Solutions—Sample Question Paper (Basic) 2025-26

SECTION A

1. (*d*) By prime factorisation, $2025 = 3 \times 3 \times 3 \times 3 \times 5 \times 5$

$$= 3^4 \times 5^2$$

$\therefore$ Exponent of 3 in the prime factorisation of 2025 is 4.

3	2025
3	675
3	225
3	75
5	25
5	5
	1

2. (*b*) We have,

$$2024\,x + 2025\,y = 1 \quad \text{...(1)}$$

and $$2025\,x + 2024\,y = -1 \quad \text{...(2)}$$

Subtracting (1) from (2), we get

$$\begin{array}{r} 2025\,x + 2024\,y = -1 \\ \underline{-\ 2024\,x \pm 2025\,y = \mp 1} \\ x - y = -2 \end{array}$$

Thus, the value of $x - y$ is -2.

3. (*d*) A polynomial whose zeros are -2 and 5 can be written as

$p(x) = k(x + 2)(x - 5)$, where $k(\neq 0)$ is any real number or $p(x) = k(x^2 - 3x - 10)$

Since k can be any real number, there are infinitely many such polynomials.

4. (*c*) We have, $(x + 2)(x - 1) = x^2 - 2x - 3$

$\Rightarrow \quad x^2 + x - 2 = x^2 - 2x - 3$

$\Rightarrow \quad 3x + 1 = 0$, which is not a quadratic equation.

Note: The equations given in other options are quadratic. For verification, students can simplify them.

5. (*a*) If $2x$, $(x + 10)$ and $(3x + 2)$ are three consecutive terms of an A.P., we must have

$2(x + 10) = 2x + (3x + 2)$ [$\because$ If a, b, c are in A.P., then $2b = a + c$]

$\Rightarrow \quad 2x + 20 = 5x + 2$

$\Rightarrow \quad 3x = 18 \quad$ or $\quad x = 6$.

6. (*b*) Given, $1 + 2 + 3 + \ldots + 50 = 25k$

We know that sum of first n natural numbers is given by

$$S_n = \frac{n(n+1)}{2}$$

$$\therefore \quad 1 + 2 + 3 + \ldots + 50 = \frac{50 \times (50+1)}{2} \quad [\because n = 50]$$

$$= \frac{50 \times 51}{2} = 25 \times 51$$

So, we have $\quad 25k = 25 \times 51$

$\Rightarrow \quad k = 51$.

7. (*d*) The given points are (cos 30°, sin 30°) and (cos 60°, – sin 60°), *i.e.*, $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

∴ Distance between the given points

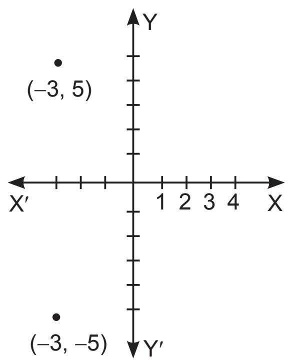
$$= \sqrt{\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2} - \frac{1}{2}\right)^2}$$

$$= \sqrt{\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)^2}$$

$$= \sqrt{\left(\frac{1}{4} + \frac{3}{4}\right) - 2\left(\frac{\sqrt{3}}{4}\right) + \left(\frac{3}{4} + \frac{1}{4}\right) + 2\left(\frac{\sqrt{3}}{4}\right)} = \sqrt{\frac{4}{4} + \frac{4}{4}} = \sqrt{1+1} = \sqrt{2}.$$

8. (*c*) The point (–3, 5) lies in second quadrant and its ordinate is 5, *i.e.*, the point is 5 units above the *x*-axis. Therefore, in its mirror image, the abscissa of the point will the same, *i.e.*, –3 but its ordinate will be 5 units below the *x*-axis, *i.e.*, –5.

So, the coordinates of the mirror image are (–3, –5) and the mirror images will be in third quadrant.

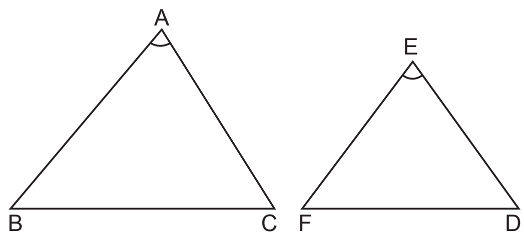


9. (*b*) In ∆s ABC and DEF, we have

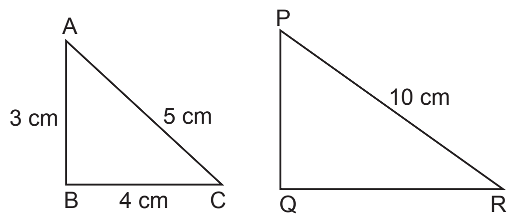
$$\frac{AB}{EF} = \frac{AC}{DE}$$

or $$\frac{AB}{AC} = \frac{EF}{ED}$$

So, by SAS Similarly Criterion the triangles will be similar if the angles included between the sides AB and AC of ∆ABC and EF and DE of ∆DEF are equal, *i.e.*, ∠A = ∠E. In this case, ∆ABC ~ ∆EFD.



10. (*b*) Given, ∆ABC ~ ∆PQR



∴ $$\frac{\text{Perimter of } \Delta ABC}{\text{Perimter of } \Delta PQR} = \frac{AC}{PR}$$

⇒ $$\frac{AB + BC + AC}{PQ + QR + PR} = \frac{5}{10} \Rightarrow \frac{3+4+5}{PQ + QR + PR} = \frac{1}{2}$$

$$\Rightarrow \frac{12}{PQ + QR + PR} = \frac{1}{2}$$

$$\Rightarrow PQ + QR + PR = 2 \times 12 = 24$$

Thus, Perimeter of ∆PQR is 24 cm.

Alternatively:

Since ∆ABC ~ ∆PQR, we have

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{1}{2}$$ [∵ AC = 5 cm and PR = 10 cm]

$$\Rightarrow \frac{3}{PQ} = \frac{1}{2} \text{ and } \frac{4}{QR} = \frac{1}{2}$$

⇒ PQ = 6 cm and QR = 8 cm

∴ Perimeter of ∆PQR = PQ + QR + PR

= 6 cm + 8 cm + 10 cm = 24 cm.

11. (*a*) Clearly, OECD is a square whose side is equal to the radius of the circle. Therefore,

$CD = CE = r$

Also, $AE = AF$ [Tangents drawn to a circle from an external point are equal.]

$\Rightarrow$ $AC - CE = AB - BF$

$\Rightarrow$ $24 - r = 25 - BD$ [$\because$ BF = BD]

$\Rightarrow$ $24 - r = 25 - (BC - CD)$

$\Rightarrow$ $24 - r = 25 - (7 - r)$ [$\because$ BC = 7 cm and CD = r]

$\Rightarrow$ $24 - r = 25 - 7 + r$

$\Rightarrow$ $24 - r = 18 + r$

$\Rightarrow$ $2r = 24 - 18 = 6$

$\Rightarrow$ $r = 3.$

12. (*b*) We know that $\text{cosec}^2 x = 1 + \cot^2 x$ is an trigonometric identity.

Therefore, $\cot^2 x - \text{cosec}^2 x = -1$. Thus, $\cot^2 x - \text{cosec}^2 x$ is not equal to unity.

13. (*c*) We draw a cumulative frequency table for the given distribution

Class (x_i)	*Frequency* (f)	*Cumulative Frequency* (cf)
0–5	11	11
5–10	12	23
10–15	13	36
15–20	9	45
20–25	11	56

From the above table, $N = \Sigma f = 56$

Therefore, $\frac{N}{2} = 28$

The cumulative frequency just greater than 28 is 36 and the class corresponding to this frequency is 10–15.

Therefore, 10–15 is the *median class* and the upper limit of this class is 15.

14. (*c*) The given empirical relationship between the three measures of central tending is

a(Median) = Mode + b(Mean)

and the actual empirical relationship is

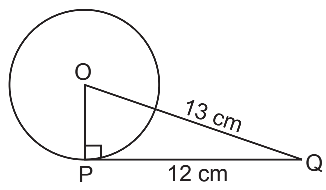
3(Median) = Mode + 2(Mean)

Comparing the two relationships, we get

$a = 3$ and $b = 2$

$\therefore$ $2b + 3a = 2 \times 2 + 3 \times 3 = 4 + 9 = 13.$

15. (*b*) In right triangle OPQ,



$$OP = \sqrt{OQ^2 - PQ^2}$$
$$= \sqrt{(13)^2 - (12)^2} = \sqrt{169 - 144}$$
$$= \sqrt{25} = 5$$

$\therefore$ Radius of the circle is 5 cm.

16. (*a*) Since AOB is the diameter of the circle, we have

$\angle AOP = 180° - \angle POB$

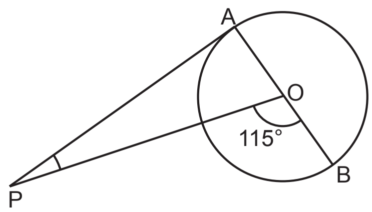
$= 180° - 115° = 65°$

Also, $\angle OAP = 90°$ [$\because$ Tangent is $\perp$ to the radius through the point of contact.]

Therefore, in right through OAP, we have

$\angle APO = 180° - (\angle OAP + \angle AOP)$

$= 180° - (90° + 65°) = 180° - 155° = 25°.$



Alternatively:

In right ΔOAP, we have

$$\angle POB = \angle APO + \angle OAP \quad \text{[By exterior angle property]}$$

$$\Rightarrow \quad 115^\circ = \angle APO + 90^\circ$$

$$\Rightarrow \quad \angle APO = 115^\circ - 90^\circ = 25^\circ.$$

17. (*c*) Let r_1 and r_2 be the radii of the two circles.

Given, ratio of the circumferences = 3 : 4

$$\therefore \quad \frac{C_1}{C_2} = \frac{3}{4} \Rightarrow \frac{2\pi r_1}{2\pi r_2} = \frac{3}{4} \quad [\because C = 2\pi r]$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{3}{4}$$

If A_1 and A_2 are the areas of the two circles, than

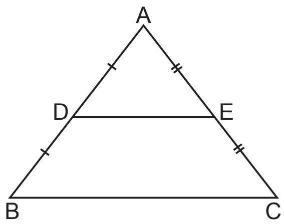
$$\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{3}{4}\right)^2 \quad \left[\because \frac{r_1}{r_2} = \frac{3}{4}\right]$$

$$= \frac{9}{16}.$$

18. (*a*) If an event is most unlikely to happen, then its probability must be the least. Out of the four options, the probability 0.0001 is the least. So, the event with the probability 0.0001 is most unlikely to happen.

19. (*a*) Assertion (A) is true:



Let in ΔABC, D and E are the mid-points of AB and AC. Then,

$$\frac{AD}{DB} = \frac{AE}{EC} = 1$$

$\Rightarrow$ DE divides AB and AC in the same ratio.

$\therefore$ By converse of Thales Theorem (BPT), DE is parallel to BC.

Thus, Assertion (A) is true.

Reason (R) is true:

The statement given in Reason (R) is the converse of Thales Theorem (BPT).

Therefore, Reason (R) is true and the correct explanation of Assertion (A).

Hence, option (*a*) is the correct answer.

20. (*d*) Assertion (A) is false:

When two coins are tossed simultaneously, possible outcomes are HH, HT, TH, TT. *i.e.* 4,

$\therefore$ P(probability of getting two heads) = $\frac{1}{4}$ [$\because$ Out of four possible outcomes, HH is only the favourable outcome.]

Thus, Assertion (A) is false.

Reason (R) is true:

The statement, "Probabilities of 'equally likely' outcomes of an experiment are always equal." is a true statement.

Therefore, Reason (R) is true.

Hence, option (*d*) is the correct answer.

SECTION B

21. (A) Let $a = 2 \times 5 \times 7 \times 11 + 11 \times 13$

$$= 11\,[2 \times 5 \times 7 + 13] = 11\,[70 + 13]$$
$$= 11 \times 83$$

Thus, the given number can be factorised into two factors 11 and 83.

Therefore, the number is a composite number.

OR

(B) The smallest number which is divisible by both 306 and 657 is the LCM of 306 and 657.

By prime factorisation,

$$306 = 2 \times 3 \times 3 \times 17$$

and $657 = 3 \times 3 \times 73$

$\therefore$ LCM of 306 and 657 $= 2 \times 3 \times 3 \times 17 \times 73$

$$= 22338$$

Thus, the required number is 22338.

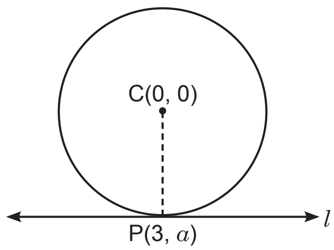
2	306
3	153
3	51
17	17
	1

3	567
3	219
73	73
	1

22. Equation of line l is: $x + y = 5$

Since point P(3, a) lies on line l, we have

$$3 + a = 5 \quad \Rightarrow \quad a = 5 - 3 = 2$$



Therefore, coordinates of P are (3, 2).

By distance formula,

Radius of the circle, CP $= \sqrt{(3-0)^2 + (2-0)^2}$

$$= \sqrt{9+4} = \sqrt{13}$$

Thus, radius of the circle is $\sqrt{13}$ units.

23. Here, $p(x) = x^2 + (a + 1)\,x + b$ is the given polynomial.

Since 2 and –3 are the zeros of the polynomial $p(x)$, we have

$$p(2) = 0 \text{ and } p(-3) = 0$$

Now, $p(2) = 0 \;\Rightarrow\; (2)^2 + (a + 1)(2) + b = 0$

$\Rightarrow\; 4 + 2a + 2 + b = 0$

$\Rightarrow\; 2a + b = -6$ ...(1)

$p(-3) = 0 \;\Rightarrow\; (-3)^2 + (a + 1)(-3) + b = 0$

$\Rightarrow\; 9 - 3a - 3 + b = 0$

$\Rightarrow\; -3a + b = -6$

or $3a - b = 6$ ...(2)

On adding (1) and (2), we get

$$5a = 0 \quad \Rightarrow \quad a = 0$$

Putting $a = 0$ in (1), we have

$$2 \times 0 + b = -6 \quad \Rightarrow \quad b = -6$$

Thus, $a = 0$ and $b = -6$ are the required values.

Alternatively:

Given polynomial is: $x^2 + (a + 1)\,x + b$ ...(1)

Comparing (1) with $Ax^2 + Bx + C$, we get

$$A = 1,\ B = (a + 1) \text{ and } C = b$$

Let $\alpha = 2$ and $\beta = -3$ be the zeros of (1).

Then,

$$\alpha + \beta = \frac{-B}{A} \qquad \left[\because \text{Sum of zeros} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}\right]$$

$$\Rightarrow \quad 2 + (-3) = \frac{-(a+1)}{1}$$

$$\Rightarrow \quad -1 = -(a+1) \quad \Rightarrow \quad a + 1 = 1 \quad \text{or} \quad a = 0$$

and

$$\alpha\beta = \frac{C}{A} \qquad \left[\because \text{Product of zeros} = \frac{\text{constant term}}{\text{coefficient of } x^2}\right]$$

$$\Rightarrow \quad (2)(-3) = \frac{b}{1} \quad \Rightarrow \quad b = -6$$

Thus, $a = 0$ and $b = -6$ are the required values.

24. The given quadratic equation is:

$$x^2 + 4x - 3\sqrt{2} = 0$$

Here, $a = 1$, $b = 4$ and $c = -3\sqrt{2}$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$$= (4)^2 - 4(1)(-3\sqrt{2})$$

$$= 16 + 12\sqrt{2} = 4(4 + 3\sqrt{2}) > 0$$

Since D > 0, the roots of the given equation are real and distinct.

25. (A) We have, $2 \sin 30° \tan 60° - 3 \cos^2 60° \sec^2 30°$

$$= 2\left(\frac{1}{2}\right)(\sqrt{3}) - 3\left(\frac{1}{2}\right)^2\left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 2 \times \frac{\sqrt{3}}{2} - 3 \times \frac{1}{4} \times \frac{4}{3} = \sqrt{3} - 1.$$

OR

(B) Given, $\sin x = \frac{7}{25}$, then

$$\sin x \cdot \cos x\,(\tan x + \cot x)$$

$$= \sin x \cdot \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= \sin x \cdot \cos x \left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)$$

$$= \frac{\sin x \cos x}{\sin x \cos x} = 1, \text{ which is a constant}$$

Since the value of $\sin x \cos x\,(\tan x + \cot x)$ is a constant, therefore it is equal to 1 for all acute angles x.

SECTION C

26. Let us assume, to the contrary, that $\sqrt{2} - \sqrt{5}$ is a rational number. Then,

$\sqrt{2} - \sqrt{5} = \frac{p}{q}$, where p and q are integers having no common factors and $q \neq 0$.

$$\Rightarrow \quad \sqrt{5} = \sqrt{2} - \frac{p}{q}$$

$$\Rightarrow \quad (\sqrt{5})^2 = \left(\sqrt{2} - \frac{p}{q}\right)^2$$

$$\Rightarrow \qquad 5 = 2 + \frac{p^2}{q^2} - 2\sqrt{2}\frac{p}{q}$$

$$\Rightarrow \qquad 2\sqrt{2}\,\frac{p}{q} = \frac{p^2}{q^2} - 3$$

$$\Rightarrow \qquad 2\sqrt{2}\,\frac{p}{q} = \frac{p^2 - 3q^2}{q^2}$$

$$\Rightarrow \qquad \sqrt{2} = \frac{p^2 - 3q^2}{2pq} \qquad \ldots(1)$$

As the square root of any prime number is always an irrational number. So, $\sqrt{2}$ is an irrational number while $\frac{p^2 - 3q^2}{2pq}$ is a rational number.

Therefore, in eq. (1), L.H.S. is an irrational number while R.H.S. is rational number.

Since an irrational number cannot be equal to a rational number, our assumption is wrong.

Hence, $\sqrt{2} - \sqrt{5}$ is an irrational number.

27. (A) The given frequency distribution is:

Area of Land (in hectares)	1–3	3–5	5–7	7–9	9–11	11–13
Number of Families	20	45	80	55	40	12

Here, frequency of the class 5–7 is 80 which is the highest. So, 5–7 is the *modal class.*

For this class, we have

$$l = 5,\ h = 2,\ f_1 = 80,\ fo = 45 \text{ and } f_2 = 55$$

$$\therefore \qquad \text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 5 + \frac{80 - 45}{2 \times 80 - 45 - 55} \times 2$$

$$= 5 + \frac{35}{160 - 100} \times 2 = 5 + \frac{35}{60} \times 2$$

$$= 5 + \frac{35}{30} = 6.17 \text{ (approx)}$$

Hence, the modal agriculture holdings of the village is 6.17 hectares (approx).

OR

(B) To calculate the value of p, we apply the 'step deviation method'. For this, we draw the following table.

Class Interval (C.I.)	*Frequency* (f_i)	*Class Mark* (x_i)	*Deviation* $\left(u_i = \frac{x_i - A}{h}\right)$	*Product* $(f_i u_i)$
0–20	7	10	–1	–7
20–40	p	30 = A	0	0
40–60	10	50	1	10
60–80	9	70	2	18
80–100	13	90	3	39
Total	$\Sigma f_i = 39 + p$			$\Sigma f_i u_i = 60$

Here, we take the class mark 30 as Assumed mean, *i.e.*, $A = 30$.

$\therefore$ By step deviation formula,

$$\text{Mean, } \bar{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$\Rightarrow \quad 54 = 30 + \frac{60}{39+p} \times 20$

$\Rightarrow \quad 54 - 30 = \frac{1200}{39+p}$

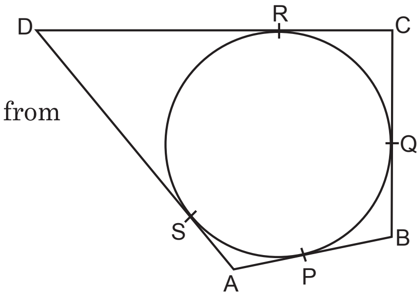
$\Rightarrow \quad 24 = \frac{1200}{39+p} \quad \Rightarrow \quad 24(39 + p) = 1200$

$\Rightarrow \quad 39 + p = 50$

$\Rightarrow \quad p = 50 - 39 = 11$

Thus, the value of p is 11.

28. In the figure, AP and AS are tangents to the circle from external point A. Therefore,



AP = AS ...(1) [∵ Tangents drawn to a circle from an external point are equal.]

Similarly, we have

BP = BQ ...(2)

CR = CQ ...(3)

and DR = DS ...(4)

On adding above equations, we have

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$\Rightarrow \quad (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$\Rightarrow \quad AB + CD = AD + BC$

$\Rightarrow \quad \frac{AB + CD}{AD + BC} = 1$ **Proved**.

29. (A) We have,

Number of people who attended the match = 50,000

Number of adults = x, number of children = y

Cost of adult ticket = ₹ 1,000, Cost of child ticket = ₹ 200

Total collection = ₹ 4,20,00,000

On the basis of above information, we have

$x + y = 50000$ ...(1)

and $1000\,x + 200\,y = 42000000$

or $5x + y = 210000$ ...(2)

From eq. (1), putting $y = 50000 - x$ in eq. (2), we get

$5x + 50000 - x = 210000$

$\Rightarrow \quad 4x = 210000 - 50000$

$\Rightarrow \quad 4x = 160000 \quad \Rightarrow \quad x = 40000$

Putting the value of x in eq. (1), we get

$40000 + y = 50000 \quad \Rightarrow \quad y = 50000 - 40000$

$\Rightarrow \quad y = 10000$

Thus, the number of adults is 40,000 and the number of children is 10,000.

(B) The given system of linear equations is:

$2x + y = 6$ ...(1)

$x + y = 5$ ...(2)

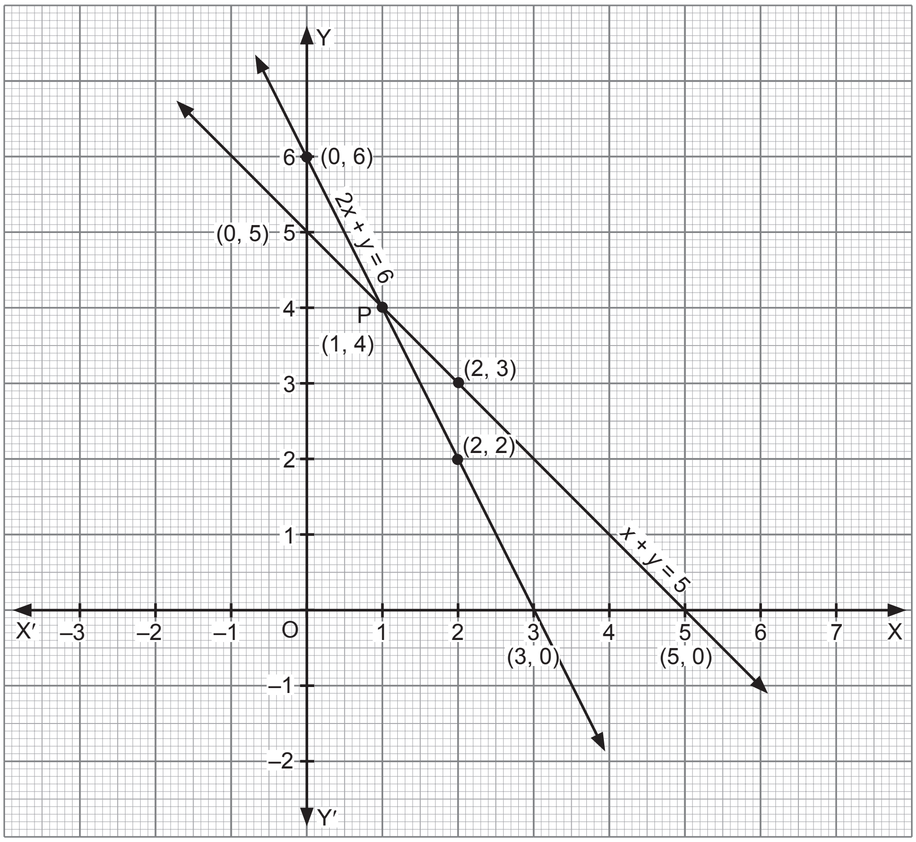
For eq. (1) the solution table is:

x	2	3	0
y	2	0	6

For eq. (2) the solution table is:

x	2	5	0
y	3	0	5

With the above values of x and y, we draw the graph of the given equations as shown below.



The two lines intersect each other at P(1, 4).

Thus, $x = 1$, and $y = 4$ is the required solution of the given equations.

30. Here, L.H.S. = $(\sin x - \cos x + 1)(\sec x - \tan x)$

$$= (\sin x - \cos x + 1)\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)$$

$$= (1 + \sin x - \cos x)\left(\frac{1 - \sin x}{\cos x}\right)$$

$$= (1 + \sin x)\left(\frac{1 - \sin x}{\cos x}\right) - \cos x\left(\frac{1 - \sin x}{\cos x}\right)$$

$$= \frac{1 - \sin^2 x}{\cos x} - (1 - \sin x)$$

$$= \frac{\cos^2 x}{\cos x} - (1 - \sin x)$$

$$= \cos x - 1 + \sin x$$

$$= \sin x + \cos x - 1 = \text{R.H.S.}$$ **Proved**.

31. Given, sum of first n terms of the A.P. is $5n^2 - n$.

$\therefore$ $S_n = 5n^2 - n$

We know that nth term of an A.P. is given by $a_n = S_n - S_{n-1}$.

So, $a_n = S_n - S_{n-1}$

$= 5n^2 - n - [5(n-1)^2 - (n-1)]$

$= 5n^2 - n - 5(n-1)^2 + (n-1)$

$= 5[n^2 - (n-1)^2] - n + n - 1$

$= 5[n^2 - n^2 + 2n - 1] - 1$

$= 5(2n-1) - 1 = 10n - 5 - 1 = 10n - 6$

Thus, nth term of the A.P. is $10n - 6$.

Alternatively:

Given, $S_n = 5n^2 - n$

Putting $n = 1$, we have

$S_1 = 5(1)^2 - 1 = 5 - 1 = 4$

So, first term of the A.P. is 4, *i.e.*, $a = 4$.

Putting $n = 2$, we have

$S_2 = 5(2)^2 - 2 = 5 \times 4 - 2$

$= 20 - 2 = 18$

So, second term of the A.P. is $S_2 - S_1 = 18 - 4 = 14$

$\therefore$ Common difference, d = second term – first term

$= 14 - 4 = 10$

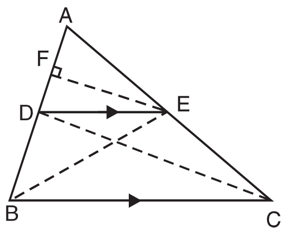
Therefore, nth term, $a_n = a + (n-1)d$

$= 4 + (n-1)10$

$= 4 + 10n - 10 = 10n - 6.$

SECTION D

32. *Given:* ΔABC in which DE ∥ BC, and intersects AB at D and AC at E.



To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE, CD and draw EF ⊥ BA.

Proof: Consider the ratio $\frac{ar(\Delta\ ADE)}{ar(\Delta\ BDE)}$

We have $\frac{ar(\Delta\ ADE)}{ar(\Delta\ BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF} = \frac{AD}{DB}$ ...(1)

Similarly, $\frac{ar(\Delta\ ADE)}{ar(\Delta\ CDE)} = \frac{AE}{EC}$ ...(2)

But $ar(\Delta BDE) = ar(\Delta CDE)$

[$\because$ ΔBDE and ΔCDE are on the same base DE and between the same parallel lines DE and BC.]

$\therefore$ $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{ar(\Delta ADE)}{ar(\Delta CDE)}$

$\Rightarrow$ $\frac{AD}{DB} = \frac{AE}{EC}$ **Proved**.

33. (A) Let x be the denominator of the given fraction. Then,

Numerator of the fraction $= x - 3$

So, the original fraction $= \frac{x-3}{x}$

On adding 2 to both of its numerator and denominator

New fraction $= \frac{x-3+2}{x+2} = \frac{x-1}{x+2}$

According to condition, we have

$$\frac{x-3}{x}+\frac{x-1}{x+2}=\frac{29}{20}$$

$\Rightarrow$ $$\frac{(x-3)(x+2)+(x-1)x}{x(x+2)}=\frac{29}{20}$$

$\Rightarrow$ $$\frac{(x^2-x-6)+x^2-x}{x^2+2x}=\frac{29}{20}$$

$\Rightarrow$ $$\frac{2x^2-2x-6}{x^2+2x}=\frac{29}{20}$$

$\Rightarrow$ $20(2x^2-2x-6)=29(x^2+2x)$

$\Rightarrow$ $40x^2-40x-120)=29x^2+58x$

$\Rightarrow$ $11x^2-98x-120=0$

$\Rightarrow$ $11x^2-110x+12x-120=0$

$\Rightarrow$ $11x(x-10)+12(x-10)=0$

$\Rightarrow$ $(x-10)(11x+12)=0$

$\Rightarrow$ $x=10$ or $x=\frac{-12}{11}$

$\Rightarrow$ $x=10$ $\left[\because x=\frac{-12}{11} \text{ is not possible.}\right]$

$\therefore$ Original fraction $=\frac{10-3}{10}$ *i.e.*, $\frac{7}{10}$

Thus, the required fraction is $\frac{7}{10}$.

OR

(B) Let the original speed of the train be x km/h.

Total distance to be covered = 300 km

$\therefore$ Time taken by the train with original speed

$$=\frac{300}{x} \text{ hours}$$

Time taken by the train with increased speed

$$=\frac{300}{x+5} \text{ hours}$$

According to the condition,

$$\frac{300}{x}-\frac{300}{x+5}=2$$

$\Rightarrow$ $$\frac{300(x+5)-300x}{x(x+5)}=2$$

$\Rightarrow$ $300x+1500-300x=2(x^2+5x)$

$\Rightarrow$ $2x^2+10x-1500=0$

$\Rightarrow$ $x^2+5x-750=0$

$\Rightarrow$ $x^2+30x-25x-750=0$

$\Rightarrow$ $x(x+30)-25(x+30)=0$

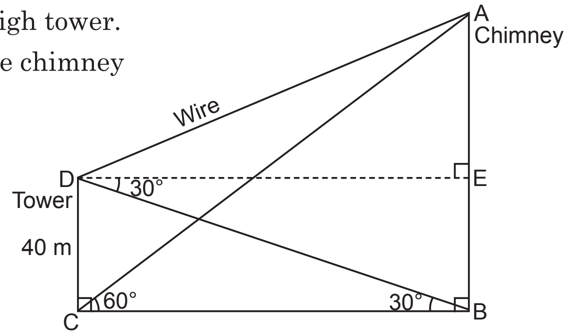
$\Rightarrow$ $(x+30)(x-25)=0$

$\Rightarrow$ $x=25$ or $x=-30$

$\Rightarrow$ $x=25$ [$\because$ $x=-30$ is not possible.]

Hence, the original speed of the train is 25 km/h.

34. (A) In the figure, AB is the chimney and CD is 40 m high tower. AD is the length of the wire tied from the top of the chimney to the top of the tower.



Here, $\angle CBD = \angle BDE = 30°$

and $\angle ACB = 60°$

In right ΔBCD, we have

$$\tan 30° = \frac{CD}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{40}{BC} \quad \Rightarrow \quad BC = 40\sqrt{3} \text{ m}$$

In right ΔABC, we have

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{AB}{40\sqrt{3}} \qquad [\because BC = 40\sqrt{3}]$$

$$\Rightarrow \quad AB = 40\sqrt{3} \times \sqrt{3} = 120 \text{ m}$$

Thus, height of the chimney is 120 m.

In right ΔAED, we have

$$AE = AB - BE$$
$$= 120 \text{ m} - 40 \text{ m} = 80 \text{ m}$$

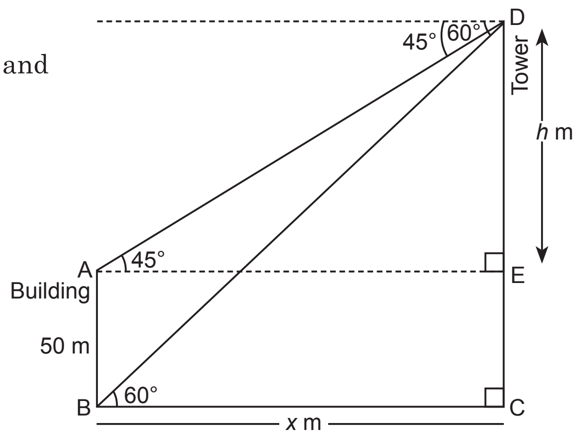
and $\quad DE = BC = 40\sqrt{3}$ m

$$\therefore AD = \sqrt{AE^2 + DE^2} = \sqrt{(80)^2 + (40\sqrt{3})^2}$$
$$= \sqrt{6400 + 4800} = \sqrt{11200} = 40\sqrt{7} \text{ m}$$

Thus, length of the wire tied from the top of the chimney to the top of the tower is $40\sqrt{7}$ m.

OR

(B) In the figure, AB is 50 m high building and CD is the tower. Let the distance between the building and the tower be x m, and DE = h m.



From the figure, we have

$\angle CBD = 60°$ and $\angle EAD = 45°$

In right ΔBCD, we have

$$\tan 60° = \frac{CD}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{DE + CE}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h + 50}{x} \quad \ldots(1) \quad [\because CE = AB = 50 \text{ m}]$$

In right ΔAED, we have

$$\tan 45° = \frac{DE}{AE}$$

$$\Rightarrow \quad 1 = \frac{h}{x} \quad \text{or} \quad h = x$$

Putting $x = h$ in (1), we get

$$\sqrt{3} = \frac{h + 50}{h}$$

$\Rightarrow \quad \sqrt{3}h = h + 50$

$\Rightarrow \quad (\sqrt{3}-1)h = 50 \quad \Rightarrow \quad h = \frac{50}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$$= \frac{50(\sqrt{3}+1)}{2}$$

$$= 25(\sqrt{3}+1)$$

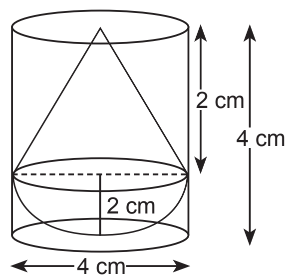
$$= 68.25 \text{ m} \qquad [\text{Taking } \sqrt{3} = 1.73]$$

$\therefore$ Height of the tower, CD = h + CE = 68.25 m + 50 m = 118.25 m and distance between the tower and the building, BC = 68.25 m.

35. If r be the radius and h be the height of the right circular cone.

Then, we have

$$r = 2 \text{ cm} \quad \text{and} \quad h = 2 \text{ cm}$$



Since radius of the base of the hemisphere is also 2 cm, we have

Volume of the toy = Volume of the cone + Volume of the hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3}\pi \times (2)^2 [2 + 2 \times 2] \text{ cu cm}$$

$$= \frac{4}{3}\pi \times 6 \text{ cu cm} = 8\pi \text{ cu cm}$$

Clearly, height of the cylinder circumscribing the toy is 4 cm and its radius is 2 cm, we have

Volume of the cylinder = πr^2H, where H is the height of the cylinder

$$= \pi(2)^2 \times 4 \text{ cu cm} = 16\pi \text{ cu cm}$$

Thus,

Difference in the volumes of the cylinder and the toy

= Volume of the cylinder – Volume of the toy

= 16π cu cm – 8π cu cm

= 8π cu cm = 8 × 3.14 cu cm = 25.12 cm cm.

SECTION E

36. Given, coordinates of A = (4, 5), coordinates of B = (6, 2) and coordinates of C = (2, 6).

(*i*) Distance between B and C = $\sqrt{(2-6)^2 + (6-2)^2}$

$$= \sqrt{(-4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ units}$$

(*ii*) The club is situated at the mid-point of the line joining the points B and C. Therefore,

Coordinates of the club = $\left(\frac{6+2}{2}, \frac{2+6}{2}\right)$ = (4, 4).

(*iii*) (A) By distance formula, we have

$$\text{OA} = \sqrt{(4-0)^2 + (5-0)^2} = \sqrt{16+25} = \sqrt{41} \text{ units}$$

$$\text{OB} = \sqrt{(6-0)^2 + (2-0)^2} = \sqrt{36+4} = \sqrt{40} \text{ units}$$

and $\quad \text{OC} = \sqrt{(2-0)^2 + (6-0)^2} = \sqrt{4+36} = \sqrt{40}$ units

Thus, society A is farthest from the office and it is at a distance of $\sqrt{41}$ units from the office.

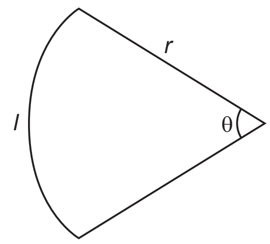
OR

(B) We have,

$$AB = \sqrt{(6-4)^2 + (2-5)^2}$$
$$= \sqrt{(2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13} \text{ units}$$
$$AC = \sqrt{(2-4)^2 + (6-5)^2}$$
$$= \sqrt{(-2)^2 + (1)^2} = \sqrt{4+1} = \sqrt{5} \text{ units}$$

Since $\sqrt{5} < \sqrt{13}$, society C is nearer to society A.

37. (*i*) If an arc of length l subtends an angle θ at the centre of a circle whose radius is r, then



$$\text{Area of sector} = \frac{\pi r^2\theta}{360°} \quad ...(1)$$

and $$\text{arc length} = \frac{2\pi r\theta}{360°}$$

or $$\frac{1}{2} \times \text{arc length} = \frac{\pi r\theta}{360°} \quad ...(2)$$

From (1), we have

$$\text{Area of sector} = \left(\frac{\pi r\theta}{360°}\right) \times r$$
$$= \frac{1}{2} \times \text{arc length} \times r \qquad \text{[Using (2)]}$$
$$= \frac{1}{2} \times \text{arc length} \times \text{radius} \qquad [\because r \text{ is the radius of the circle.}]$$

Thus, in terms of arc length, area of the sector $= \frac{1}{2} \times$ arc length $\times$ radius.

(*ii*) Here, watered region forms a sector of a circle whose radius is 21 m and angle θ is complementary angle of 10°, *i.e.*, 90° – 10° = 80°.

$$\therefore \text{Area of watered region} = \frac{\pi r^2\theta}{360°} = \frac{\pi \times 21 \times 21 \times 80°}{360°} \text{ sq m}$$
$$= \frac{\pi \times 21 \times 21 \times 2}{9} \text{ sq m}$$
$$= 98\pi \text{ sq m.}$$

(*iii*) (A) When radius changes to 28 m and area of water region remains the same, we have

$$\text{Area of watered region} = \frac{\pi \times 28 \times 28 \times \theta}{360°}$$
$$\Rightarrow \quad 98\pi = \frac{\pi \times 28 \times 28 \times \theta}{360°}$$
$$\Rightarrow \quad \theta = \frac{98 \times 360°}{28 \times 28} = 45°$$

Thus, the angle θ becomes 45° when radius changes to 28 m.

OR

(B) When radius increased to 28 m and angle remains the same, we have

$$\text{Area of the watered region} = \frac{\pi \times 28 \times 28 \times 80°}{360°} \text{ sq m}$$
$$= \frac{1568}{9} \pi \text{ sq m}$$

$\therefore$ Increase in area of watered region

$$= \left(\frac{1568}{9}\pi - 98\pi\right) \text{ sq m}$$

$$= \left(\frac{1568\pi - 882\pi}{9}\right) \text{ sq m}$$

$$= \frac{686}{9}\pi \text{ sq m}$$

$$= \frac{686}{9} \times \frac{22}{7} \text{ sq m}$$

$$= \frac{98}{9} \times 22 \text{ sq m} = 239.56 \text{ sq m.}$$

38. (*i*) From the given data, we have

$$x + 30 + 8 + 24 + 6 + 18 + 1 + 3 = 100$$

$\Rightarrow$ $x + 90 = 100 \quad \Rightarrow \quad x = 100 - 90 = 10$

Thus, the value of x is 10.

(*ii*) From the data, the number of persons (in %) who have a Rhesus negative blood type

$$= 10 + 8 + 6 + 1 = 25$$

Total number of persons (in %) = 100

$\therefore$ P(person has a Rhesus negative blood type)

$$= \frac{25}{100} = \frac{1}{4}.$$

(*iii*) (A) The person selected is Rhesus positive but neither blood type A nor B = Person selected is Rhesus positive blood type O or AB

$$= 30 + 3 = 33$$

Total number persons (in %) = 100

$\therefore$ P(person selected is a Rhesus positive but neither blood type A nor B) $= \frac{33}{100}$.

OR

(B) From the data, we have

People with blood type AB positive (AB^+) = 3

People with blood type O negative (O^-) = 10 ($\because x = 10$)

$\therefore$ Number of people with blood type AB positive or O negative = 3 + 10 = 13

i.e., Number of people who are either universal recipient or universal donor = 13

$\therefore$ Number of people who are neither universal recipient nor universal donor = 100 – 13 = 87

$\therefore$ P(person selected is neither universal recipient nor universal donor $= \frac{87}{100}$.

Solutions—Sample Question Paper (Standard) 2025-26

SECTION A

1. (*c*) Given, $a = 2^2 \times 3^x$

$b = 2^2 \times 3 \times 5$

$c = 2^2 \times 3 \times 7$

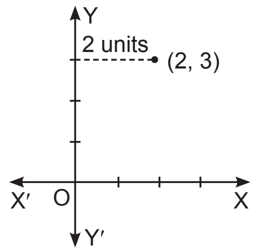
$\therefore$ LCM $(a, b, c) = 2^2 \times 3^x \times 5 \times 7$

$\Rightarrow \quad 3780 = 140 \times 3^x$

$\Rightarrow \quad 3^x = 27 \quad \Rightarrow \quad 3^x = 3^3 \quad \Rightarrow \quad x = 3.$

2. (*a*) The shortest distance of a point from the y-axis is its x-coordinate, *i.e.*, abscissa.

So, the shortest distance of the point (2, 3) from the y-axis is 2 units.



3. (*b*) If the lines $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ and not parallel, then they must be either intersecting or coincident.

For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \quad \frac{3}{2} \neq \frac{2k}{5} \quad \Rightarrow k \neq \frac{15}{4}$

For coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{3}{2} = \frac{2k}{5} = \frac{-2}{1}$

Since $\frac{3}{2} \neq \frac{-2}{1}$, the lines are not coincident.

So, the given lines intersect each other for $k \neq \frac{15}{4}$.

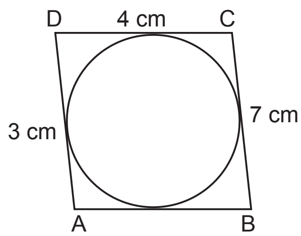
4. (*c*) Given, a quadrilateral ABCD circumscribes a circle. Then,

AB + CD = AD + BC

$\Rightarrow$ AB + 4 cm = 3 cm + 7 cm

$\Rightarrow$ AB + 4 cm = 10 cm $\quad \Rightarrow \quad$ AB = 10 cm – 4 cm

or $\quad$ AB = 6 cm.



5. (*d*) By the identity, $\sec^2\theta = 1 + \tan^2\theta$, we have

$$\sec^2\theta - \tan^2\theta = 1$$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$$\Rightarrow \quad \sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$$

$$\Rightarrow \quad \sec\theta - \tan\theta = \frac{1}{x}. \qquad [\because \sec\theta + \tan\theta = x]$$

6. (*d*) Simplifying the equation $(x + 2)(x + 1) = x^2 + 2x + 3$, we have

$$x^2 + 3x + 2 = x^2 + 2x + 3$$

$\Rightarrow \quad x - 1 = 0$, which is not a quadratic equation.

Note: The equations given in other options are quadratic equations. Students can simplify and check these equations themselves.

7. (*d*) Since radius of each circle is 1 cm and the length of the chords formed by joining the point of intersection of two circles is also 1 cm, the area of 4 dotted regions is same.

In the left most circle, by joining the two ends of chord AB with the centre O, we get an equilateral triangle of side 1 cm.

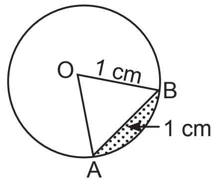
∴ Area of dotted region enclosed by the circle

$$= \text{Area of sector AOB} - \text{Area of equilateral } \Delta\text{AOB}$$

$$= \frac{\pi r^2 \theta}{360°} - \frac{\sqrt{3}}{4} r^2$$

$$= \left[\frac{\pi (1)^2\ 60°}{360°} - \frac{\sqrt{3}}{4}(1)^2\right] \text{sq cm} \qquad [\because r = 1 \text{ and } \theta = 60°]$$

$$= \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) \text{sq cm}$$



Since each of the four chords encloses 2 such regions, therefore

Total area of all the dotted regions

$$= 8\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) \text{sq cm.}$$

8. (*b*) When a pair of dice is thrown, the possible outcomes are

(1, 1), (1, 2), ..., (1, 6)

(2, 1), (2, 2), ..., (2, 6)

....

(6, 1), (6, 2), ..., (6, 6)

So, there are 36 possible outcomes in all.

Out of these, the outcomes (2, 6), (6, 2), (3, 5), (5, 3) and (4, 4) give the sum eight.

∴ Number of outcomes of getting eight = 5

So, probability of getting sum eight $= \frac{5}{36}$

∴ Probability of not getting sum eight $= 1 - \frac{5}{36} = \frac{31}{36}$.

9. (*b*) We have,

$$2 \sin 5x = \sqrt{3},\ 0° \le x \le 90°$$

$$\Rightarrow \quad \sin 5x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \sin 5x = \sin 60° \quad \Rightarrow \quad 5x = 60° \quad \text{or} \quad x = 12°.$$

10. (*c*) Since HCF of the two numbers is 81, the two numbers will in the from $81x$ and $81y$, where x and y are coprime to each other.

$$\therefore \quad 81x + 81y = 1215$$

$$\Rightarrow \quad x + y = 15$$

So, all coprime pairs having sum 15 are (1, 14), (2, 13), (4, 11) and (7, 8).

Thus, there are 4 possible pairs of such numbers.

11. (*d*) Given, area of the base of the cone, $\pi r^2 = 51$ cm^2

and volume of the cone, $\frac{1}{3}\pi r^2 h = 85$ cm^3

$$\Rightarrow \quad \frac{1}{3} \times 51 \times h = 85 \qquad [\because \pi r^2 = 51 \text{ cm}^2]$$

$$\Rightarrow \quad h = \frac{85 \times 3}{51} \times 5$$

Thus, height of the cone is 5 cm.

12. (*d*) Given, the zeros of the quadratic polynomial $ax^2 + bx + c$ $(a, c \neq 0)$ are equal.

Let α, α be the zeros of the polynomial, then

$$\alpha + \alpha = \frac{-b}{a} \quad \text{and} \quad \alpha \cdot \alpha = \frac{c}{a}$$

$$\Rightarrow \quad 2a = \frac{-b}{a} \quad \text{and} \quad \alpha^2 = \frac{c}{a}$$

Since α^2 is always a positive number, $\frac{c}{a}$ will be positive only when both c and a have the same signs.

Alternatively:

The roots of the polynomial equation $ax^2 + bx + c = 0$ $(a, c \neq 0)$ will be equal, if $b^2 - 4ac = 0$.

$$\therefore \quad b^2 - 4ac = 0 \quad \Rightarrow \quad ac = \frac{b^2}{4}$$

Since $\frac{b^2}{4}$ is always a positive number, the product ac will be positive provided both a and c have the same signs.

13. (*c*) Here, arc length = 22 cm

$$\Rightarrow \quad \frac{2\pi r\theta}{360°} = 22 \text{ cm}$$

$$\Rightarrow \quad \frac{\pi r\theta}{360°} = 11 \text{ cm}$$

$$\therefore \text{ Area of sector} = \frac{\pi r^2\theta}{360°} = \frac{\pi r\theta}{360°} \times r$$

$$= 11 \text{ cm} \times 21 \text{ cm} \qquad [\because r = 21 \text{ cm}]$$

$$= 231 \text{ cm}^2.$$

14. (*c*) Since ΔABC ~ ΔDEF, we have

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}$$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{AB + BC + CA}{DE + EF + FD}$$

$$\Rightarrow \quad \frac{6}{9} = \frac{AB + BC + CA}{9 + 6 + 12}$$

$$\Rightarrow \quad \frac{2}{3} = \frac{AB + BC + CA}{27}$$

$$\Rightarrow \quad AB + BC + CA = \frac{2 \times 27}{3} = 18 \text{ cm}$$

Thus, perimeter of ΔABC is 18 cm.

15. (*b*) Total number of letters in the word 'Mathematics' = 11

Number of vowels in the word 'Mathematics' = 4

$$\therefore \text{ Probability (the letter chosen is a vowel)} = \frac{4}{11}$$

So, we have $\frac{2}{2x+1} = \frac{4}{11}$

$$\Rightarrow \quad 2x + 1 = \frac{2 \times 11}{4}$$

$$\Rightarrow \quad 2x + 1 = \frac{11}{2}$$

$$\Rightarrow \quad 2x = \frac{11}{2} - 1 = \frac{9}{2}$$

$$\Rightarrow \quad x = \frac{9}{4}.$$

16. (*c*) We know diagonals of a parallelogram bisect each other. So, we check the vertices for the parallelogram.

$\therefore$ Mid-point of AC = $\left(\frac{9+(-9)}{2}, \frac{0+0}{2}\right)$ = (0, 0)

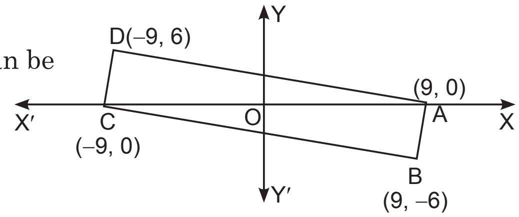
Mid-point of BD = $\left(\frac{9+(-9)}{2}, \frac{(-6)+6}{2}\right)$ = (0, 0)

Since mid-points of AC and BD are same. AC and BD bisect each other. So the given points are the vertices of a parallelogram.

Also, by distance formula, we can check AC ≠ BD, *i.e.*, the diagonals are not equal therefore it is neither a square nor a rectangle. The given vertices cannot form a trapezium as the length of opposite sides are equal.

Alternatively:

By plotting the given points in a coordinate plane it can be easily seen that the figure formed is parallelogram.

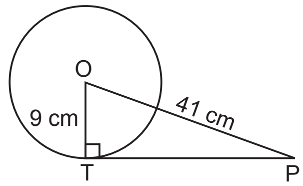


17. (*a*) Given that median of 9 distinct observations is 20.5. Since median is the middle most observation, therefore out of 9 observations, 5th observation will be the median, *i.e.*, 5th observation = 20.5.

Now, when each of the observations increases by 2, the 5th observation will also increase by 2. Therefore, new median = 20.5 + 2, *i.e.*, 22.5.

18. (*a*) In the figure, OT is the radius of the circle and P is a point 41 cm from the centre O of the circle. PT is tangent to the circle at T.

In ΔOTP, OT ⊥ PT



[∵ Tangent is ⊥ to the radius through the point of contact.]

⇒ ΔOTP is right triangle.

$\therefore \quad PT = \sqrt{OP^2 - OT^2} = \sqrt{(41)^2 - (9)^2}$

$= \sqrt{1681 - 81} = \sqrt{1600} = 40$

Thus, the length of tangent PT is 40 cm.

19. (*a*) Assertion (A) is true:

A number ends with the digit 0 if 10 is a factor of the number, *i.e.*, 2 and 5 both are the factors in its prime factorisation.

The number 5^n, where *n* is a natural number will always ends with digit 5.

So, the number 5^n cannot end with the digit 0.

Therefore, Assertion (A) is true.

Reason (R) is true:

The statement given in Reason (R) is a true statement and the correct explanation of Assertion (A).

Hence, option (*a*) is the correct answer.

20. (*a*) Assertion (A) is true:

Given, $\cos A + \cos^2 A = 1$

$\Rightarrow \quad \cos A = 1 - \cos^2 A$

$\Rightarrow \quad \cos A = \sin^2 A$ and $\cos^2 A = \sin^4 A$

$\therefore \quad \sin^2 A + \sin^4 A = \cos A + \cos^2 A$

or $\quad \sin^2 + \sin^4 A = 1$ [$\because \cos A + \cos^2 A = 1$]

Therefore, Assertion (A) is true.

Reason (R) is true:

Since $\sin^2 A + \cos^2 A = 1$ is a basic trigonometric identity, Reason (R) is true and the correct explanation of Assertion (A).

Hence, option (*a*) is the correct answer.

SECTION B

21. (A) The given A.P. is 8, 10, 12, ... up to 60 terms.

Here, first term, $a = 8$; common difference, $d = 10 - 8 = 2$ and number of terms, $n = 60$

$\therefore$ 51st term, $a_{51} = 8 + (51 - 1) \times 2$ $[\because a_n = a + (n-1)d]$

$= 8 + 50 \times 2 = 8 + 100 = 108$

and 60th term, $a_{60} = 8 + (60 - 1) \times 2$

$= 8 + 59 \times 2 = 8 + 118 = 126$

$\therefore$ Sum of last 10 terms = Sum of a_{51} to a_{60} terms

$= a_{51} + a_{52} + \ldots + a_{60}$

Using the formula, $S_n = \frac{n}{2}[a + l]$, we have

$$S_{10} = \frac{10}{2}[108 + 126] \quad [\because a = a_{51} = 108 \text{ and } l = a_{60} = 126]$$

$= 5 \times 234 = 1170$

Thus, sum of the last 10 terms of the A.P. is 1170.

Alternatively:

By using the formula, $S_n = \frac{n}{2}[2a + (n-1)d]$, we have

Sum of first 60 terms, $S_{60} = \frac{60}{2}[2 \times 8 + (60-1) \times 2]$

$= 30[16 + 59 \times 2]$

$= 30[16 + 118] = 30 \times 134 = 4020$

Sum of first 50 terms, $S_{50} = \frac{50}{2}[2 \times 8 + (50 - 1) \times 2]$

$= 25[16 + 49 \times 2]$

$= 25[16 + 98] = 25 \times 114 = 2850$

$\therefore$ Sum of last 10 terms = Sum of first 60 terms – Sum of first 50 terms

$= 4020 - 2850 = 1170.$

Note: From part (A), $a_{60} = 126$, so the given A.P. in reverse order is:

126, 124, 122, ..., 10, 8

Now we can find the sum of first 10 terms of the above A.P. which will be the sum of last 10 terms of the given A.P.

OR

(B) The given A.P. is 6, 13, 20, ..., 230.

For the above A.P., we have

First term, $a = 6$; common difference, $d = 20 - 13 = 7$ and last term, $l = 230$

$\therefore$ Last term, $l = a + (n-1)d$

$\Rightarrow$ $230 = 6 + (n-1) \times 7$

$\Rightarrow$ $230 - 6 = (n-1) \times 7$

$\Rightarrow$ $(n-1) \times 7 = 224$

$\Rightarrow$ $n - 1 = \frac{224}{7} = 32$

$\Rightarrow$ $n = 33$

So, there are 33 terms in the given A.P.

$\therefore$ Middle term $= \frac{n+1}{2} = \frac{33+1}{2} = 17$

By using the formula, $a_n = a + (n-1)d$, we have

Middle term, $a_{17} = 6 + (17-1) \times 7$

$= 6 + 16 \times 7 = 6 + 112 = 118$

Thus, middle term of the A.P. is 118.

22. Given, sin (A + B) = 1 and cos (A–B) = $\frac{\sqrt{3}}{2}$, 0° < A, B < 90°

$\therefore \quad \sin(A + B) = 1$

$\Rightarrow \quad \sin(A + B) = \sin 90°$

$\Rightarrow \quad A + B = 90° \quad ...(1)$

and $\quad \cos(A - B) = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \cos(A - B) = \cos 30°$

$\Rightarrow \quad A - B = 30° \quad ...(2)$

On adding eqs. (1) and (2), we get

$$2A = 120° \quad \Rightarrow \quad A = 60°$$

Putting A = 60° in eq. (1), we get

$60° + B = 90°$

$\Rightarrow \quad B = 90° - 60° = 30°.$

Thus, the measures of angles A and B respectively are 60° and 30°.

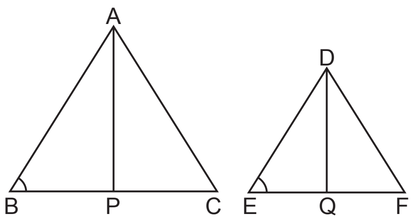
23. Given, AP and DQ are medians of triangles ABC and DEF, where $\Delta ABC \sim \Delta DEF$.

Therefore, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

So, $\frac{AB}{DE} = \frac{BC}{EF} \Rightarrow \frac{AB}{DE} = \frac{2BP}{2EQ}$ [$\because$ P and Q are mid-points of BC and EF respectively.]

$\Rightarrow \frac{AB}{DE} = \frac{BP}{EQ}$

or $\frac{AB}{BP} = \frac{DE}{EQ}$



Now, in Δs ABP and DEQ

$\Rightarrow \quad \frac{AB}{BP} = \frac{DE}{EQ}$ [Proved above]

and $\quad \angle B = \angle E$ [$\because \Delta ABC \sim \Delta DEQ$]

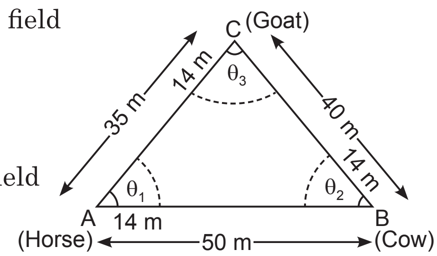
$\therefore$ By SAS Similarity Criterion, $\Delta ABP \sim \Delta DEQ$

$$\frac{AB}{DE} = \frac{AP}{DQ}$$ **Proved**.

24. (A) Let θ_1, θ_2 and θ_3 be angles at vertices A, B and C of the grass field. Then,

Area of the grass field grazed by horse tied at vertex A of the field

$$= \frac{\pi(14)^2\theta_1}{360°}$$



Similarly,

Area of the grass field grazed by cow tied at vertex B of the field

$$= \frac{\pi(14)^2\theta_2}{360°}$$

Also,

Area of the grass field grazed by goat tied at vertex C of the field

$$= \frac{\pi(14)^2\theta_3}{360°}$$

$\therefore$ Total area of the grass field grazed by horse, cow and goat

$$= \frac{\pi(14)^2\theta_1}{360°} + \frac{\pi(14)^2\theta_2}{360°} + \frac{\pi(14)^2\theta_3}{360°}$$

$$= \frac{\pi(14)^2}{360°}(\theta_1 + \theta_2 + \theta_3)$$

$$= \frac{\pi(14)^2}{360°} \times 180° \qquad [\because \theta_1 + \theta_2 + \theta_3 = 180°]$$

$$= \frac{22}{7} \times 14 \times 14 \times \frac{1}{2}$$

$$= 308 \text{ m}^2.$$

OR

(B) Let chord AB subtends an angle of 90° at the centre O of the circle.

$\therefore$ Area of minor segment = Area of sector AOB – Area of right ΔAOB

$$= \left(\frac{\pi(5)^2 90°}{360°} - \frac{1}{2} \times 5 \times 5\right) \text{cm}^2 \qquad \left(\because \text{Area of sector} = \frac{\pi r^2\theta}{360°}\right)$$

$$= \left(\frac{25\pi}{4} - \frac{25}{2}\right) \text{cm}^2$$

Now,

Area of major segment = Area of the circle – Area of minor segment

$$= \left[\pi(5)^2 - \left(\frac{25\pi}{4} - \frac{25}{2}\right)\right] \text{cm}^2$$

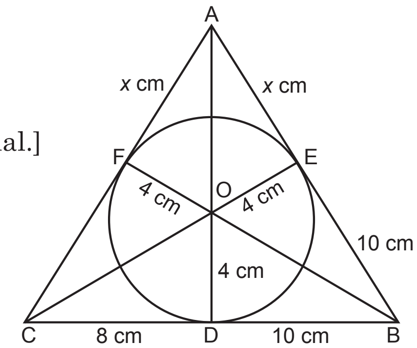
$$= \left(25\pi - \frac{25\pi}{4} + \frac{25}{2}\right) \text{cm}^2$$

$$= \left(\frac{100\pi - 25\pi}{4} + \frac{25}{2}\right) \text{cm}^2$$

$$= \left(\frac{75\pi}{4} + \frac{25}{2}\right) \text{cm}^2$$

Thus, area of the major segment is $\left(\frac{75\pi}{4} + \frac{25}{2}\right) \text{cm}^2$.

25. In the figure, we have



OD = OE = OF = 4 cm [$\because$ Radii of the same circle]

BD = BE = 10 cm [$\because$ Tangents drawn to a circle from an external point are equal.]

CD = CF = 8 cm

Let AF = AE = x cm

Also, OD $\perp$ BC, OE $\perp$ AB and OF $\perp$ AC

[$\because$ Tangent is $\perp$ to radius through the point of contact.]

Area of ΔABC = Area of ΔBOC + Area of ΔAOB + Area of ΔAOC

$$= \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AB \times OE + \frac{1}{2} \times AC \times OF$$

$\Rightarrow$ $90 = \frac{1}{2} \times (10 + 8) \times 4 + \frac{1}{2} \times (10 + x) \times 4 + \frac{1}{2} \times (8 + x) \times 4$ [$\because$ $ar(\Delta ABC = 90 \text{ cm}^2]$

$\Rightarrow$ $90 = 36 + 2(10 + x) + 2(8 + x)$

$\Rightarrow$ $90 = 36 + 20 + 16 + 4x$

$\Rightarrow$ $90 = 72 + 4x$

$\Rightarrow \quad 4x = 90 - 72 = 18$

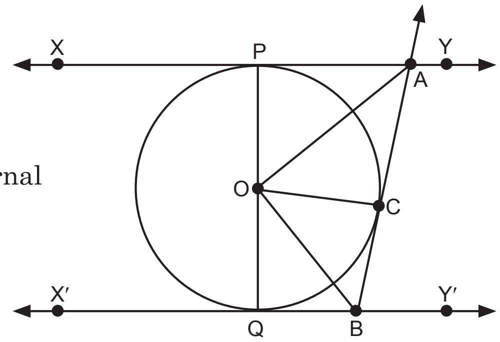
$\Rightarrow \quad x = 4.5$ cm

Thus, AB = 10 + x = (10 + 4.5) cm = 14.5 cm

and AC = 8 + x = (8 + 4.5) cm = 12.5 cm.

SECTION C

26. Given, XY and X′Y′ are two parallel tangents to a circle with centre O. Tangent AB touches the circle at C and intersect XY and X′Y′ at A and B respectively.



In Δs AOP and AOC, we have

AP = AC [$\because$ Tangents to a circle from an external point are equal.]

OP = OC [Radii of same circle.]

and OA = OA [Common]

$\therefore$ By SSS Congruence Criterion, $\Delta AOP \cong \Delta AOC$

$\Rightarrow \quad \angle AOP = \angle AOC$ ...(1) [By CPCT]

Similarity, we can prove that $\Delta BOQ \cong \Delta BOC$

$\Rightarrow \quad \angle BOQ = \angle BOC$ ...(2)

Now, POQ is diameter of the circle. Therefore,

$\angle AOP + \angle AOC + \angle BOC + \angle BOQ = 180°$

$\Rightarrow \quad 2\angle AOC + 2\angle BOC = 180°$ [Using (1) and (2)]

$\Rightarrow \quad \angle AOC + \angle BOC = 90°$

$\Rightarrow \quad \angle AOB = 90°$ **Proved**.

27. Given, number of teachers of English, Hindi and Science are 36, 60 and 84.

For same number of teachers to be seated in each room, we need to find HCF of 36, 60 and 84.

$\therefore$ $36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$

So, HCF (36, 60, 84) = $2^2 \times 3 = 12$

Thus, 12 teachers are to be seated in each room.

Also, number of rooms required for English teachers

$$= \frac{\text{Number of English teachers}}{\text{Number of teachers in each room}}$$

$$= \frac{36}{12} = 3$$

Similarly, number of rooms required for Hindi teachers

$$= \frac{60}{12} = 5$$

and number of rooms required for Science teachers

$$= \frac{84}{12} = 7$$

Thus, minimum number rooms required for all teachers

$$= 3 + 5 + 7 = 15.$$

28. The given quadratic polynomial is:

$$p(x) = 2x^2 - (1 + 2\sqrt{2})x + \sqrt{2}$$

By factorisation, we have

$$2x^2 - (1 + 2\sqrt{2})x + \sqrt{2} = 2x^2 - x - 2\sqrt{2}x + \sqrt{2}$$

$$= 2x^2 - 2\sqrt{2}x - x + \sqrt{2}$$
$$= 2x(x - \sqrt{2}) - 1\,(x - \sqrt{2})$$
$$= (x - \sqrt{2})(2x - 1)$$

Thus, $p(x) = (x - \sqrt{2})(2x - 1)$

The zeros of $p(x)$ are given by $p(x) = 0$.

$\therefore \quad p(x) = 0 \Rightarrow (x - \sqrt{2})(2x - 1) = 0$

$\Rightarrow x - \sqrt{2} = 0$ or $2x - 1 = 0$

$\Rightarrow x = \sqrt{2}$ or $x = \frac{1}{2}$

So, $x = \sqrt{2}$ and $x = \frac{1}{2}$ are the zeros of the given polynomial.

Verification:

For a quadratic polynomial,

$$\text{Sum of zeros} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\Rightarrow \quad \sqrt{2} + \frac{1}{2} = \frac{-\{-(1 + 2\sqrt{2})\}}{2} = \frac{1 + 2\sqrt{2}}{2} = \frac{1}{2} + \sqrt{2}$$

and
$$\text{Product of zeros} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\Rightarrow \quad \sqrt{2}\left(\frac{1}{2}\right) = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Hence, the zeros are verified.

29. (A) We have, $\sin\theta + \cos\theta = \sqrt{3}$

Squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3$

$\Rightarrow 2\sin\theta\cos\theta = 2$ or $\sin\theta\cos\theta = 1$ ...(1)

Now,
$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\sin\theta\cos\theta} = 1 \quad \text{[Using (1)]}$$

Thus, $\tan\theta + \cot\theta = 1$ **Proved**.

OR

(B) We have,
$$\text{L.H.S.} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$
$$= \frac{(\cos A - \sin A + 1)(\cos A + \sin A + 1)}{(\cos A + \sin A - 1)(\cos A + \sin A + 1)}$$
$$= \frac{\{(\cos A + 1) - \sin A\}\{(\cos A + 1) + \sin A\}}{\{(\cos A + \sin A) - 1\}\{(\cos A + \sin A) + 1\}}$$
$$= \frac{(\cos A + 1)^2 - \sin^2 A}{(\cos A + \sin A)^2 - 1}$$

$$= \frac{\cos^2 A + 2\cos A + 1 - \sin^2 A}{\cos^2 A + \sin^2 A + 2\sin A\cos A - 1}$$

$$= \frac{2\cos^2 A + 2\cos A}{1 + 2\sin A\cos A - 1} \qquad [\because 1 - \sin^2 A = \cos^2 A]$$

$$= \frac{2\cos A(\cos A + 1)}{2\sin A\cos A} = \frac{\cos A + 1}{\sin A} = \frac{\cos A}{\sin A} + \frac{1}{\sin A}$$

$= \cot A + \text{cosec } A$ = R.H.S. **Proved**.

30. When a coin is tossed three times, the all possible outcomes are HHH, HHT, HTH, THH, HTT, THT, TTH, TTT.

$\therefore$ Total number of possible outcomes = 8

According to condition,

1. Vidhi drive the car if she gets two heads in a row. So, favourable outcomes are HHT, THH, HHH.

   $\therefore$ Number of favourable outcomes = 3

   Thus, P(Vidhi drives the car) $= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

   $= \frac{3}{8}$

2. Unnati drive the car if she gets a head immediately followed by a tail

   So, favourable outcomes are THT, THH, HTH, TTH.

   $\therefore$ Number of favourable outcomes = 4

   Thus, P(Unnati drives the car) $= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

   $= \frac{4}{8}$

   Since $\frac{4}{8} > \frac{3}{8}$, Unnati has greater probability to drive the car.

31. (A) Let monthly incomes of Aryan and Babban be $3x$ and $4x$ respectively and their monthly expenditures be $5y$ and $7y$ respectively.

Given that each saves ₹ 15,000 per month. Therefore,

Monthly saving of Aryan = $3x - 5y$

$\Rightarrow \quad 3x - 5y = 15000 \quad \ldots(1)$

Monthly saving Babban = $4x - 7y$

$\Rightarrow \quad 4x - 7y = 15000 \quad \ldots(2)$

Multiplying eq. (1) by 7, we get

$\Rightarrow \quad 21x - 35y = 105000 \quad \ldots(3)$

Multiplying eq. (2) by 5, we get

$\Rightarrow \quad 20x - 35y = 75000 \quad \ldots(4)$

Subtracting eq. (4) from (3), we get

$\Rightarrow \quad x = 30000$

So, the value of x is ₹ 30,000

Thus, monthly income of Aryan = $3x = 3 \times$ ₹ 30,000 = ₹ 90,000

and monthly income of Babban = $4x = 4 \times$ ₹ 30,000 = ₹ 1,20,000.

OR

(B) The given system of equations is:

$2x + y = 6 \quad \ldots(1)$

and $2x - y = 2 \quad \ldots(2)$

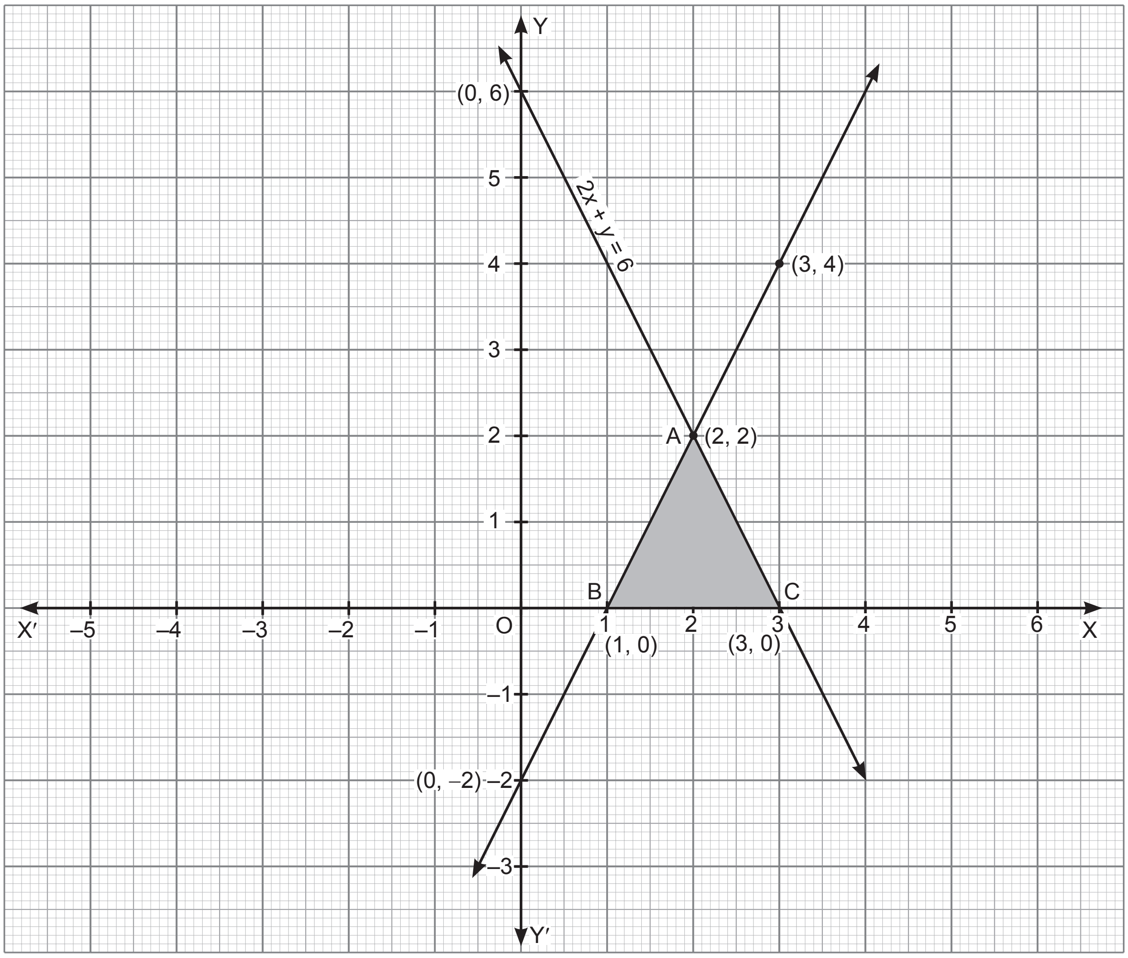
The solution table for eq. (1) is:

x	3	0	2
y	0	6	2

The solution table for eq. (2) is:

x	1	0	3
y	0	–2	4

With the above values of x and y, we draw the graph of the given lines as shown below:



In the above graph, the two lines intersect each other at (2, 2).

Thus, $x = 2$, $y = 2$ is the solution of the given equations.

The triangle formed with the two lines and the x-axis is ΔABC whose base BC is 2 units and corresponding height is also 2 units.

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 2 \times 2 = 2$ sq units.

SECTION D

32. Let original average speed of the train be x km/h. Then,

Time taken by the train to travel 63 km with original speed $= \frac{63}{x}$ hours

Time taken by the train to travel 72 km with increased speed $= \frac{72}{x+6}$ hours

Given that the train completes the total journey in 3 hours

$\therefore \quad \frac{63}{x} + \frac{72}{x+6} = 3$

$\Rightarrow \quad \frac{63(x+6)+72x}{x(x+6)} = 3$

$\Rightarrow \quad 63x + 378 + 72x = 3x(x + 6)$

$\Rightarrow \quad 135x + 378 = 3x^2 + 18x$

$\Rightarrow \quad 3x^2 + 18x - 135x - 378 = 0$

$\Rightarrow \quad 3x^2 - 117x - 378 = 0$

$\Rightarrow \quad x^2 - 39x - 126 = 0$

$\Rightarrow \quad x^2 - 42x + 3x - 126 = 0$

$\Rightarrow \quad x(x - 42) + 3(x - 42) = 0$

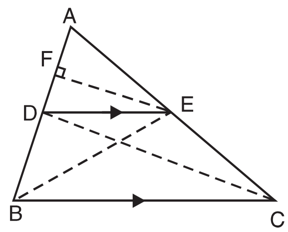
$\Rightarrow \quad (x - 42)(x + 3) = 0$

$\Rightarrow \quad x = 42 \quad \text{or} \quad x = -3$

$\Rightarrow \quad x = 42$ [$\because x = -3$ is not possible.]

Thus, the original average speed of the train is 42 km/h.

33. *Given:* ΔABC in which DE ∥ BC, and intersects AB at D and AC at E.



To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE, CD and draw EF ⊥ BA.

Proof: Consider the ratio $\frac{ar(\Delta\ ADE)}{ar(\Delta\ BDE)}$

We have $\frac{ar(\Delta\ ADE)}{ar(\Delta\ BDE)} = \frac{\frac{1}{2}\times AD\times EF}{\frac{1}{2}\times DB\times EF} = \frac{AD}{DB}$ ...(1)

Similarly, $\frac{ar(\Delta\ ADE)}{ar(\Delta\ CDE)} = \frac{AE}{EC}$ ...(2)

But $ar(\Delta BDE) = ar(\Delta CDE)$

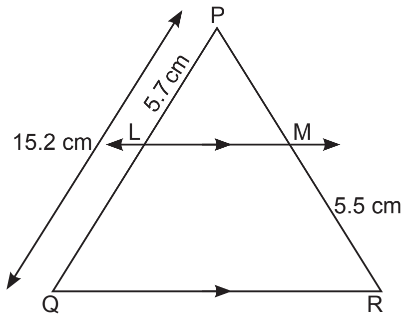
[$\because$ ΔBDE and ΔCDE are on the same base DE and between the same parallel lines DE and BC.]

$\therefore \quad \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{ar(\Delta ADE)}{ar(\Delta CDE)}$

$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$ **Proved**.

Given that in ΔPQR, LM || QR.

Therefore, by Basic Proportionality Theorem,



$\frac{PL}{LQ} = \frac{PM}{MR}$

$\Rightarrow \quad \frac{PL}{PQ - PL} = \frac{PM}{MR}$

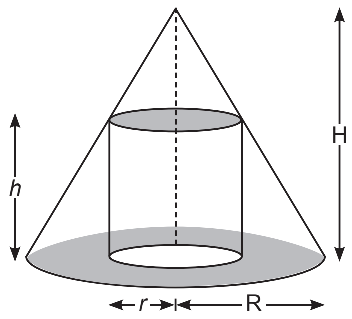
$\Rightarrow \quad \frac{5.7}{15.2 - 5.7} = \frac{PM}{5.5}$

$\Rightarrow \quad \frac{5.7}{9.5} = \frac{PM}{5.5}$

$\Rightarrow \quad PM = \frac{5.7 \times 5.5}{9.5} = 3.3$

Thus, the length of PM is 3.3 cm.

34. (A) Let H be the height and R be radius of base of cone and h and r respectively be the height and base radius of the cylinder hollowed out. Then,



We have, H = 6 cm, R = 12 cm

and $h = 3$ cm, $r = 4$ cm

Also, slant height of the cone, $L = \sqrt{H^2 + R^2} = \sqrt{(6)^2 + (12)^2}$

$= \sqrt{36 + 144} = \sqrt{180} = 6\sqrt{5}$ cm

$\therefore$ Curved surface area of the cone = πRL

$= \pi \times 12 \times 6\sqrt{5} = 72\sqrt{5}\,\pi$ cm^2

Curved surface area of the cylinder

$= 2\pi rh$

$= 2\pi \times 4 \times 3 = 24\pi$ cm^2

Area of base of the cone (shaded region in figure)

$= \pi R^2 - \pi r^2$

$= \pi(12)^2 - \pi(4)^2 = 144\pi - 16\pi$

$= 128\pi$ cm^2

Area of base of the cylinder (shaded region in figure)

$= \pi r^2$

$= \pi(4)^2 = 16\pi$ cm^2

$\therefore$ Surface area of the remaining solid

= Curved surface area of the cone + Curved surface area of the cylinder + Area of base of the cone + Area of base of the cylinder

$= 72\sqrt{5}\,\pi$ cm^2 + 24π cm^2 + 128π cm^2 + 16π cm^2

$= 72\sqrt{5}\,\pi$ cm^2 + 168π cm^2 = $24(3\sqrt{5} + 7)\,\pi$ cm^2.

OR

(B) Here, height of the cone, $h = 12$ cm

and radius of base of the cone, $r = 3$ cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (3)^2 \times 12 = 36\pi$ cm^3

Given that $\left(\frac{1}{6}\right)$th part of the cone is unfilled (empty). Therefore,

Volume of ice-cream in the cone $= \frac{5}{6}$ th part of volume of the cone

$= \frac{5}{6} \times 36\,\pi$ cm^3

$= 30\pi$ cm^3

Since radius of hemispherical part and conical part are same, we have

Volume of ice-cream in the hemispherical top

$= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times (3)^3$ cm^3

$= 18\pi$ cm^3

∴ Volume of the ice-cream = Volume of ice-cream in the cone

+ Volume of ice-cream in the hemispherical top

$= 30\pi \text{ cm}^3 + 18\pi \text{ cm}^3$

$= 48\pi \text{ cm}^3 = 48 \times \frac{22}{7} \text{ cm}^3 = 150.86 \text{ cm}^3$ (approx).

35. (A) Given that mode of the distribution is 55 and 55 lies in the class interval 45–60.

Therefore, *modal class* is 45–60.

According to the modal class, 45–60, we have

$l = 45, f_0 = x, f_1 = 15, f_2 = 10$ and $h = 15$

Using the formula, mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$, we have

$$\text{Mode} = 45 + \frac{15 - x}{30 - x - 10} \times 15$$

$$\Rightarrow \quad 55 = 45 + \frac{15 - x}{20 - x} \times 15$$

$$\Rightarrow \quad 55 - 45 = \frac{15 - x}{20 - x} \times 15$$

$$\Rightarrow \quad 10 = \frac{15 - x}{20 - x} \times 15 \quad \Rightarrow \quad \frac{15 - x}{20 - x} = \frac{10}{15}$$

$$\Rightarrow \quad \frac{15 - x}{20 - x} = \frac{2}{3}$$

$$\Rightarrow \quad 45 - 3x = 40 - 2x$$

$$\Rightarrow \quad 45 - 40 = 3x - 2x$$

$$\text{or} \quad x = 5$$

Thus, the value of x is 5.

Now, to find the mean, we put $x = 5$ in the given distribution and form the following table.

Class Interval (C.I.)	*Frequency* (f_i)	*Class Mark* (x_i)	*Deviation* $\left(u_i = \frac{x_i - A}{h}\right)$	*Product* (f_iu_i)
0–15	10	7.5	–2	–20
15–30	7	22.5	–1	–7
30–45	5	37.5 = A	0	0
45–60	15	52.5	1	15
60–75	10	67.5	2	20
75–90	12	82.5	3	26
Total	$\Sigma f_i = 59$			$\Sigma f_iu_i = 44$

Here, assmed mean, A is taken as 37.5.

$$\therefore \quad \text{Mean, } \bar{x} = A + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h$$

$$= 37.5 + \frac{44}{59} \times 15$$

$$= 48.68 \text{ (approx)}$$

Thus, mean of the given distribution is 48.68 (approx).

OR

(B) From the given distribution, we form the following continuous frequency distribution:

Height (in cm)	*Frequency* (f_i)	*Cumulative Frequency (cf)*
135–140	4	4
140–145	7	11
145–150	18	29
150–155	11	40
155–160	6	46
160–165	5	51
Total	$\Sigma f_i = 51$	

Here, $N = \Sigma f_i = 51$

$\therefore \quad \frac{N}{2} = \frac{51}{2} = 25.5$

The cumulative frequency just greater than 25.5 is 29 and the class corresponding to this frequency is 145–150.

So, 145–150 is the *median class*.

$\therefore$ For the median class 145–150, we have

$l = 145, f = 18, c = 11$ and $h = 5$

$$\therefore \text{ Median} = l + \frac{\left(\frac{N}{2} - c\right)}{f} \times h$$

$$= 145 + \frac{25.5 - 11}{18} \times 5$$

$$= 145 + \frac{14.5}{18} \times 5 = 145 + \frac{72.5}{18}$$

$$= 145 + 4.03 = 149.03$$

Thus, median height of the girls is 149.03 cm.

Now, by empirical formula,

$3 \times \text{Median} = \text{Mode} + 2 \times \text{Mean}$, we have

$3 \times 149.03 = 148.05 + 2 \times \text{Mean}$ [$\because$ Mode = 148.05]

$\Rightarrow \quad 447.09 = 148.05 + 2 \times \text{Mean}$

$\Rightarrow \quad 229.04 = 2 \times \text{Means}$

or $\quad \text{Mean} = \frac{299.04}{2} = 149.52$

Thus, mean of the given distribution is 149.52 cm.

SECTION E

36. (*i*) A.P. written by Aryan is –5, –2, 1, 4, …

A.P. written by Roshan is 187, 184, 181, …

Let d_1 and d_2 respectively be the common difference of the two A.Ps.

Then, $d_1 = -2 - (-5) = -2 + 5 = 3$

$d_2 = 184 - 187 = -3$

$\therefore \quad d_1 + d_2 = 3 - 3 = 0$

So, sum of the common difference of two A.Ps. is 0.

(*ii*) For the A.P. 187, 184, 181, …

First term, $a = 187$; common difference, $d = -3$

$\therefore$ 34th term, $a_{34} = 187 + (34 - 1)(-3)$

$= 187 + 33 \times (-3) = 187 - 99$

$= 88$

So, 34th term of the A.P. is 88.

(*iii*) (A) For the A.P. –5, –2, 1, 4 ...

First term, $a = -5$; common difference, $d = 3$ and number of terms, $n = 10$

$\therefore$ Sum of first 10 terms, $S_{10} = \frac{10}{2}[2 \times (-5) + (10-1)(3)]$

$= 5[-10 + 9 \times 3]$

$= 5(-10 + 27) = 5 \times 17 = 85$

Thus, sum of the first 10 terms of the A.P. is 85.

OR

(B) Let nth term of the two A.Ps. have the same value. Then,

nth term of the first A.P., $a_n = -5 + (n-1)(3)$

$= -5 + 3n - 3$

$= 3n - 8$

nth term of the second A.P., $a'_n = 187 + (n-1)(-3)$

$= 187 - 3n + 3$

$= 190 - 3n$

$\therefore a_n = a'_n \Rightarrow 3n - 8 = 190 - 3n$

$\Rightarrow 6n = 198$

or $n = 33$

Thus, 33rd term of the two A.Ps. have the same value.

37. (*i*) From the given diagram, we have

Coordinates of P = (2, 5) and coordinates of R = (8, 3)

$\therefore$ By distance formula, $d = \sqrt{(8-2)^2 + (3-5)^2}$

$= \sqrt{(6)^2 + (-2)^2} = \sqrt{36+4} = \sqrt{40}$

or $2\sqrt{10}$

Thus, the distance between P and R is $2\sqrt{10}$ units.

(*ii*) Mid-point of PR $= \left(\frac{2+8}{2}, \frac{5+3}{2}\right)$

$= \left(\frac{10}{2}, \frac{8}{2}\right) = (5, 4)$

From the diagram, coordinates of Q = (4, 4)

So, Q is not the mid-point of PR.

(*iii*) (A) Let T (x, 0) be the point on x-axis which is equidistant from P and Q.

$\therefore$ PT = QT

$\Rightarrow \sqrt{(x-2)^2 + (0-5)^2} = \sqrt{(x-4)^2 + (0-4)^2}$

$\Rightarrow \sqrt{(x-2)^2 + 25} = \sqrt{(x-4)^2 + 16}$

$\Rightarrow (x-2)^2 + 25 = (x-4)^2 + 16$

$\Rightarrow x^2 - 4x + 4 + 25 = x^2 - 8x + 16 + 16$

$\Rightarrow x^2 - 4x + 29 = x^2 - 8x + 32$

$\Rightarrow -4x + 8x = 32 - 29$

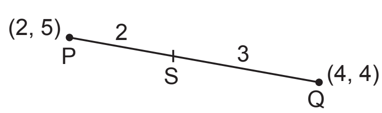
$\Rightarrow 4x = 3$ or $x = \frac{3}{4}$

Therefore, the required point is T$\left(\frac{3}{4}, 0\right)$.

OR

(B) Given that the point S divides the line joining PQ in the ratio 2 : 3.

$\therefore$ By section formula, $(x, y) = \left(\dfrac{mn_2 + nx_1}{m+n}, \dfrac{my_2 + ny_1}{m+n}\right)$



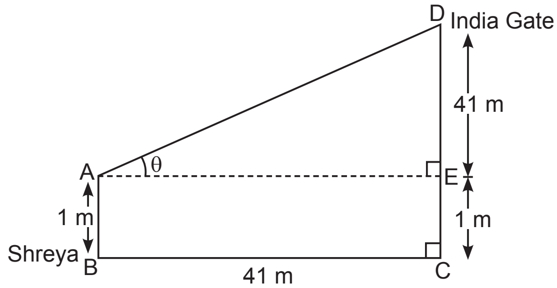
Replacing (x_1, y_1) and (x_2, y_2) with the coordinates (2, 5) and (4, 4) respectively and $m : n = 2 : 3$, we have

$$(x, y) = \left(\frac{2\times4+3\times2}{2+3}, \frac{2\times4+3\times5}{2+3}\right) = \left(\frac{8+6}{5}, \frac{8+15}{5}\right) = \left(\frac{14}{5}, \frac{23}{5}\right)$$

Thus, the coordinates of S are $\left(\dfrac{14}{5}, \dfrac{23}{5}\right)$.

38. (*i*) In the adjoining diagram, AB is the position of Shreya and CD denotes 42 m high India Gate. Let θ be the angle of elevation of the top of India Gate and BC = AE = 41 m be the distance between Shreya and India Gate.

$\therefore$ In right ΔAED, we have



$$\tan\theta = \frac{DE}{AE}$$

$$\Rightarrow \quad \tan\theta = \frac{41}{41}$$

$$\Rightarrow \quad \tan\theta = 1 \quad \Rightarrow \quad \tan\theta = \tan 45°$$

$$\text{or} \quad \theta = 45°$$

Thus, the angle of elevation is 45°.

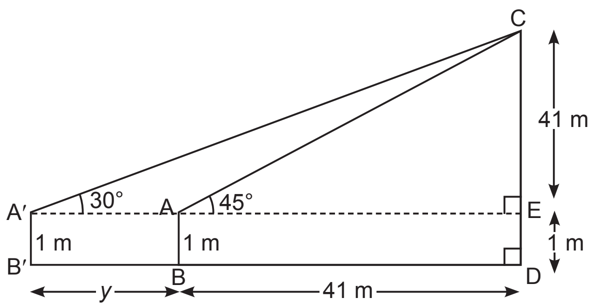
(*ii*) In the figure drawn in part (*i*), let AE = x m, and θ = 60°, we have

$$\tan 60° = \frac{DE}{AE}$$

$$\Rightarrow \quad \sqrt{3} = \frac{41}{x} \quad \Rightarrow \quad x = \frac{41}{\sqrt{3}} \text{ m} \quad \text{or} \quad x = \frac{41\sqrt{3}}{3}$$

Thus, Shreya is standing at a distance of $\dfrac{41\sqrt{3}}{3}$ m from the India Gate.

(*iii*) (A) Let Shreya moves y m back from her original position and reached to new position A′B′. Then, In right ΔA′EC, we have



$$\tan 30° = \frac{CE}{A'E}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{41}{y+41}$$

[∵ A′E = A′A + AE]

$$\Rightarrow \quad y + 41 = 41\sqrt{3}$$

$$\Rightarrow \quad y = 41\sqrt{3} - 41 \quad \text{or} \quad y = 41(\sqrt{3} - 1) \text{ m}$$

So, the distance she moved back is $41(\sqrt{3} - 1)$ m.

OR

(B) In the figure drawn in part (*i*), let AE = $\dfrac{41}{\sqrt{3}}$ m, then

$$\tan\theta = \frac{41}{\frac{41}{\sqrt{3}}}$$

$$\Rightarrow \tan\theta = \frac{41\times\sqrt{3}}{41} \quad \Rightarrow \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \quad \tan\theta = \tan 60°$$

$$\text{or} \quad \theta = 60°$$

So, in this case angle of elevation is 60°.