2\ S_5$ and common difference, $d = ?$

Now, $$S_8 = 2\ S_5$$

$\Rightarrow$ $$\frac{8}{2}[2 \times 3 + (8-1)d] = 2 \times \frac{5}{2}[2 \times 3 + (5-1)d] \qquad \left[\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right]$$

$\Rightarrow$ $$4[6 + 7d] = 5[6 + 4d]$$

$\Rightarrow$ $$24 + 28d = 30 + 20d$$

$\Rightarrow$ $$8d = 6 \text{ or } d = \frac{3}{4}$$

Thus, common difference of the given A.P. is $\frac{3}{4}$.

(*iii*) Here, $a = q - r$, $b = r - p$ and $c = p - q$.

For equal roots, $b^2 = 4ac \Rightarrow (r-p)^2 = 4(q-r)(p-q)$

$\Rightarrow$ $$r^2 + p^2 - 2pr = 4(pq - q^2 - pr + qr)$$

$\Rightarrow$ $$r^2 + p^2 - 2pr + 4pr = 4(pq - q^2 + qr)$$

$\Rightarrow$ $$(p + r)^2 = 4[q(p + r) - q^2]$$

$\Rightarrow$ $$(p + r)^2 - 4q(p + r) + 4q^2 = 0$$

Putting $(p + r) = y$, the above equation reduces to

$$y^2 - 4qy + 4q^2 = 0$$

$\Rightarrow$ $$(y - 2q)^2 = 0$$

$\Rightarrow$ $$y - 2q = 0 \quad \text{or} \quad p + r = 2q \qquad [\because y = p + r].$$

Hence, $2q = p + r$, that is, p, q and r are in A.P.

Question 10

(*i*) A car travels a distance of 72 km at a certain average speed of x km per hour and then travels a distance of 81 km at an average speed of 6 km per hour more than its original average speed. If it takes 3 hours to complete the total journey, then form a quadratic equation and solve it to find its original average speed. (3)

(*ii*) Given matrix, $X = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ prove that $X^2 = 4X + 5I$. (3)

(*iii*) Use ruler and compasses for the following question taking a scale of 10 m = l cm.

A park in a city is bounded by straight fences AB, BC, CD and DA.

Given that AB = 50 m, BC = 63 m, ∠ABC = 75°. D is a point equidistant from the fences AB and BC. If ∠BAD = 90°, construct the outline of the park ABCD. Also, locate a point P on the line BD for the flag post which is equidistant from the corners of the park A and B. (4)

Solution:

(*i*) Let the original average speed of the car be x km/h. Then,

Time taken to cover 72 km with average speed $= \frac{72}{x}$ hours

Time taken to cover 81 km with increased speed $= \frac{81}{x+6}$ hours

According to problem, $\frac{72}{x} + \frac{81}{x+6} = 3$

$$\Rightarrow \frac{24}{x} + \frac{27}{x+6} = 1 \Rightarrow \frac{24(x+6)+27x}{x(x+6)} = 1$$

$$\Rightarrow 51x + 144 = x^2 + 6x$$

$$\Rightarrow x^2 - 45x - 144 = 0$$

$$\Rightarrow (x - 48)(x + 3) = 0$$

$$\Rightarrow x = 48 \quad \text{or} \quad x = -3$$

$$\Rightarrow x = 48 \qquad [\because x = -3 \text{ is not possible.}]$$

Thus, original average speed of the car is 48 km/h.

(*ii*) We have, $X^2 = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 1\times1+1\times8 & 1\times1+1\times3 \\ 8\times1+3\times8 & 8\times1+3\times3 \end{bmatrix} = \begin{bmatrix} 1+8 & 1+3 \\ 8+24 & 8+9 \end{bmatrix}$

$$\therefore \quad X^2 = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} \qquad \text{...(1)}$$

Also, $4X = 4\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$ and $5I = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

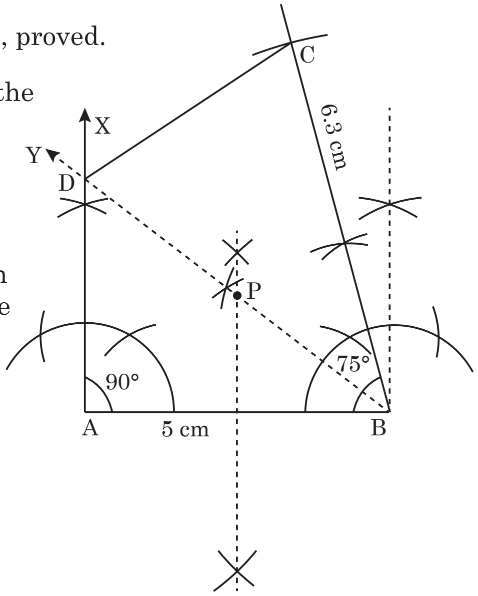
$$\therefore 4X + 5I = \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} \qquad \text{...(2)}$$

Thus, from (1) and (2) $X^2 = 4X + 5I$ Hence, proved.

(*iii*) Construction of the outline of the park ABCD is drawn in the figure alongside.

Point D is equidistant from the fences AB and BC, so it will be the point of intersection of ray AX and bisector BY of ∠ABC.

Point P is equidistant from the corners A and B and lies on the line BD, so it will be the point of intersection of the line BD and perpendicular bisector of AB.



ICSE 2024 SPECIMEN QUESTIONS WITH SOLUTIONS

Question 1

Choose the correct answers to the questions from the given options: (15)

(*i*) If A = [–1 2] and B = $\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$.

Which of the following operation is possible?

(*a*) A – B. (*b*) A + B. (*c*) AB. (*d*) BA.

(*ii*) If $x^2 + kx + 6 = (x - 2)(x - 3)$ for all values of x, then the value of k is

(*a*) – 5. (*b*) – 3. (*c*) – 2. (*d*) 5.

(*iii*) A retailer purchased an item for ₹ 1,500 from a wholesaler and sells it to a customer at 10% profit. The sales are intra-state and the rate of GST is 10%.

The amount of GST paid by the customer is

(*a*) ₹ 15. (*b*) ₹ 30. (*c*) ₹ 150. (*d*) ₹ 165.

(*iv*) If the roots of equation $x^2 - 6x + k = 0$ are real and distinct, then value of k is

(*a*) > – 9. (*b*) > – 6. (*c*) < 6. (*d*) < 9.

(*v*) Which of the following is/are an Arithmetic Progression (A.P.)?

1. 1, 4, 9, 16, ...
2. $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}, \ldots$
3. 8, 6, 4, 2, ...

(*a*) Only 1. (*b*) Only 2. (*c*) Only 2 and 3. (*d*) All 1, 2 and 3.

(*vi*) The table shows the values of x and y, where x is proportional to y.

x	6	12	N
y	M	18	6

What are the values of M and N?

(*a*) M = 4, N = 9. (*b*) M = 9, N = 3. (*c*) M = 9, N = 4. (*d*) M = 12, N = 0.

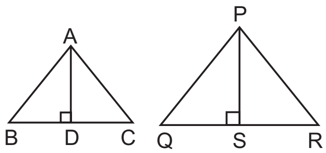
(*vii*) In the given diagram, ΔABC ~ ΔPQR and $\frac{AD}{PS} = \frac{3}{8}$. The value of AB : PQ is

(*a*) 8 : 3.

(*b*) 3 : 5.

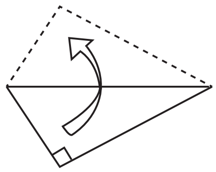
(*c*) 3 : 8.

(*d*) 5 : 8.



(*viii*) A right angle triangle shaped piece of hard board is rotated completely about its hypotenuse, as shown in the diagram. The solid so formed is always

1. a single cone 2. a double cone



Which of the statement is valid?

(*a*) Only 1. (*b*) Only 2.

(*c*) Both 1 and 2. (*d*) Neither 1 nor 2.

(*ix*) **Event A:** The sun will rise from east tomorrow.

Event B: It will rain on Monday.

Event C: February month has 29 days in a leap year.

Which of the above event(s) has probability equal to 1?

(*a*) All events A, B and C. (*b*) Both events A and B.

(*c*) Both events B and C. (*d*) Both events A and C.

(*x*) The three vertices of a scalene triangle are always equidistant from a fixed point. The point is:

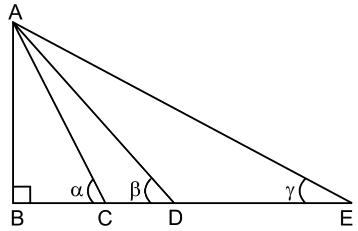
(*a*) Orthocentre of the triangle. (*b*) Incentre of the triangle.

(*c*) Circumcentre of the triangle. (*d*) Centroid of the triangle.

(*xi*) In a circle with radius R, the shortest distance between two parallel tangents is equal to

(*a*) R (*b*) 2R (*c*) 2πR (*d*) πR

(*xii*) An observer at point E, which is at a certain distance from the lamp post AB, finds the angle of elevation of top lamp post from positions C, D and E as α, β and γ. It is given that B, C, D and E are along a straight line.



Which of the following condition is satisfied?

(*a*) $\tan \alpha > \tan \beta$

(*b*) $\tan \beta < \tan \gamma$

(*c*) $\tan \gamma > \tan \alpha$

(*d*) $\tan \alpha < \tan \beta$

(*xiii*) 1. Shares of company A, paying 12%, ₹ 100 shares are at ₹ 80.

2. Shares of company B, paying 12%, ₹ 100 shares are at ₹ 100.

3. Shares of company C, paying 12%, ₹ 100 shares are at ₹ 120.

Shares of which company are at premium?

(*a*) Company A (*b*) Company B

(*c*) Company C (*d*) Companies A and C

(*xiv*) Which of the following equations represent a line passing through origin?

(*a*) $3x - 2y + 5 = 0$ (*b*) $2x - 3y = 0$ (*c*) $x = 5$ (*d*) $y = -6$

(*xv*) For the given 25 variables: $\boldsymbol{x_1, x_2, x_3 \ldots \ldots \ldots \ldots x_{25}}$

Assertion (A): To find median of the given data, the variate needs to be arranged in ascending or descending order.

Reason (R): The median is the central most term of the arranged data.

(*a*) A is true, R is false. (*b*) A is false, R is true.

(*c*) Both A and R are true. (*d*) Both A and R are false.

Solution

(*i*) (*c*) Matrix A is of order 1 × 2, Matrix B is of order 2 × 2. Since number of columns in Matrix A is same as the number of rows in Matrix B, Matrix AB is defined.

(*ii*) (*a*) Given, $x^2 + kx + 6 = (x - 2)(x - 3)$

$\Rightarrow \quad x^2 + kx + 6 = x^2 - 5x + 6$

$\Rightarrow \quad k = -5.$ [Equating coefficients of x on both sides]

(*iii*) (*d*) Sale price of the item = ₹ 1500 + 10% of ₹ 1500

= ₹ 1500 + ₹ 150 = ₹ 1650

∴ Amount of GST = 10% of ₹ 1650 = ₹ 165.

(*iv*) (*d*) Comparing the given equation $x^2 - 6x + k = 0$ with the general quadratic equation $ax^2 + bx + c = 0$, we have $a = 1$, $b = -6$, $c = k$.

Roots of the equation will be real and distinct, if $b^2 - 4ac > 0$.

$\Rightarrow \quad (-6)^2 - 4(1)(k) > 0$

$\Rightarrow \quad 4k < 36 \text{ or } k < 9.$

(*v*) (*c*) Here, $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$ is an A.P., with $a = \sqrt{3}$ and $d = \sqrt{3}$

Also, 8, 6, 4, 2, ... is an A.P., with $a = 8$ and $d = -2$.

But 1, 4, 9, 16, ... is not an A.P., as $4 - 1 \neq 9 - 4$.

(*vi*) (*c*) Since x is proportional to y, we have

$$\frac{6}{M} = \frac{12}{18} = \frac{N}{6} \Rightarrow \frac{6}{M} = \frac{2}{3} \text{ and } \frac{N}{6} = \frac{2}{3}$$

$$\Rightarrow M = 9 \text{ and } N = 4.$$

(*vii*) (*c*) Since $\Delta ABC \sim \Delta PQR$,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

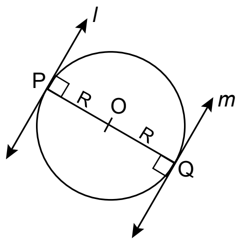
$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PS} = \frac{3}{8} \text{ or } AB : PQ = 3 : 8.$$

(*viii*) (*b*) When rotated about hypotenuse, one cone is generated along the perpendicular and another along the base of the right triangular board.

(*ix*) (*d*) Event A and C are sure. So, their probability is equal to 1.

The event B is not sure. So, its probability is not equal to 1.

(*x*) (*c*) By the definition of orthocentre of a triangle.



(*xi*) (*b*) Clearly, $OP \perp l$ and $OQ \perp m$

and O lies on PQ $\Rightarrow$ PQ is a diameter of the circle.

Hence, PQ = 2R.

(*xii*) (*a*) Here, $\tan\alpha = \frac{AB}{BC}$, $\tan\beta = \frac{AB}{BD}$ and $\tan\gamma = \frac{AB}{BE}$

Since $\quad BC < BD < BE,$

$\Rightarrow \quad \tan\alpha > \tan\beta > \tan\gamma.$

(*xiii*) (*c*) Shares of company A are available at a discount of ₹ 20.

Shares of company B are available at par.

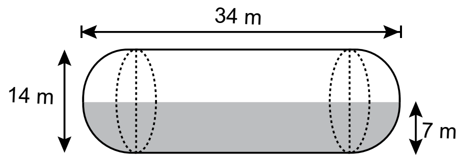
Shares of company C are available at a premium of ₹ 20.

(*xiv*) (*b*) Equation $2x - 3y = 0$ represents a line passing through the origin (0, 0), as $2(0) - 3(0) = 0$.

(*xv*) (*c*)

Question 2

(*i*) Shown alongside is a horizontal water tank composed of a cylinder and two hemispheres. The tank is filled up to a height of 7 m. Find the surface area of the tank in contact with water. Use $\pi = \frac{22}{7}$. (4)



(*ii*) In a recurring deposit account for 2 years, the total amount deposited by a person is ₹ 9,600. If the interest earned by him is one-twelfth of his total deposit, then find:
(*a*) the interest he earns. (*b*) his monthly deposit. (*c*) the rate of interest. (4)

(*iii*) Find:
(*a*) $(\sin\theta + \operatorname{cosec}\theta)^2$ (*b*) $(\cos\theta + \sec\theta)^2$
Using the above results prove the following trigonometric identity.
$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$ (4)

Solution

(*i*) From the given figure, we have

Radius of the cylindrical portion (r) = Radius of the two hemispherical ends (r) = 7 m

Length of the cylindrical portion (h) = 34 – (2 × 7) = 20 m

$$\text{Surface area of the whole tank} = 2\pi rh + 2(2\pi r^2)$$
$$= 2\pi r(h + 2r)$$
$$= 2 \times \frac{22}{7} \times 7\,(20 + 2 \times 7) = 44 \times 34 = 1496 \text{ sq m}$$

Surface area of the tank in contact with water

$$= \frac{1}{2} \times \text{Surface area of the whole tank}$$
$$= \frac{1}{2} \times 1496 \text{ sq m} = 748 \text{ sq m.}$$

(*ii*) Here, n = 2 years, *i.e.,* = 24 months and total amount deposited = ₹ 9,600.

(*a*) Given, Interest earned = $\frac{1}{12}$ of total deposit = $\frac{1}{12} \times 9600$ = ₹ 800

Thus, the interest he earns is ₹ 800.

(*b*) Let the amount of monthly deposit be ₹ P. Then,

Total amount deposited = ₹ 9600

$$\Rightarrow \quad P \times n = 9600$$
$$\Rightarrow \quad P \times 24 = 9600 \quad \Rightarrow \quad P = 400$$

Thus, his monthly deposit is ₹ 400.

(*c*) If r% denotes the rate of interest, then

$$\text{Interest (I)} = 400 \times \frac{24 \times 25}{2 \times 12} \times \frac{r}{100} \qquad \left[\because I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}\right]$$
$$\Rightarrow \quad 800 = 400 \times \frac{r}{4} \quad \Rightarrow \quad r = 8\%$$

Thus, the rate of interest of the R.D. is 8% p.a.

(*iii*) (*a*)
$$(\sin\theta + \operatorname{cosec}\theta)^2 = \sin^2\theta + 2\sin\theta\operatorname{cosec}\theta + \operatorname{cosec}^2\theta$$
$$= \sin^2\theta + 2 + \operatorname{cosec}^2\theta$$

(*b*)
$$(\cos\theta + \sec\theta)^2 = \cos^2\theta + 2\cos\theta\sec\theta + \sec^2\theta$$
$$= \cos^2\theta + 2 + \sec^2\theta$$

Now,
$$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = [\sin^2\theta + 2 + \operatorname{cosec}^2\theta] + [\cos^2\theta + 2 + \sec^2\theta]$$
$$= (\sin^2\theta + \cos^2\theta) + 4 + (\operatorname{cosec}^2\theta + \sec^2\theta)$$
$$= 5 + (\operatorname{cosec}^2\theta + \sec^2\theta) \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$
$$= 5 + [(1 + \cot^2\theta) + (1 + \tan^2\theta)]$$
$$= 7 + \tan^2\theta + \cot^2\theta.$$

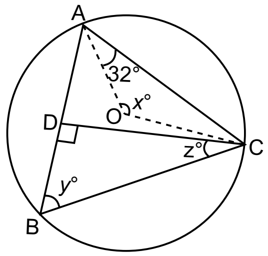
Hence, $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$.

Question 3

(*i*) If a, b and c are in continued proportion, then prove that:

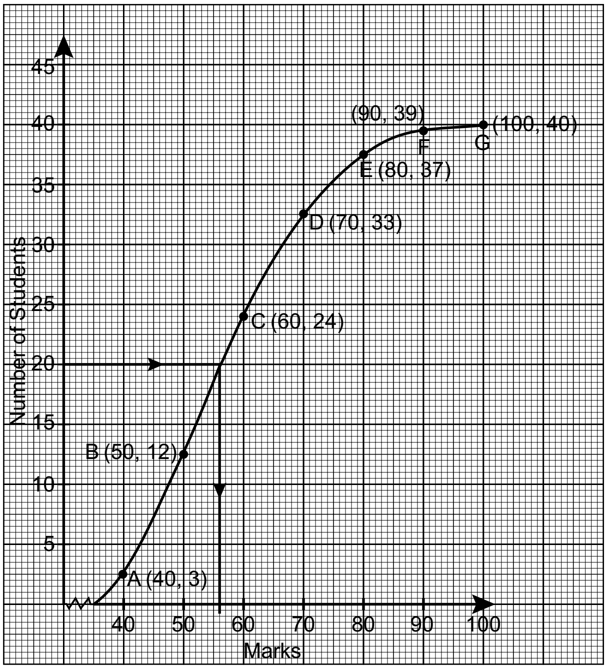
$$\frac{3a^2 + 5ab + 7b^2}{3b^2 + 5bc + 7c^2} = \frac{a}{c}. \quad (4)$$

(*ii*) In the given diagram, O is the centre of circle circumscribing the ΔABC. CD is perpendicular to chord AB. $\angle$OAC = 32°. Find each of the unknown angles x, y and z. (4)



(*iii*) Study the graph and answer each of the following.

(*a*) Name the curve plotted.
(*b*) Total number of students.
(*c*) The median marks.
(*d*) Number of students scoring between 50 and 80 marks. (5)



Solution

(*i*) Since a, b and c are in continued proportion, we have

$$\frac{a}{b} = \frac{b}{c} = k \text{ (say)}$$

$\Rightarrow$ $a = bk$ and $b = ck$

$\Rightarrow$ $a = ck^2$ and $b = ck$

Now, $$\frac{3a^2 + 5ab + 7b^2}{3b^2 + 5bc + 7c^2} = \frac{3c^2k^4 + 5c^2k^3 + 7c^2k^2}{3c^2k^2 + 5c^2k + 7c^2}$$

$$= \frac{k^2(3c^2k^2 + 5c^2k + 7c^2)}{3c^2k^2 + 5c^2k + 7c^2} = k^2 = \frac{a}{c} \qquad [\because a = ck^2]$$

Hence, $$\frac{3a^2 + 5ab + 7b^2}{3b^2 + 5bc + 7c^2} = \frac{a}{c}.$$

(*ii*) Here, in ΔAOC, we have

$\angle$OCA = $\angle$OAC = 32° [$\because$ OA = OC = Radii of the same circle]

Now, $\angle$OAC + $\angle$AOC + $\angle$OCA = 180°

$32° + x° + 32° = 180° \Rightarrow 64° + x° = 180° \Rightarrow x° = 180° - 64°$

$\Rightarrow x° = 116°$

Further, $\angle ABC = \frac{1}{2}\angle AOC$ [The angle which the arc AC of the circle subtends at the centre O is double that which it subtends at B of the circle.]

$\Rightarrow \quad y° = \frac{1}{2}x°$

$\Rightarrow \quad y° = \frac{1}{2} \times 116° = 58°$

Also, in right triangle BDC, $y° + z° = 90°$

$\Rightarrow \quad z° = 90° - y° \quad \Rightarrow \quad z° = 90° - 58° = 32°$

Hence, $x = 116$, $y = 58$ and $z = 32$ are the required values.

(*iii*) (*a*) Cumulative frequency curve or ogive

(*b*) 40 students

(*c*) 56 marks

(*d*) 37 – 12, *i.e.,* 25 students.

Question 4

(*i*) If $A = \begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix}$, find A^2. If $A^2 = p$ A, then find the value of p. (3)

(*ii*) Solve the given equation $x^2 - 4x - 2 = 0$ and express your answer correct to two places of decimal. (You may use mathematical tables for this question.) (3)

(*iii*) In the given diagram, ΔABC is right-angled at B. BDFE is a rectangle.

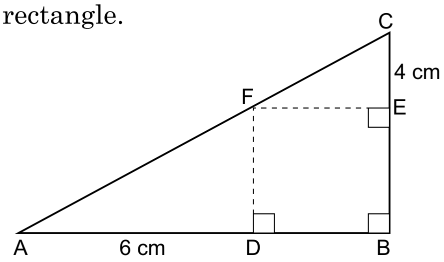
AD = 6 cm, CE = 4 cm and BC = 12 cm.

(*a*) Prove that ΔADF ~ ΔFEC.

(*b*) Prove that ΔADF ~ ΔABC.

(*c*) Find the length of FE.

(*d*) Find area ΔADF : area ΔABC. (4)



Solution

(*i*) Here,

$$A^2 = A \cdot A = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}\begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 32 & -32 \\ -32 & 32 \end{bmatrix}$$

So, $A^2 = pA$ gives

$$\begin{bmatrix} 32 & -32 \\ -32 & 32 \end{bmatrix} = p\begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 32 & -32 \\ -32 & 32 \end{bmatrix} = \begin{bmatrix} 4p & -4p \\ -4p & 4p \end{bmatrix}$$

$$\Rightarrow \quad 4p = 32 \quad \Rightarrow \quad p = 8.$$

(*ii*) The given equation is $x^2 - 4x - 2 = 0$.

So, using the quadratic formula, we have

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2 \times 1}$$

$$= \frac{4 \pm \sqrt{16+8}}{2} = \frac{4 \pm 2\sqrt{6}}{2}$$

$$= 2 \pm \sqrt{6} = 2 \pm 2.45 \quad \text{[Taking } \sqrt{6} = 2.45\text{]}$$

Thus, $x = 4.45$ or -0.45.

(*iii*) (*a*) In ∆s ADF and FEC, we have

$\angle A = \angle F$ [Corresponding angles]

and $\angle D = \angle E$ [Each 90°]

So, by AA Similarity Criterion, ∆ADF ~ ∆FEC.

(*b*) In ∆s ADF and ABC, we have

$\angle A = \angle A$ [Common angle]

$\angle D = \angle B$ [Each 90°]

So, by AA Similarity Criterion, ∆ADF ~ ∆ABC.

(*c*) Since ∆ADF ~ ∆FEC, we have

$$\frac{AD}{FE} = \frac{DF}{EC} \Rightarrow \frac{6}{FE} = \frac{BE}{4} \quad [\because DF = BE]$$

$$\Rightarrow \frac{6}{FE} = \frac{BC - CE}{4}$$

$$\Rightarrow \frac{6}{FE} = \frac{12-4}{4} \Rightarrow \frac{6}{FE} = 2 \Rightarrow FE = 3 \text{ cm.}$$

(*d*) Since ∆ADF ~ ∆ABC, we have

$$\frac{\text{area }(\Delta ADF)}{\text{area }(\Delta ABC)} = \frac{AD^2}{AB^2} \Rightarrow \frac{\text{area }(\Delta ADF)}{\text{area }(\Delta ABC)} = \frac{AD^2}{(AD+DB)^2} = \frac{6^2}{(6+3)^2} = \frac{36}{81} = \frac{4}{9}$$

Hence, area (∆ADF) : area (∆ABC) = 4 : 9.

Question 5

(*i*) Shown below is a table illustrating the monthly income distribution of a company with 100 employees.

Monthly Income (in ₹ 10,000)	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24
Number of Employees	55	15	6	8	12	4

Using step-deviation method, find the mean monthly income of an employee. (3)

(*ii*) The following bill shows the GST rate and the marked price of articles:

Vidhyut Electronics				
S.No.	*Item*	*Marked Price*	*Quantity*	*Rate of GST*
(*a*)	LED TV set	₹ 12,000	1	28%
(*b*)	MP4 player	₹ 5,000	1	18%

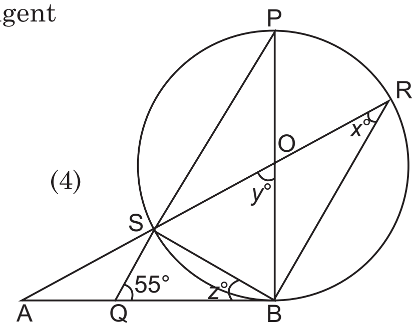
Find the total amount to be paid (including GST) for the above bill. (3)

(*iii*) In the given figure, O is the centre of the circle and AB is a tangent to the circle at B.

If ∠PQB = 55°,

(*a*) find the values of the angles *x*, *y* and *z*.

(*b*) prove that RB is parallel to PQ. (4)



Solution

(*i*) Let the assumed mean (*A*) be 14.

The frequency table for the calculation of mean is:

Monthly Income (in ₹ 10,000)	*Frequency* (f_i)	*Class Mark* (x_i)	*Step Deviation* $\left(u_i = \frac{x_i - A}{h}\right)$	*Product* (f_iu_i)
0 – 4	55	2	– 3	– 165
4 – 8	15	6	– 2	– 30
8 – 12	6	10	– 1	– 6
12 – 16	8	14	0	0
16 – 20	12	18	1	12
20 – 24	4	22	2	8
Total	$\Sigma f_i = 100$			$\Sigma f_iu_i = -181$

$$\therefore \quad \text{Mean} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 14 + \frac{-181}{100} \times 4 = 14 - 7.24 = 6.76$$

Thus, mean monthly income of an employee is ₹ 6.76 × 10000 = ₹ 67,600

(*ii*) Amount of GST on LED TV set = 28% of ₹ 12000 = ₹ 3360

∴ Total amount to be paid for LED TV set = ₹ (12000 + 3360) = ₹ 15360

Amount of GST on MP4 player = 18% of ₹ 5000 = ₹ 900

∴ Total amount to be paid for MP4 player = ₹ (5000 + 900) = ₹ 5900

So, total amount of bill to be paid = ₹ (15360 + 5900) = ₹ 21,260.

(*iii*) (*a*) In the figure, $\angle PSB = 90°$ [Angle in semicircle]

$\Rightarrow$ $\angle QSB = 90°$

Therefore, in ΔQSB, $z° = 180° - (90° + 55°) = 35°$

Now, $\angle BRS = \angle QBS$ [Angles in alternate segments]

$\Rightarrow$ $x° = z° = 35°$

Also, $\angle BOS = 2\angle BRS$ $\Rightarrow$ $y° = 2x° = 2 \times 35° = 70°$

Thus, $x = 35$, $y = 70$ and $z = 35$.

(*b*) We have, $\angle OBR = \angle ORB = x°$ $\Rightarrow$ $\angle PBR = x°$

and $\angle BPS = \angle BRS = x°$ [Angles in the same segment]

Thus, $\angle PBR = \angle BPS$ and these are alternate angles.

So, RB is parallel to PS, *i.e.*, RB is parallel to PQ.

Question 6

(*i*) There are three positive numbers in a Geometric Progression (G.P.) such that:

(*a*) their product is 3375;

(*b*) the result of the product of first and second number added to the product of second and third number is 750.

Find the numbers. (3)

(*ii*) The table given below shows the ages of members of a society.

Age (in years)	*Number of Members of the Society*
25 – 35	5
35 – 45	32
45 – 55	69
55 – 65	80
65 – 75	61
75 – 85	13

Use graph sheet for this question.

Take 2 cm = 10 years along one axis and 2 cm = 10 members along the other axis.

(*a*) Draw a histogram representing the above distribution.

(*b*) Hence; find the modal age of the members. (3)

(*iii*) A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 7 m each and the total height of the tent is 14 m. Find the:

(*a*) quantity of air contained inside the tent;

(*b*) radius of a sphere whose volume is equal to the quantity of air-inside the tent. $\left(\text{Use } \pi = \frac{22}{7}\right)$

(4)

Solution

(*i*) Let $\frac{a}{r}$, a and ar be any three positive numbers in the G.P. Then,

According to the question

$$\frac{a}{r} \times a \times ar = 3375 \quad \Rightarrow \quad a^3 = 3375 \qquad \ldots(1)$$

and $$\left(\frac{a}{r} \times a\right) + (a \times ar) = 750 \quad \Rightarrow \quad a^2\left(\frac{1}{r} + r\right) = 750 \qquad \ldots(2)$$

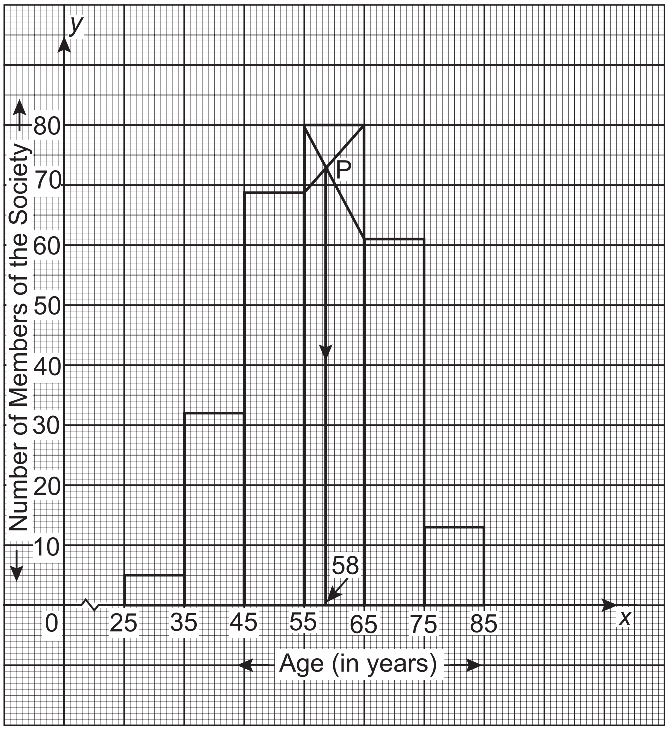
From (1), we get $$a = \sqrt[3]{3375} = 15$$

From (2), we get $$r + \frac{1}{r} = \frac{750}{225} = \frac{10}{3} \quad \Rightarrow \quad r + \frac{1}{r} = 3 + \frac{1}{3}$$

$$\Rightarrow \qquad r = 3$$

Thus, the three positive numbers are $\frac{15}{3}, 15, 15 \times 3$, *i.e.,* 5, 15, 45.

(*ii*) (*a*) The required histogram is represented in the following graph.



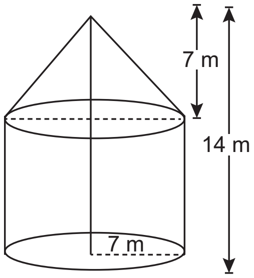
Scale:
x-axis 1 cm = 10 years
y-axis 1 cm = 10 members

Note : Scale is revised to represent the graph in permissible size.

(*b*) Perpendicular from P meets the *x*-axis at 58.

Therefore, modal age of the members of the society is 58 years.

(*iii*) (*a*) Quantity of air inside the tent



$$= \text{Volume of the tent}$$
$$= \text{Volume of cylindrical part + Volume of conical part}$$
$$= \pi r^2 h + \frac{1}{3}\pi r^2 h$$
$$= \pi (7)^2 (7) + \frac{1}{3}\pi (7)^2 (7) \text{ cu m}$$
$$= \frac{4\pi}{3} \times (7)^2 \times 7 \text{ cu m} = \frac{4312}{3} \text{ cu m}$$

(*b*) Let 'R' metre be the radius of the sphere. Then,

Volume of the sphere = Quantity of air inside the tent

$$\frac{4}{3}\pi R^3 = \frac{4312}{3}$$

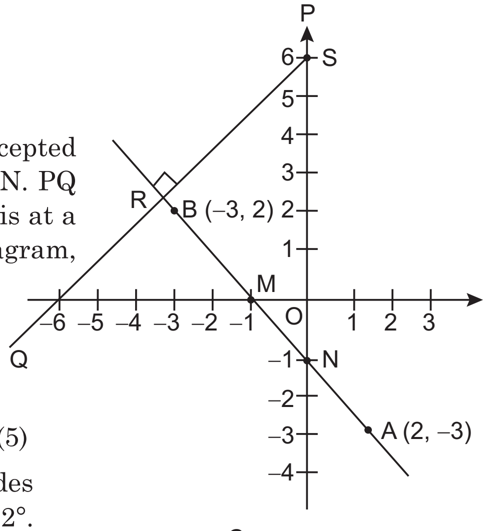
$\Rightarrow$ $$R^3 = \frac{4312}{3} \times \frac{3}{4\pi} = 343$$

or $$R = 7 \text{ m}$$

Thus, the radius of the sphere is 7 m.

Question 7

(*i*) The line segment joining A(2, –3) and B(–3, 2) is intercepted by the *x*-axis at the point M and the *y*-axis at the point N. PQ is perpendicular to AB produced at R and meets the *y*-axis at a distance of 6 units from the origin O, as shown in the diagram, at S. Find the:



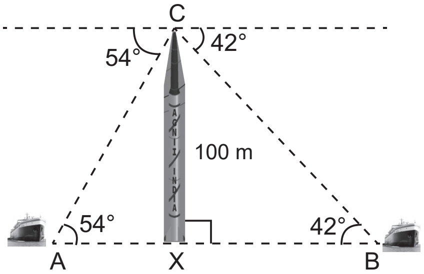
(*a*) coordinates of M and N

(*b*) coordinates of S

(*c*) slope of AB.

(*d*) equation of line PQ. (5)

(*ii*) The angle of depression of two ships A and B on opposite sides of a lighthouse of height 100 m are respectively 54° and 42°. The line joining the two ships passes through the foot of the lighthouse.



(*a*) Find the distance between the two ships A and B.

(*b*) Give your final answer correct to the nearest whole number.

(Use mathematical tables for this question.) (5)

Solution

(*i*) (*a*) Coordinates of M and N respectively are (– 1, 0) and (0, –1), *i.e.*, M(–1, 0) and N(0, –1).

(*b*) Point S lies on *y*-axis at a distance of 6 units from the origin. Therefore, coordinates of S are (0, 6) *i.e.*, S(0, 6).

(*c*) Slope of the line AB passing through A(2, –3) and B(–3, 2) $= \frac{2-(-3)}{-3-2} = -1.$

(*d*) Since line PQ is perpendicular to line AB, we have

$$\text{Slope of line PQ} = \frac{-1}{\text{Slope of line AB}} = -\left(\frac{1}{-1}\right) = 1$$

Also, line PQ passes through S(0, 6).

Hence, equation of line PQ is $y - 6 = 1(x - 0)$ *i.e.*, $x - y + 6 = 0$.

(*ii*) (*a*) In right triangle AXC, we have $\frac{CX}{AX} = \tan 54°$ $\Rightarrow$ $\frac{CX}{AX} = 1.3764$

$\Rightarrow$ $AX = \frac{100}{1.3764}$ $\Rightarrow$ $AX = 72.65$

In right triangle BXC, we have $\frac{CX}{XB} = \tan 42°$ $\Rightarrow$ $\frac{CX}{XB} = 0.9004$ [Using tables]

$\Rightarrow$ $XB = \frac{100}{0.9004}$ $\Rightarrow$ $XB = 111.06$

Hence, distance between two ships, AB = AX + XB = 72.65 + 111.06 = 183.71 metres

(*b*) Correct to the nearest whole number, the distance between two ships is 184 metres.

Question 8

(*i*) Solve the following inequation. Write the solution set and represent it on the real number line.

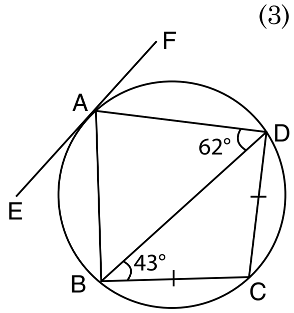
$$3 - 2x \geq x + \frac{1-x}{3} > \frac{2x}{5}, x \in R$$ (3)

(*ii*) ABCD is a cyclic quadrilateral in which BC = CD and EF is a tangent at A. ∠CBD = 43° and ∠ADB = 62°. Find:

(*a*) ∠ADC.

(*b*) ∠ABD.

(*c*) ∠FAD. (3)



(*iii*) A (*a*, *b*), B (–4, 3) and C (8, –6) are the vertices of a ΔABC. Point D is on BC such that BD : DC is 2 : 1 and M (6, 0) is mid-point of AD. Find:

(*a*) coordinates of point D;

(*b*) coordinates of point A;

(*c*) equation of a line passing through M and parallel to line BC. (4)

Solution

(*i*) Here, $3 - 2x \geq x + \frac{1-x}{3} > \frac{2x}{5}$

$\Rightarrow$ $45 - 30x \geq 15x + 5(1 - x) > 3(2x)$ [Multiplying the inequation by 15]

$\Rightarrow$ $45 - 30x \geq 15x + 5 - 5x > 6x$

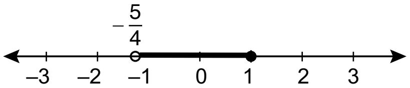
$\Rightarrow$ $45 - 30x \geq 10x + 5 > 6x$

$\Rightarrow$ $45 - 30x \geq 10x + 5$ and $10x + 5 > 6x$

$\Rightarrow$ $40x \leq 40$ and $4x > -5$

$\Rightarrow$ $x \leq 1$ and $x > -\frac{5}{4}$

Thus, solution set $= \left\{x : x \in R \text{ such that } \frac{-5}{4} < x \leq 1\right\}$.



(*ii*) In ΔBCD, we have BC = CD [Given]

So, ∠BDC = ∠DBC = 43°

Therefore, ∠BCD = 180° – (43° + 43°) = 94°

(*a*) In the figure, ∠ADC = ∠ADB + ∠BDC

= 62° + 43° = 105°.

(*b*) Since ABCD is a cyclic quadrilateral

$$\angle BAD + \angle BCD = 180°$$

$\Rightarrow$ $\angle BAD = 180° - 94° = 86°$ [$\because \angle BCD = 94°$]

Hence, in ΔABD, we have

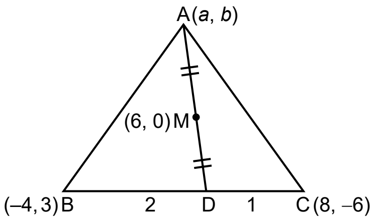
$$\angle ABD = 180° - (\angle BAD + \angle ADB)$$
$$= 180° - (86° + 62°)$$
$$= 180° - 148° = 32°.$$

(*c*) We have, $\angle FAD = \angle ABD = 32°$. [Angles in the alternate segments]

(*iii*) (*a*) Using section formula, coordinates of D are:

$$D\left(\frac{2\times 8 + 1\times(-4)}{2+1}, \frac{2\times(-6)+1\times 3}{2+1}\right), \text{ i.e. } D(4, -3)$$

Thus, the coordinates of point D are (4, –3).



(*b*) Since M (6, 0) is the mid-point of AD,

$$6 = \frac{a+4}{2}, 0 = \frac{b-3}{2}$$

$\Rightarrow$ $a = 8$ and $b = 3$

Thus, the coordinates of point A are (8, 3).

(*c*) Slope of the line BC $= \frac{-6-3}{8-(-4)} = \frac{-9}{12} = -\frac{3}{4}$

Since the required line is parallel to BC, therefore slope of the required line $= -\frac{3}{4}$

So, the equation of the required line is

$$y - 0 = -\frac{3}{4}(x-6), \text{ i.e., } 3x + 4y = 18.$$

Question 9

(*i*) Using componendo and dividendo, find the value of x, when:

$$\frac{x^3+3x}{3x^2+1} = \frac{14}{13}.$$ (3)

(*ii*) The total expenses of a trip for certain number of people is ₹ 18,000. If three more people join them, then the share of each reduces by ₹ 3,000. Taking x to be the original number of people, form a quadratic equation in x and solve it to find the value of x. (3)

(*iii*) Using ruler and compass only, construct $\angle ABC = 60°$, AB = 6 and BC = 5 cm.

(*a*) construct the locus of points equidistant from AB and BC.

(*b*) construct the locus of points equidistant from A and B.

(*c*) Mark the point which satisfies both the conditions (*a*) and (*b*) as P.

Hence, construct a circle with centre P and passing through A and B. (4)

Solution

(*i*) Given, $\frac{x^3+3x}{3x^2+1} = \frac{14}{13}$

Applying the componendo and dividendo, we have

$$\frac{x^3+3x+3x^2+1}{x^3+3x-3x^2-1} = \frac{14+13}{14-13}$$

$\Rightarrow$ $$\frac{(x+1)^3}{(x-1)^3} = 27 = 3^3$$

$\Rightarrow$ $\frac{x+1}{x-1} = 3 \Rightarrow x + 1 = 3x - 3 \Rightarrow 2x = 4$ or $x = 2$.

(*ii*) Let x denote the original number of people proposed to go on a trip. Then,

Expenses for 1 person = ₹ $\left(\frac{18000}{x}\right)$

In case of 3 more people joining the group.

Expenses for 1 person = ₹ $\left(\frac{18000}{x+3}\right)$

As per question, $$\frac{18000}{x} - 3000 = \frac{18000}{x+3}$$

$$\Rightarrow \quad 18000\left(\frac{1}{x} - \frac{1}{x+3}\right) = 3000$$

$$\Rightarrow \quad \frac{1}{x} - \frac{1}{x+3} = \frac{1}{6} \quad \Rightarrow \quad \frac{x+3-x}{x(x+3)} = \frac{1}{6}$$

$$\Rightarrow \quad x(x+3) = 18$$

$$\Rightarrow \quad x^2 + 3x - 18 = 0$$

$$\Rightarrow \quad x^2 + 6x - 3x - 18 = 0$$

$$\Rightarrow \quad (x+6) - 3(x+6) = 0$$

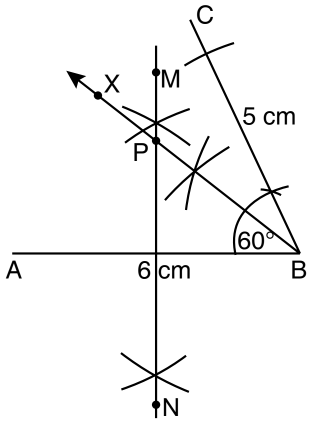
$$\Rightarrow \quad (x-3)(x+6) = 0$$

$$\Rightarrow \quad x - 3 = 0 \text{ or } x = 3. \qquad [\because x + 6 \neq 0]$$

(*iii*) (*a*) Ray BX, the angular bisector of ∠ABC is the required locus.

(*b*) Line MN, the right bisector of line segment AB is the required locus.

(*c*) The point which satisfy both the conditions (*a*) and (*b*) is the point of intersection of BX and MN (marked as P), is the required point.



Question 10

(*i*) Using remainder and factor theorem, factorise completely the given polynomial:

$$2x^3 - 9x^2 + 7x + 6$$ (3)

(*ii*) Each of the letter of the word 'HOUSEWARMING' is written on cards and put in a bag. If a card is drawn at random from the bag after shuffling, what is the probability that the letter on the card is:

(*a*) a vowel.

(*b*) one of the letters of the word SEWING.

(*c*) not a letter from the word WEAR. (3)

(*iii*) Use graph sheet for this question. Take 2 cm = 1 unit along the axes.

(*a*) Plot A(1, 2), B(1, 1) and C(2, 1)

(*b*) Reflect A, B and C about y-axis and name them as A′, B′ and C′.

(*c*) Reflect A, B, C, A′, B′ and C′ about x-axis and name them as A″, B″, C″, A‴, B‴ and C‴ respectively.

(*d*) Join A, B, C, C″, B″, A″, A‴, B‴, C‴, C′, B′, A′ and A to form a closed figure. (4)

Solution

$$\begin{array}{r|l} & 2x^2 - 5x - 3 \\ \hline x - 2 & 2x^3 - 9x^2 + 7x + 6 \\ & 2x^3 - 4x^2 \\ & - \quad + \\ \hline & \quad - 5x^2 + 7x + 6 \\ & \quad - 5x^2 + 10x \\ & \quad + \quad\quad - \\ \hline & \quad\quad - 3x + 6 \\ & \quad\quad - 3x + 6 \\ & \quad\quad + \quad - \\ \hline & \quad\quad\quad 0 \end{array}$$

(*i*) Let the given polynomial be denoted by $p(x)$. Then,

$$p(x) = 2x^3 - 9x^2 + 7x + 6$$

Here, $p(2) = 2(2)^3 - 9(2)^2 + 7(2) + 6$

$= 16 - 36 + 14 + 6 = 0.$

So, $x = 2$ is a root of $p(x)$, or ($x - 2$ is a factor of $p(x)$.

On dividing $p(x)$ by $(x - 2)$, we get

$$p(x) = (x - 2)(2x^2 - 5x - 3)$$

$$= (x - 2)\{(x - 3)(2x + 1)\}$$

Thus, $2x^3 - 9x^2 + 7x + 6 = (x - 2)(x - 3)(2x + 1).$

(*ii*) (*a*) The word 'HOUSEWARMING' contains 12 different letters, of which 5 are vowels, namely O, U, E, A and I.

∴ Number of favourable outcomes = 5; and Number of total outcomes = 12

So, P(a vowel) $= \frac{5}{12}$.

(*b*) The word 'SEWING' contains 6 different letters.

∴ Number of favourable outcomes = 6; and Number of total outcomes = 12

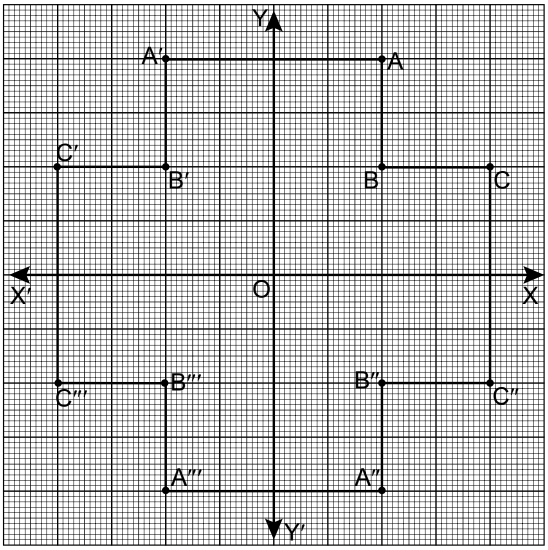
So, P(a letter of the word SEWING) $= \frac{6}{12}$ or $\frac{1}{2}$.

(*c*) Leaving the letters of the word 'WEAR', we have 8 remaining letters.

∴ Number of favourable outcomes = 8; and Number of total outcomes = 12

So, P (not a letter of the word WEAR) $= \frac{8}{12}$ or $\frac{2}{3}$.

(*iii*) The required graph is drawn as follows:



Examination Questions (with Solutions) 2024 for Practice

Question 1

Choose the correct answers to the questions from the given options: (15)

(*i*) For an intra-state sale, the CGST paid by a dealer to the Central government is ₹ 120. If the marked price of the article is ₹ 2,000, the rate of GST is

(*a*) 6%. (*b*) 10%. (*c*) 12%. (*d*) 16.67%.

(*ii*) What must be subtracted from the polynomial $x^3 + x^2 - 2x + 1$, so that the result is exactly divisible by $(x - 3)$?

(*a*) – 31 (*b*) – 30 (*c*) 30 (*d*) 31

(*iii*) The roots of the quadratic equation $px^2 - qx + r = 0$ are real and equal if

(*a*) $p^2 = 4qr$. (*b*) $q^2 = 4pr$. (*c*) $-q^2 = 4pr$. (*d*) $p^2 > 4qr$.

(*iv*) If matrix $A = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$ and $A^2 = \begin{bmatrix} 4 & x \\ 0 & 4 \end{bmatrix}$, then the value of x is

(*a*) 2. (*b*) 4. (*c*) 8. (*d*) 10.

(*v*) The median of the following observations arranged in ascending order is 64. Find the value of x.

27, 31, 46, 52, x, $x + 4$, 71, 79, 85, 90

(*a*) 60 (*b*) 61 (*c*) 62 (*d*) 66

(*vi*) Points A (x, y), B $(3, -2)$ and C $(4, -5)$ are collinear. The value of $\boldsymbol{y}$ in terms of $\boldsymbol{x}$ is

(*a*) $3x - 11$. (*b*) $11 - 3x$. (*c*) $3x - 7$. (*d*) $7 - 3x$.

(*vii*) The given table shows the distance covered and the time taken by a train moving at a uniform speed along a straight track.

Distance (in m)	60	90	y
Time (in sec)	2	x	5

The values of x and y are

(*a*) $x = 4, y = 150$. (*b*) $x = 3, y = 100$. (*c*) $x = 4, y = 100$. (*d*) $x = 3, y = 150$.

(*viii*) The 7th term of the given Arithmetic Progression (A.P.):

$$\frac{1}{a}, \left(\frac{1}{a}+1\right), \left(\frac{1}{a}+2\right), \ldots \text{ is}$$

(*a*) $\left(\frac{1}{a}+6\right)$. (*b*) $\left(\frac{1}{a}+7\right)$. (*c*) $\left(\frac{1}{a}+8\right)$. (*d*) $\left(\frac{1}{a}+7^7\right)$.

(*ix*) The sum invested to purchase 15 shares of a company of nominal value ₹ 75 available at a discount of 20% is

(*a*) ₹ 60. (*b*) ₹ 90. (*c*) ₹ 1,350. (*d*) ₹ 900.

(*x*) The circumcentre of a triangle is the point which is

(*a*) at equal distance from the three sides of the triangle.
(*b*) at equal distance from the three vertices of the triangle.
(*c*) the point of intersection of the three medians.
(*d*) the point of intersection of the three altitudes of the triangle.

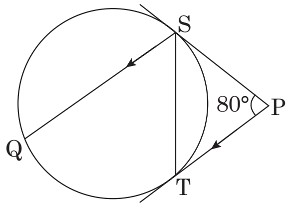
(*xi*) **Statement 1**: $\sin^2\theta + \cos^2\theta = 1$

Statement 2: $\text{cosec}^2\theta + \cot^2\theta = 1$

Which of the following is valid?

(*a*) Both the statements are true.
(*b*) Both the statements are false.
(*c*) Statements 1 is true, and Statement 2 is false.
(*d*) Statements 1 is false, and Statement 2 is true. (*Modified*)

(*xii*) In the given diagram, PS and PT are the tangents to the circle. SQ || PT and $\angle$SPT = 80°. The value of $\angle$QST is



(*a*) 140°.
(*b*) 90°.
(*c*) 80°.
(*d*) 50°.

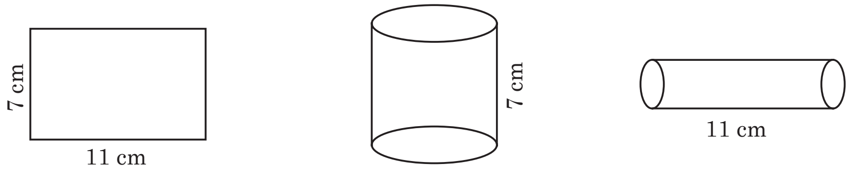
(*xiii*) **Assertion (A):** A die is thrown once and the probability of getting an even number is $\frac{2}{3}$.

Reason (R): The sample space for even numbers on a die is {2, 4, 6}.

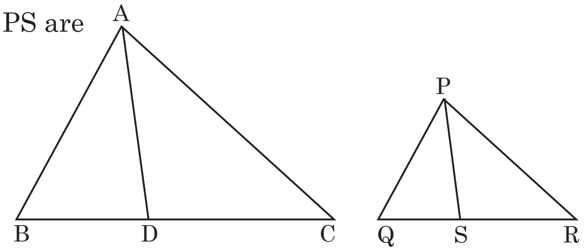
(*a*) A is true, R is false.
(*b*) A is false, R is true.
(*c*) Both A and R are true, and R is the correct reason for A.
(*d*) Both A and R are true, and R is incorrect reason for A. (*Modified*)

(*xiv*) A rectangular sheet of paper of size 11 cm × 7 cm is first rotated about the side 11 cm and then about the side 7 cm to form a cylinder, as shown in the diagram. The ratio of their curved surface areas is

(*a*) 1 : 1. (*b*) 7 : 11. (*c*) 11 : 7. (*d*) $\frac{11\pi}{7} : \frac{7\pi}{11}$.



(*xv*) In the given diagram, ΔABC ~ ΔPQR. If AD and PS are bisectors of $\angle$BAC and $\angle$QPR respectively, then



(*a*) ΔABC ~ ΔPQS.
(*b*) ΔABD ~ ΔPQS.
(*c*) ΔABD ~ ΔPSR.
(*d*) ΔABC ~ ΔPRS.

Solution:

(*i*) (*c*) Here, CGST is ₹ 120 $\Rightarrow$ SGST is also ₹ 120.

So, total GST = ₹ 240

$$\therefore \text{ Rate of GST} = \frac{\text{Total GST}}{\text{M.P. of the article}} \times 100\% = \frac{240}{2000} \times 100\% = 12\%.$$

(*ii*) (*d*) When $p(x) = x^3 + x^2 - 2x + 1$ is divided by $x - 3$, the remainder

$$p(3) = (3)^3 + (3)^2 - 2(3) + 1 = 27 + 9 - 6 + 1 = 31$$

$\therefore$ We need to subtract 31, so that the result is exactly divisible by $(x - 3)$.

(*iii*) (*b*) The roots of the quadratic equation $px^2 - qx + r = 0$ will be real and equal if $(-q)^2 - 4(p)(r) = 0$, *i.e.*, $q^2 = 4pr$.

(*iv*) (*c*) Given, $A = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} \Rightarrow A^2 = A \times A = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 0 & 4 \end{bmatrix}$

$\therefore \begin{bmatrix} 4 & 8 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 4 \end{bmatrix} \Rightarrow x = 8.$

(*v*) (*c*) Here, median = mean of 5th and 6th observations

$\Rightarrow \quad 64 = \frac{x + x + 4}{2} \Rightarrow 2x + 4 = 128 \Rightarrow x = 62.$

(*vi*) (*d*) Here, slope of AB $= \frac{-2 - y}{3 - x} = \frac{y + 2}{x - 3}$ and slope of BC $= \frac{-5 - (-2)}{4 - 3} = -3$

Since A, B and C are collinear, slope of AB = slope of BC

$\therefore \frac{y + 2}{x - 3} = -3 \Rightarrow y + 2 = -3x + 9$ or $y = 7 - 3x$.

(*vii*) (*d*) In a uniform speed, the ratio of distance and time is constant.

$\therefore \frac{60}{2} = \frac{90}{x} \Rightarrow x = 3$ and $\frac{60}{2} = \frac{y}{5} \Rightarrow y = 150.$

(*viii*) (*a*) Here, first term is $\frac{1}{a}$ and common difference is 1, therefore 7th term $= \frac{1}{a} + (7-1)(1) = \frac{1}{a} + 6.$

(*ix*) (*d*) Market value of one share = 80% of ₹ 75 = ₹ 60

∴ Sum required to purchase 15 shares = 15 × ₹ 60 = ₹ 900.

(*x*) (*b*) The circumcentre of a triangle is at equal distance from the three vertices of the triangle.

(*xi*) (*c*) For every value of θ, $\sin^2\theta + \cos^2\theta = 1$ is a true identity.

So, Statement 1 is true.

$\text{cosec}^2\theta + \cot^2\theta \neq 1$. In fact, the true identity is $\text{cosec}^2\theta - \cot^2\theta = 1$.

So, Statement 2 is false.

Hence, Statement 1 is true and Statement 2 is false.

(*xii*) (*d*) In the given diagram, PS = PT ⇒ ∠PTS = ∠PST

∵ ∠PTS = 80° ⇒ ∠PTS = ∠PST = 50°

Also, ∠QST = ∠PST [Alternate angles, between parallel lines SQ and PT]

⇒ ∠QST = 50°.

(*xiii*) (*b*) There are three even numbers (2, 4 and 6) on a die.

So, probability of getting an even number is $\frac{3}{6}$, *i.e.*, $\frac{1}{2}$.

Thus, Assertion (A) is false.

The sample space for even numbers on a die is {2, 4, 6} which is a true statement.

Thus, Reason (R) is true.

Hence, A is false, R is true.

(*xiv*) (*a*) In first case, $2pr = 11$ cm and $h = 7$ cm

∴ Curved surface area $= 2\pi rh = 11 \text{ cm} \times 7 \text{ cm} = 77 \text{ cm}^2$

In second case, $2\pi r = 7$ cm and $h = 11$ cm

∴ Curved surface area $= 2\pi rh = 7 \text{ cm} \times 11 \text{ cm} = 77 \text{ cm}^2$

Thus, required ratio = 1 : 1.

(*xv*) (*b*) $\Delta ABC \sim \Delta PQR \Rightarrow \angle BAC = \angle QPR$

$\Rightarrow \angle BAD = \angle QPS$ [$\because$ AD and PS bisect $\angle BAC$ and $\angle QPR$ respectively.]

$\therefore$ In Δs ABD and PQS,

$\angle BAD = \angle QPS$ and $\angle ABD = \angle PQS$

Hence, by AA Similarity Criterion, $\Delta ABD \sim \Delta PQS$.

Question 2

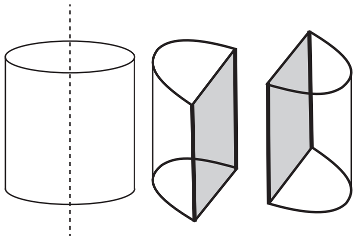
(*i*) $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$.

Find the values of x and y, if AB = C. (4)

(*ii*) A solid metallic cylinder is cut into two identical halves along its height (as shown in the diagram). The diameter of the cylinder is 7 cm and the height is 10 cm. Find:

(*a*) The total surface area (both the halves).

(*b*) The total cost of painting the two halves at the rate of ₹ 30 per cm². $\left(\text{Use } \pi = \frac{22}{7}\right)$ (4)



(*iii*) 15, 30, 60, 120... are in G.P. (Geometric Progression).

(*a*) Find the nth term of this G.P. in terms of n.

(*b*) How many terms of the above G.P. will give the sum 945? (4)

Solution:

(*i*) Given, $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$. Then,

AB = C gives $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4x & 0 \\ 4+y & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$

Equating the corresponding elements on both sides, we get

$4x = 4$ and $4 + y = x$

$\Rightarrow x = 1$ and $y = -3$.

Thus, the values of x and y respectively are 1 and -3.

(*ii*) (*a*) Given, radius of cylinder $(r) = \frac{7}{2}$ cm and height $(h) = 10$ cm.

Total surface area (both the halves)

= 2 × [Area of half curved surface + Area of two semicircular ends + Area of rectangular surface]

$= 2 \times [\pi rh + \pi r^2 + 2rh]$

$= 2 \times \left[\left(\frac{22}{7} \times \frac{7}{2} \times 10\right) + \frac{22}{7} \times \left(\frac{7}{2}\right)^2 + 2\left(\frac{7}{2}\right)(10)\right]$ sq cm

$= 2 \times [110 + 38.5 + 70]$ sq cm

$= 2 \times 218.5$ sq cm $= 437$ sq cm.

(*b*) Cost of painting the two halves = ₹ [30 × 437] = ₹ 13,110.

(*iii*) (*a*) The given G.P. is 15, 30, 60, 120, ...

Here, first term, $a = 15$ and common ratio, $r = 2$

So, nth term of the G.P., $a_n = ar^{n-1} = 15(2)^{n-1}$

(*b*) Sum of first *n* terms of a G.P., $S_n = \frac{a(r^n - 1)}{r - 1}$ $(r > 1)$

$$\Rightarrow \quad 945 = \frac{15\,[(2^n - 1]}{2 - 1} \qquad [\because S_n = 945]$$

$$\Rightarrow \quad 2^n - 1 = 63$$

$$\Rightarrow \quad 2^n = 64$$

$$\Rightarrow \quad 2^n = 2^6$$

$$\Rightarrow \quad n = 6$$

So, sum of first 6 terms of the G.P. is 945.

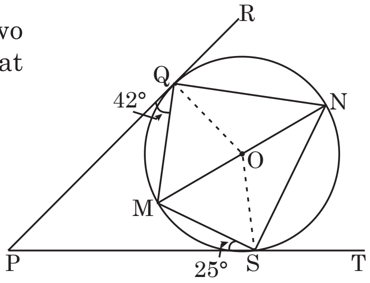
Question 3

(*i*) Factorise: $\sin^3\theta + \cos^3\theta$

Hence, prove the following identity:

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta = 1. \qquad (4)$$

(*ii*) In the given diagram, O is the centre of the circle. PR and PT are two tangents drawn from the external point P and touching the circle at Q and S respectively. MN is a diameter of the circle.
Given $\angle PQM = 42^\circ$ and $\angle PSM = 25^\circ$.



Find:

(*a*) $\angle OQM$

(*b*) $\angle QNS$

(*c*) $\angle QOS$

(*d*) $\angle QMS$. (4)

(*iii*) Use graph sheet for this question. Take 2 cm = 1 unit along the axes.

(*a*) Plot A(0, 3), B(2, 1) and C(4, –1).

(*b*) Reflect points B and C in *y*-axis and name their images as B' and C' respectively. Plot and write coordinates of the points B' and C'.

(*c*) Reflect point A in the line BB' and name its images as A'.

(*d*) Plot and write coordinates of point A'.

(*e*) Join the points ABA'B' and give the geometrical name of the closed figure so formed. (5)

Solution:

(*i*) Here, $\sin^3\theta + \cos^3\theta = (\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)$ $[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$

$= (\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$ ...(1) $[\because \sin^2\theta + \cos^2\theta = 1]$

Now,

$$\text{L.H.S.} = \frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta = \frac{(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)}{\sin\theta + \cos\theta} + \sin\theta\cos\theta \quad [\text{Using (1)}]$$

$$= 1 - \sin\theta\cos\theta + \sin\theta\cos\theta$$

$$= 1 = \text{R.H.S.}$$

Hence proved.

(*ii*) (*a*) We have, $\angle OQP = 90^\circ$ [$\because$ Tangent is $\perp$ to the radius through the point of contact.]

$\therefore \quad \angle OQM = \angle OQP - \angle MQP = 90^\circ - 42^\circ = 48^\circ$

(*b*) Using (*a*), we have

$$\angle OQN = \angle MQN - \angle OQM = 90^\circ - 48^\circ = 42^\circ$$

$\Rightarrow \quad \angle QNO = 42^\circ$ [$\because$ ON = OQ = radius of the circle]

Also, $\angle OSM = 90° - 25° = 65°$; and

$\angle OSN = \angle MSN - \angle OSM = 90° - 65° = 25°$ [$\because \angle MSN = 90°$]

$\Rightarrow$ $\angle ONS = 25°$

Hence, $\angle QNS = \angle QNO + \angle ONS = 42° + 25° = 67°$.

(*c*) $\angle QOS = 2 \times \angle QNS$ [Angle at the centre is double the angle at the remaining part of the circle]

$= 2 \times 67° = 134°$.

(*d*) In cyclic quad. MSNQ, $\angle QMS = 180° - \angle QNS$ [$\because$ Opposite angles of a cyclic quad. are supplementary.]

$= 180° - 67° = 113°$.

(*iii*) (*a*) The points A, B and C are plotted in the graph alongside.

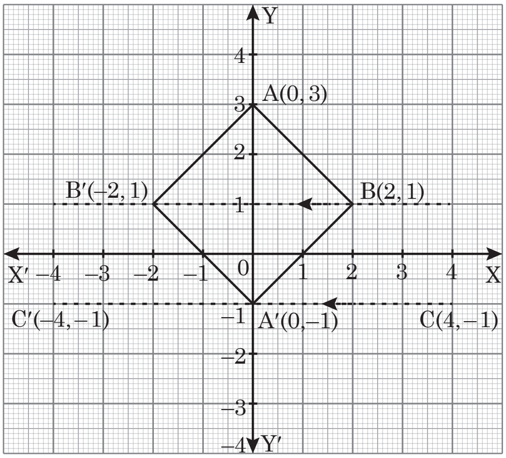
(*b*) R_y [B(2, 1)] $\rightarrow$ B′(–2, 1)

R_y [C(4, –1)] $\rightarrow$ C′(–4, –1)

(*c*) The point A′ is plotted in the graph alongside.

(*d*) A′(0, –1).

(*e*) Quad. ABA′B′ is a square.



Question 4

(*i*) Suresh has a recurring deposit account in a bank. He deposits ₹ 2,000 per month and the bank pays interest at the rate of 8% per annum. If he gets ₹ 1,040 as interest at the time of maturity, find in years total time for which the account was held. (3)

(*ii*) The following table gives the duration of movies in minutes.

Duration (in minutes)	100 –110	110 –120	120 –130	130 –140	140 –150	150 –160
No. of Movies	5	10	17	8	6	4

Using step-deviation method, find the mean duration of the movies. (3)

(*iii*) If $\frac{(a+b)^3}{(a-b)^3} = \frac{64}{27}$

(*a*) Find $\frac{a+b}{a-b}$.

(*b*) Hence using properties of proportion, find $a : b$. (4)

Solution:

(*i*) Here, P = ₹ 2,000, r = 8% p.a., and interest = ₹ 1,040. n = ?

We know that interest on a R.D. is calculated as:

$$\text{Interest} = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow \quad 1040 = 2000 \times \frac{n(n+1)}{24} \times \frac{8}{100}$$

$$\Rightarrow \quad n(n+1) = \frac{1040 \times 24}{8 \times 20} \quad \Rightarrow \quad n(n+1) = 156$$

$$\Rightarrow \quad n^2 + n - 156 = 0$$

$$\Rightarrow \quad n^2 + 13n - 12n - 156 = 0$$

$$\Rightarrow \quad n(n+13) - 12(n+13) = 0$$

$$\Rightarrow \quad (n+13)(n-12) = 0$$

$$\Rightarrow \quad n - 12 \text{ or } n = -13$$

$$\Rightarrow \quad n = 12$$

[$\because n = -13$ is inadmissible]

Therefore, in 12 months Suresh will get ₹ 1,040 as interest on his R.D.

Thus, total time in years is 1 year.

(*ii*) To calculate mean using step-deviation method, we draw the following table:
Let's take assumed mean (A) = 125 and class size (h) = 10.

Duration (Class Interval)	*No. of Movies* (f_i)	*Class Mark* (x_i)	*Step Deviation* $\left(u_i = \frac{x_i - A}{h}\right)$	*Product* ($f_i u_i$)
100–110	5	105	–2	–10
110–120	10	115	–1	–10
120–130	17	125	0	0
130–140	8	135	1	8
140–150	6	145	2	12
150–160	4	155	3	12
Total	$\Sigma f_i = 50$			$\Sigma f_i u_i = 12$

Therefore, $$\text{mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 125 + \frac{12}{50} \times 10 = 125 + 2.4 = 127.4$$

Thus, the mean duration of the movies is 127.4 minutes.

(*iii*) (*a*) Given, $$\frac{(a+b)^3}{(a-b)^3} = \frac{64}{27} \quad \Rightarrow \quad \frac{(a+b)^3}{(a-b)^3} = \frac{4^3}{3^3}$$

$$\Rightarrow \quad \frac{a+b}{a-b} = \frac{4}{3}$$ [Taking cube root on both sides]

(*b*) From (*a*), we have $$\frac{a+b}{a-b} = \frac{4}{3}$$

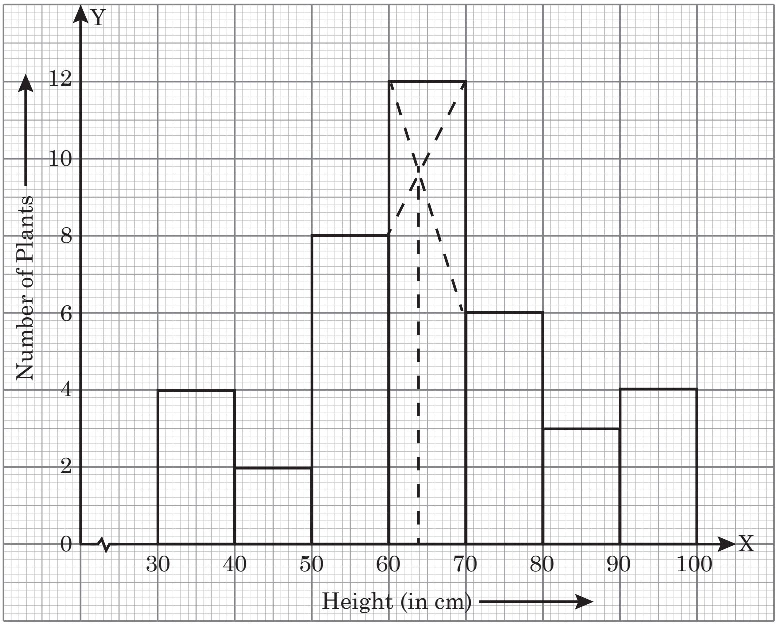
$$\Rightarrow \quad \frac{(a+b)+(a-b)}{(a+b)-(a-b)} = \frac{4+3}{4-3}$$ [By componendo and dividendo]

$$\Rightarrow \quad \frac{2a}{2b} = \frac{7}{1}$$

$$\Rightarrow \quad a : b = 7 : 1.$$

Question 5

(*i*) The given graph with a histogram represents the number of plants of different heights grown in a school campus. Study the graph carefully and answer the following questions:



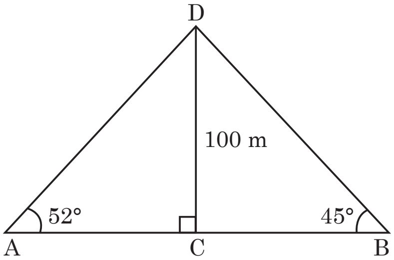
(*a*) Make a frequency table with respect to the class boundaries and their corresponding frequencies.

(*b*) State the modal class.

(*c*) Identify and note down the mode of the distribution.

(*d*) Find the number of plants whose height range is between 80 cm and 90 cm. (5)

(*ii*) The angle of elevation of the top of a 100 m high tree from two points A and B on the opposite side of the tree are 52° and 45° respectively. Find the distance AB, to the nearest metre. (5)



Solution:

(*i*) (*a*) Corresponding to the given histogram, the required frequency table is drawn as below:

Class Boundary	*Frequency*
30 – 40	4
40 – 50	2
50 – 60	8
60 – 70	12
70 – 80	6
80 – 90	3
90 – 100	4

(*b*) Height of the rectangle between class interval 60 and 70 is 12, which is the highest, therefore modal class is 60 – 70.

(*c*) The dotted line in the highest rectangle meets the x-axis at 64, so mode of the distribution is 64 cm.

(*d*) In the given histogram, the height of the rectangle between class interval 80 and 90 is 3. Therefore, required number of plants is 3.

(*ii*) In the figure, CD is a 100 m high tree and A and B are the two points opposite side of the tree such that ∠CAD = 52 and ∠CBD = 45°.

From right ΔACD, we have

$$\frac{100}{AC} = \tan 52°$$

$$\Rightarrow \quad AC = \frac{100}{\tan 52°} = \frac{100}{1.2799} \qquad [\because \tan 52° = 1.2799]$$

$$\Rightarrow \quad AC = 78.13 \text{ m}$$

Also, from right ΔBCD, we have

$$\frac{100}{BC} = \tan 45°$$

$$\Rightarrow \quad BC = 100 \text{ m} \qquad [\because \tan 45° = 1]$$

So, AB = AC + CB = (78.13 + 100) m = 178.13 m ≃ 178 m

Thus, the distance AB to the nearest metre is 178 m.

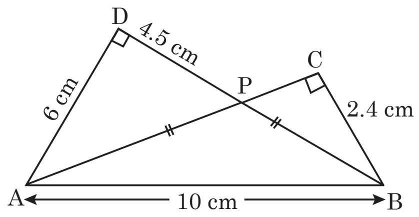
Question 6

(*i*) Solve the following quadratic equation for x and give your answer correct to three significant figures: $2x^2 - 10x + 5 = 0$.
(Use mathematical tables, if necessary) (3)

(*ii*) The nth term of an Arithmetic Progression (A.P.) is given by the relation $T_n = 6(7 - n)$. Find:
(*a*) its first term and common difference.
(*b*) sum of its first 25 terms. (3)

(*iii*) In the given diagram, ΔADB and ΔACB are two right-angled triangles with ∠ADB = ∠BCA = 90°. If AB = 10 cm, AD = 6 cm, BC = 2.4 cm and DP = 4.5 cm.

(*a*) Prove that ΔAPD ~ ΔBPC.
(*b*) Find the length of BD and PB.
(*c*) Hence, find the length of PA.
(*d*) Find area ΔAPD : area ΔBPC. (4)



Solution:

(*i*) Given quadratic equation is $2x^2 - 10x + 5 = 0$.

Using quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ we have

$$x = \frac{10 \pm \sqrt{(10)^2 - 4 \times 2 \times 5}}{2 \times 2}$$

$$= \frac{10 \pm \sqrt{100 - 40}}{4}$$

$$= \frac{10 \pm \sqrt{60}}{4}$$

$$= \frac{10 \pm 7.746}{4} = \frac{17.746}{4} \text{ or } \frac{2.254}{4}$$

Thus, $x = 4.436$ or $x = 0.564$ is the required solution of the given quadratic equation.

(*ii*) Given, nth term, $T_n = 6(7 - n)$

So, first term, $T_1 = 6(7 - 1) = 36$ and second term, $T_2 = 6(7 - 2) = 30$.

Thus,

(*a*) First term is 36 and common difference is $T_2 - T_1 = 30 - 36 = (-6)$.

(*b*) Sum of first 25 terms, $S_{25} = \frac{25}{2}[2 \times 36 + (25 - 1)(-6)]$

$$= \frac{25}{2}[72 - 144] = -900.$$

Thus, sum of the first 25 terms is (– 900).

(*iii*) (*a*) In Δs APD and BPC, we have

$\angle ADP = \angle BCP = 90°$ [Given]

and $\angle APD = \angle BPD$ [Vertically opp. angles]

∴ By AA Similarity Criterion, ΔAPD ~ ΔBPC.

(*b*) In right ΔADB, $BD = \sqrt{AB^2 - AD^2} = \sqrt{100 - 36} = \sqrt{64} = 8$ cm

⇒ BP = BD – PD = (8 cm – 4.5) cm = 3.5 cm.

(*c*) In right ΔADP, $AP = \sqrt{AD^2 + PD^2} = \sqrt{6^2 + (4.5)^2}$

$$= \sqrt{36 + 20.25} = \sqrt{56.25} = 7.5 \text{ cm}.$$

(*d*) Since Δs APD and BPC are similar, we have

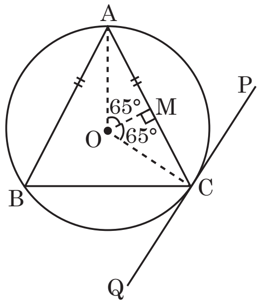
$$\frac{\text{Area of } \Delta APD}{\text{Area of } \Delta BPC} = \frac{AD^2}{BC^2} = \frac{(6)^2}{(2.4)^2}$$

$$= \frac{6 \times 6}{2.4 \times 2.4} = \frac{1 \times 1}{0.4 \times 0.4} = \frac{10 \times 10}{4 \times 4} = \frac{25}{4}$$

∴ *ar*(ΔAPD): *ar*(ΔBPC) = 25 : 4.

Question 7

(*i*) In the given diagram, an isosceles ΔABC is inscribed in a circle with centre O. PQ is a tangent to the circle at C. OM is perpendicular to chord AC and ∠COM = 65°.



Find:

(*a*) ∠ABC

(*b*) ∠BAC

(*c*) ∠BCQ. (3)

(*ii*) Solve the following inequation, write down the solution set and represent it on the real number line.

$$-3 + x \le \frac{7x}{2} + 2 < 8 + 2x, \ x \in I$$ (3)

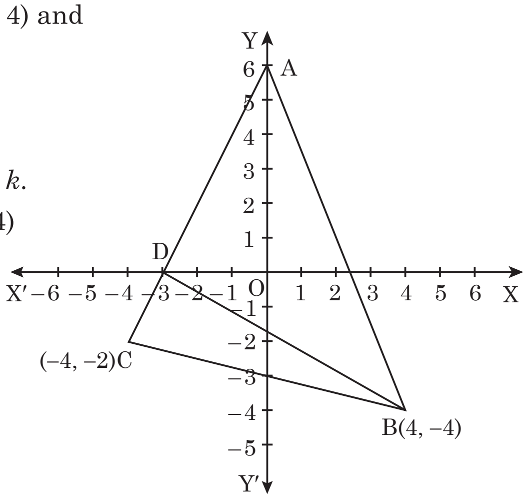
(*iii*) In the given diagram, ABC is a triangle, where B(4, – 4) and C(–4, –2). D is a point on AC.

(*a*) Write down the coordinates of A and D.

(*b*) Find the coordinates of the centroid of ΔABC.

(*c*) If D divides AC in the ratio *k* : 1, find the value of *k*.

(*d*) Find the equation of the line BD. (4)



Solution:

(*i*) (*a*) From the figure, $\angle AOC = 2\angle ABC$ [Angle at the centre is double the angle drawn on the same arc in the remaining part of the circle.]

$$\Rightarrow \quad \angle ABC = \frac{1}{2} \times \angle AOC = \frac{1}{2} \times (65° + 65°) = 65°.$$

(*b*) In triangle BAC, it is given that AB = AC.

So, $\angle ABC = \angle ACB = 65°$ [Angles opposite to equal sides are equal.]

$\Rightarrow \quad \angle BAC = 180° - (65° + 65°) = 50°.$

(*c*) We have, $\angle BCQ = \angle BAC$ [Angles in the alternate segments are equal.]

$\Rightarrow \quad \angle BCQ = 50°.$

(*ii*) (*a*) Given inequation is:

$$-3 + x \le \frac{7x}{2} + 2 < 8 + 2x, \; x \in I$$

$\Rightarrow \quad -6 + 2x \le 7x + 4 < 16 + 4x$

$\Rightarrow \quad -10 + 2x \le 7x < 12 + 4x$

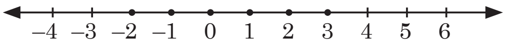
$\Rightarrow \quad -10 + 2x \le 7x$ and $7x < 12 + 4x$

$\Rightarrow \quad 5x \ge -10$ and $3x < 12$

$\Rightarrow \quad x \ge -2$ and $x < 4$

$\Rightarrow -2 \le x < 4$, which is the required solution.

Its representation on the number line is:



(*iii*) (*a*) The coordinates of A and D are A(0, 6) and D(–3, 0).

(*b*) The coordinates of vertices of ΔABC are A(0, 6), B(4, –4) and C(–4, –2).

Therefore, centroid of $\Delta ABC = \left(\frac{0+4-4}{3}, \frac{6-4-2}{3}\right) = (0, 0)$

Thus, coordinates of the centroid is (0, 0), *i.e.*, the origin.

(*c*) Here, AD : DC = k : 1, therefore by section formula, $D\left(\frac{-4k+0}{k+1}, \frac{-2k+6}{k+1}\right)$

Since point D lies on x-axis, its ordinate is zero.

$\therefore$ Equating $\frac{-2k+6}{k+1}$ equal to zero, we get $\frac{-2k+6}{k+1} = 0 \quad \Rightarrow \quad k = 3$

Thus, the value of k is 3.

(*d*) Equation of a line passing through (x_1, y_1) and (x_2, y_2) is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.

Therefore, equation of the line BD passing through B(4, –4) and D(–3, 0) is

$$\Rightarrow \quad y - (-4) = \frac{0-(-4)}{-3-4}(x - 4)$$

$$\Rightarrow \quad y + 4 = \frac{4}{-7}(x - 4)$$

$$\Rightarrow \quad -7(y + 4) = 4(x - 4) \quad \Rightarrow \quad 4x + 7y + 12 = 0.$$

Question 8

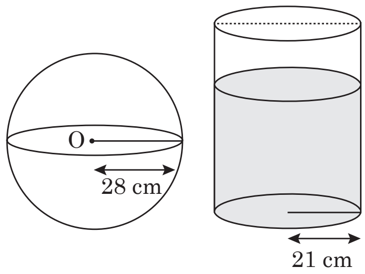
(*i*) The polynomial $3x^3 + 8x^2 - 15x + k$ has $(x - 1)$ as a factor. Find the value of k. Hence, factorise the resulting polynomial completely. (3)

(*ii*) The following letters A, D, M, N, O, S, U, Y of the English alphabet are written on separate cards and put in a box. The cards are well shuffled and one card is drawn at random. What is the probability that the card drawn is a letter of the word

(*a*) MONDAY?
(*b*) which does not appear in MONDAY?
(*c*) which appears both in SUNDAY and MONDAY? (3)

(*iii*) Oil is stored in a spherical vessel occupying 3/4 of its full capacity. Radius of this spherical vessel is 28 cm. This oil is then poured into a cylindrical vessel with a radius of 21 cm. Find the height of the oil in the cylindrical vessel (correct to the nearest cm).

$\left(\text{Take } \pi = \frac{22}{7}\right)$ (4)



Solution:

(*i*) Since $(x - 1)$ is a factor of $p(x) = 3x^3 + 8x^2 - 15x + k$, by Factor Theorem $p(1) = 0$.

Therefore, $p(1) = 0 \Rightarrow 3(1)^3 + 8(1)^2 - 15(1) + k = 0$

$\Rightarrow 3 + 8 - 15 + k = 0$

$\Rightarrow -4 + k = 0$ or $k = 4$

Thus, the value of k is 4.

By long division,

$$
\begin{array}{r|l}
 & 3x^2 + 11x - 4 \\
x - 1 & 3x^3 + 8x^2 - 15x + 4 \\
 & \underline{3x^3 - 3x^2} \\
 & 11x^2 - 15x \\
 & \underline{11x^2 - 11x} \\
 & -4x + 4 \\
 & \underline{-4x + 4} \\
 & 0
\end{array}
$$

So, $p(x) = 3x^3 + 8x^2 - 15x + 4$

$= (x - 1)[3x^2 + 11x - 4]$

$= (x - 1)[3x^2 + 12x - x - 4]$

$= (x - 1)[3x(x + 4) - 1(x + 4)]$

$= (x - 1)(x + 4)(3x - 1)$

Thus, $3x^3 + 8x^2 - 15x + 4 = (x - 1)(x + 4)(3x - 1)$.

(*ii*) (*a*) Number of letters in the word MONDAY = 6

$\therefore$ Number of favourable outcomes = 6

Total number of letters = 8

$\therefore$ Total number of possible outcomes = 8

So, $\text{P(Monday)} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{6}{8} = \frac{3}{4}$.

(*b*) $\text{P(not Monday)} = 1 - \text{P(Monday)} = 1 - \frac{3}{4} = \frac{1}{4}$.

(*c*) Letters appear both in SUNDAY and MONDAY are N, D, A, Y, *i.e.*, 4

$\therefore$ Number of favourable outcomes = 4

Total number of letters = 8

$\therefore$ Total number of possible outcomes = 8

So, P(Sunday and Monday) $= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \dfrac{4}{8} = \dfrac{1}{2}.$

(*iii*) Given, radius of spherical vessel (R) = 28 cm and radius of cylindrical vessel (r) = 21 cm

Full capacity of the spherical vessel $= \dfrac{4}{3}\pi R^3 = \dfrac{4}{3}\pi(28)^3$ cu cm

$\therefore$ Volume of oil in the spherical vessel $= \dfrac{3}{4}\left[\dfrac{4}{3}\pi(28)^3\right]$ cu cm

$= 68992$ cu cm

Let h cm be the height of oil in the cylindrical vessel. Then,

Volume of oil in the cylindrical vessel $= \pi r^2 h = \pi(21)^2 h$ cu cm

Now,

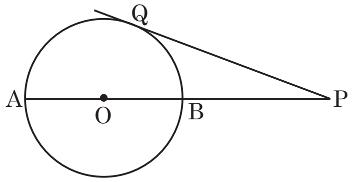
$\therefore$ Volume of oil in the spherical vessel = Volume of oil in the cylindrical vessel

$\Rightarrow \quad \pi(21)^2 h = 68992$

$\Rightarrow \quad h = \dfrac{68992 \times 7}{22 \times 21 \times 21} = 49.78 \approx 50$ cm. (correct to the nearest cm)

Question 9

(*i*) The figure shows a circle of radius 9 cm with O as the centre. The diameter AB produced meets the tangent PQ at P. If PA = 24 cm, find the length of tangent PQ. (3)



(*ii*) Mr. Gupta invested ₹ 33,000 in buying ₹ 100 shares of a company at 10% premium. The dividend declared by the company is 12%. Find:

(*a*) the number of shares purchased by him.

(*b*) his annual dividend. (3)

(*iii*) A life insurance agent found the following data for distribution of ages of 100 policy holders:

Age in years	*Policy Holders (Frequency)*	*Cumulative frequency*
20–25	2	2
25–30	4	6
30–35	12	18
35–40	20	38
40–45	28	66
45–50	22	88
50–55	8	96
55–60	4	100

On a graph sheet draw an ogive using the given data. Take 2 cm = 5 years along one axis and 2 cm = 10 policy holders along the other axis. Use your graph to find:

(*a*) The median age.

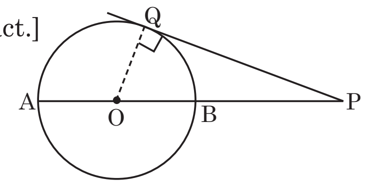
(*b*) Number of policy holders whose age is above 52 years. (4)

Solution:

(*i*) Join O to Q.

Since OQ is radius and PQ is tangent to the circle.

$\angle OQP = 90°$ [Radius is $\perp$ to tangent through the point of contact.]



Given, OA = OQ = 9 cm and PA = 24 cm

$\therefore$ OP = PA – OA = (24 – 9) cm = 15 cm

Now, in right ΔOQP, we have

$$PQ = \sqrt{OP^2 - OQ^2} = \sqrt{15^2 - 9^2}$$

$$= \sqrt{225 - 81} = \sqrt{144} = 12 \text{ cm.}$$

(*ii*) Face value of a share = ₹ 100

Market value of the share at 10% premium = ₹ 100 + 10% of ₹ 100

= ₹ 100 + ₹ 10 = ₹ 110

(*a*) No. of shares purchased = $\frac{\text{Total investment}}{\text{Market value of one share}} = \frac{33000}{110}$ = 300 shares.

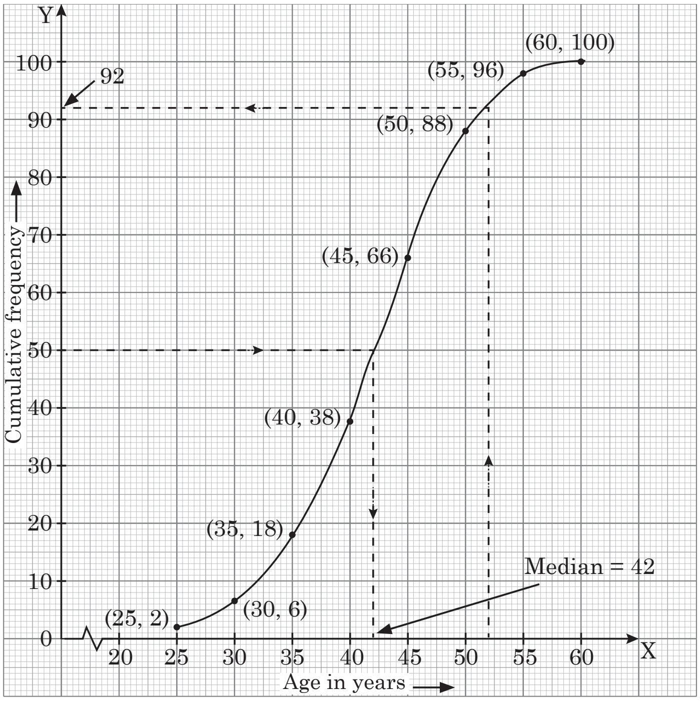
(*b*) Annual dividend = No. of shares × Rate of dividend × Face value of a share

$= ₹\ 300 \times \frac{12}{100} \times 100 = ₹\ 3{,}600.$

(*iii*) (*a*) As per the graph, the median age of policy holders is 42 years.

(*b*) From the graph, 100 – 92 = 8.

Therefore, there are 8 policy holders whose age is above 52 years.



Scale:
x-axis: 2 cm = 5 years
y-axis: 2 cm = 10 policy holders

Question 10

(*i*) Rohan bought the following eatables for his friends :

Soham Sweet Mart : Bill				
S.No.	*Item*	*Price*	*Quantity*	*Rate of GST*
1	Laddu	₹ 500 per kg	2 kg	5%
2	Pastries	₹ 100 per piece	12 pieces	18%

Calculate :

(a) Total GST paid.

(b) Total bill amount including GST. (3)

(*ii*) (*a*) If the lines $kx - y + 4 = 0$ and $2y = 6x + 7$ are perpendicular to each other, find the value of k.

(*b*) Find the equation of a line parallel to $2y = 6x + 7$ and passing through (–1, 1). (3)

(*iii*) Use ruler and compass to answer this question.
Construct $\angle ABC = 90^o$, where AB = 6 cm, BC = 8 cm.

(*a*) Construct the locus of points equidistant from B and C.

(*b*) Construct the locus of points equidistant from A and B.

(*c*) Mark the point which satisfies both the conditions (*a*) and (*b*) as O. Construct the locus of points keeping a fixed distance OA from the fixed point O.

(*d*) Construct the locus of points which are equidistant from BA and BC (4)

Solution:

(*i*) (*a*) Cost of 2 kg of laddu = 2 × ₹ 500 = ₹ 1000

Cost of 12 pieces of pastries = 12 × ₹ 100 = ₹ 1200

GST paid for 2 kg of laddu = 5% of ₹ 1000 = ₹ 50

GST paid for 12 pieces of pastries = 18% of ₹ 1200 = ₹ 216

∴ Total GST paid = ₹ 50 + ₹ 216 = ₹ 266.

(*b*) Total bill = Cost of 2 kg of laddu + Cost of 12 pieces of pastries + Total GST paid

= ₹ 1000 + ₹ 1200 + ₹ 266 = ₹ 2,466.

(*ii*) (*a*) Equation of first line is: $kx - y + 4 = 0$ ...(1)

Writing the line (1) in $y = mx + c$ form, we have

$$y = kx + 4$$

∴ Slope (m_1) of line (1) is k.

Equation of second line is: $2y = 6x + 7$ ...(2)

Writing the line (2) in $y = mx + c$ form, we have

$$y = 3x + \frac{7}{2}$$

∴ Slope (m_2) of line (2) is 3.

Since the two lines are perpendicular, $m_1 m_2 = -1$

$$\Rightarrow \quad 3k = -1 \quad \text{or} \quad k = -\frac{1}{3}.$$

(*b*) Equation of given line is $2y = 6x + 7$. ...(1)

Then, equation of a line parallel to line (1) is $2y = 6x + k$ ...(2)

[∵ Slopes of parallel lines are equal.]

As this line passes through (–1, 1), the point (–1, 1) will satisfy line (2).

Putting $x = -1$ and $y = 1$ in equation (2), we get

$$2(1) = 6(-1) + k \Rightarrow k = 8$$

Substituting $k = 8$ in (2), equation of the required line is

$$2y = 6x + 8 \quad \text{or} \quad y = 3x + 4.$$

(*iii*) (*a*) The locus of points equidistant from B and C is the ⊥ bisector of BC, *i.e.*, line MPN.

(*b*) The locus of points equidistant from A and B is the ⊥ bisector of AB, *i.e.*, line EFG.

(*c*) The point which satisfy both the condition (*a*) and (*b*) is the point O, *i.e.*, the point of intersection of lines MPN and EFG.
The locus of the points is the circle with centre O and radius OA.

(*d*) The locus of points equidistant from BA and BC is the angular bisector of $\angle ABC$, *i.e.*, BS.