SOLUTIONS OF ICSE EXAMINATION 2020 QUESTIONS

Section A

Solution 1

(*a*) Here, $x^2 - 7x + 3 = 0$

Using quadratic formula, we have

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{7 \pm \sqrt{49-12}}{2}$$

$$= \frac{7 \pm \sqrt{37}}{2}$$

$$= \frac{7 \pm 6.083}{2}$$

$$= \frac{13.083}{2} \text{ or } \frac{0.917}{2}$$

$= 6.54$ or 0.46. (correct to two decimal places)

(*b*) Here, $A^2 = 3I$

$$\Rightarrow \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}\begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x^2+3y & 3x+9 \\ xy+3y & 3y+9 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Corresponding elements of above matrices must be equal.

$\therefore$ We have, $x^2 + 3y = 3$ ...(1)

$3x + 9 = 0$ ...(2) $\Rightarrow x = -3.$

$xy + 3y = 0$ ...(3)

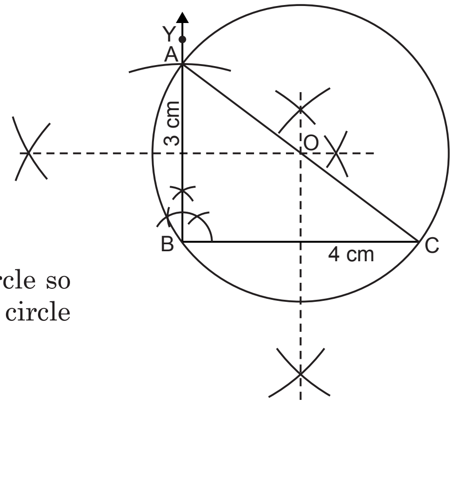
$3y + 9 = 3$ ...(4) $\Rightarrow y = -2.$

These values of x and y also satisfy equations (1) and (3).

Hence, $x = -3$ and $y = -2$ are the required values.

(*c*) **Steps of Construction**

1. Draw BC = 4 cm.
2. At B, construct $\angle$YBC = 90°.
3. From BY cut off BA = 3 cm.
4. Join AC. We get ΔABC with the given data.
5. Draw perpendicular bisectors of BC and BA. Let these bisectors meet at point O.
6. With O as centre and radius OA, draw a circle. The circle so drawn passes through A, B and C and is the required circle circumscribing ΔABC.



Here, measure of the radius of circle is 2.5 cm.

Solution 2

(*a*) Let $f(x) = 6x^3 + 17x^2 + 4x - 12$.

Here, $f(-2) = 6(-2)^3 + 17(-2)^2 + 4(-2) - 12$

$= -48 + 68 - 8 - 12 = 0$

$\therefore$ By factor theorem, $(x + 2)$ is factor of $f(x)$.

On dividing $f(x)$ by $(x + 2)$, we get $6x^2 + 5x - 6$ as the quotient and remainder as 0.

$$
\begin{array}{r|l}
 & 6x^2 + 5x - 2 \\
\hline
x + 2 & 6x^3 + 17x^2 + 4x - 12 \\
 & 6x^3 + 12x^2 \\
 & - \quad - \\
\hline
 & 5x^2 + 4x - 12 \\
 & 5x^2 + 10x \\
 & - \quad - \\
\hline
 & -6x - 12 \\
 & -6x - 12 \\
 & + \quad + \\
\hline
 & 0
\end{array}
$$

$\therefore$ The other factors of $f(x)$ are the factors of $6x^2 + 5x - 6$.

Now, $6x^2 + 5x - 6 = (2x + 3)(3x - 2)$

Hence, $f(x) = 6x^3 + 17x^2 + 4x - 12$

$= (x + 2)(2x + 3)(3x - 2)$.

(*b*) Here, $\dfrac{3x}{5} + 2 < x + 4 \le \dfrac{x}{2} + 5$ is equivalent to

$\dfrac{3x}{5} + 2 < x + 4$ and $x + 4 \le \dfrac{x}{2} + 5$

$\Rightarrow$ $3x + 10 < 5x + 20$ and $2x + 8 \le x + 10$

$\Rightarrow$ $3x - 5x < 20 - 10$ and $2x - x \le 10 - 8$

$\Rightarrow$ $-2x < 10$ and $x \le 2$

$\Rightarrow$ $x > -5$ and $x \le 2$

The solution set is $\{x: -5 < x \le 2, x \in \mathrm{R}\}$

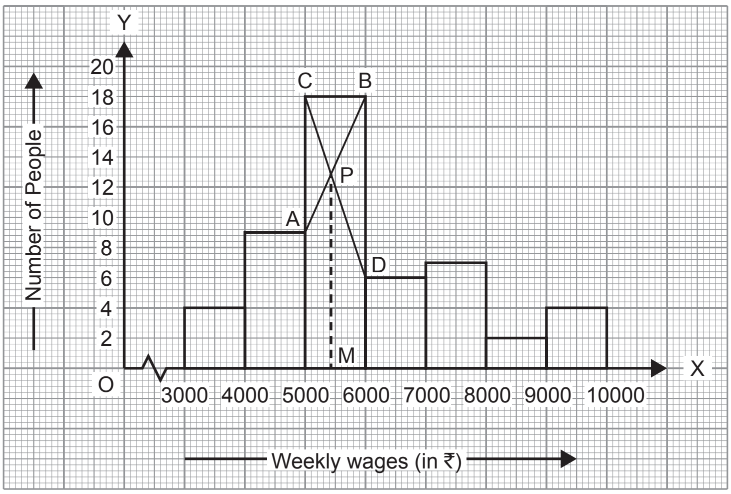
The solution on real number line is represented as

(*c*) (*i*) Construct rectangles corresponding to the given data. The required histogram is shown in the figure below.

(*ii*) In the highest rectangle, draw two straight lines AB and CD from corners of rectangles on either side of the highest rectangle to the opposite corners of the highest rectangle. Let P be the point of intersection of AB and CD.

(*iii*) Through P, draw a vertical line to meet the *x*-axis at M. The abscissa of point M represents 5400.

$\therefore$ The required mode is ₹ 5,400.



Solution 3

(*a*) (*i*) Here, $\angle ABC = \frac{1}{2}\angle AOC$ [Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.]

$= \frac{1}{2}(72°)$

$= 36°$.

(*ii*) Since, $AC = BD$ [Equal chords of a circle subtends equal angles at any point on the major (or minor) arcs of the circle.]

$\therefore$ $\angle ABC = \angle BAD$

$\therefore$ $\angle BAD = 36°$.

(*iii*) In ΔABD, $\angle ADB = 90°$ [Angle in a semicircle]

$\therefore$ $\angle ABD = 180° - (\angle ADB + \angle BAD)$

$= 180° - (90° + 36°)$

$= 54°$.

(*b*) Here, $\frac{\sin A}{1+\cot A} - \frac{\cos A}{1+\tan A}$

$$= \frac{\sin A}{1+\frac{\cos A}{\sin A}} - \frac{\cos A}{1+\frac{\sin A}{\cos A}}$$

$$= \frac{\sin^2 A}{\sin A + \cos A} - \frac{\cos^2 A}{\cos A + \sin A}$$

$$= \frac{\sin^2 A - \cos^2 A}{\sin A + \cos A}$$

$$= \frac{(\sin A - \cos A)(\sin A + \cos A)}{\sin A + \cos A}$$

$= \sin A - \cos A$.

(*c*) Let y-axis ($x = 0$) divide the line joining the points P(5, 3) and Q(–5, 3) in the ratio $m_1 : m_2$ at the point R, *i.e.*, PR : RQ = $m_1 : m_2$.

$\therefore$ Coordinates of R are $\left(\frac{-5m_1 + 5m_2}{m_1 + m_2}, \frac{3m_1 + 3m_2}{m_1 + m_2}\right)$

As the point R lies on y-axis ($x = 0$), we have

$$\frac{-5m_1 + 5m_2}{m_1 + m_2} = 0$$

$\Rightarrow$ $-5m_1 + 5m_2 = 0$

$\Rightarrow$ $m_1 = m_2$ or $m_1 : m_2 = 1 : 1$

Thus, R is mid-point of line joining points P(5, 3) and Q(–5, 3).

Hence, the coordinates of point of intersection are $\left(\frac{5+(-5)}{2}, \frac{3+3}{2}\right)$, *i.e.*, (0, 3).

Solution 4

(*a*) Here, r (radius of solid spherical ball) = 6 cm

Let r_1 be the radius of each spherical marble recast.

Now,

64 (Volume of spherical marble) = Volume of solid spherical ball

$\Rightarrow$ $64 \cdot \left(\frac{4}{3}\pi r_1^3\right) = \frac{4}{3}\pi r^3$

$$\Rightarrow \qquad 64{r_1}^3 = r^3$$

$$\Rightarrow \qquad 64{r_1}^3 = (6)^3$$

$$\Rightarrow \qquad {r_1}^3 = \left(\frac{6}{4}\right)^3$$

$$\Rightarrow \qquad r_1 = \frac{6}{4} = 1.5$$

Hence, the radius of each spherical marble is 1.5 cm.

(*b*) Since the 10 letters of the word 'AUTHORIZES' are written on 10 identical circular discs and put in a bag.

So the total number of possible outcomes is 10.

(*i*) Here, the number of favourable outcomes to the event that the 'letter is a vowel' is 5 (since all 5 vowels A, E, I, O, U appear in the word AUTHORIZES).

$$\therefore \quad \text{P(letter is a vowel)} = \frac{5}{10} = \frac{1}{2}.$$

(*ii*) The first 9 letters of English alphabets are

A, B, C, D, E, F, G, H, I.

Out of these only A, E, H, I appear in the word AUTHORIZES.

So, the number of favourable outcomes in this case is 4.

∴ P(one of the first 9 letters of English alphabet which appears in the given word)

$$= \frac{4}{10} = \frac{2}{5}.$$

(*iii*) The last 9 letters of English alphabet are

R, S, T, U, V, W, X, Y, Z.

Out of these only U, T, R, Z, S appear in the word 'AUTHORIZES'.

So, the number of favourable outcomes in this case is 5.

∴ P(one of the last 9 letters of English alphabet which appears in the given word)

$$= \frac{5}{10} = \frac{1}{2}.$$

(*c*) The purchase value of medicines is ₹ 950 and the rate of GST is 5%.

$$\therefore \qquad \text{GST} = 5\% \text{ of ₹ } 950$$
$$= \text{₹ } 47.50$$

The purchase value of pair of shoes is ₹ 3000 and the rate of GST is 18%.

$$\therefore \qquad \text{GST} = 18\% \text{ of ₹ } 3000$$
$$= \text{₹ } 540$$

The purchase value of laptop bag

$$= \text{₹ } 1000\left(1 - \frac{30}{100}\right)$$
$$= \text{₹ } 700$$

Since the rate of GST is 18%.

$$\therefore \qquad \text{GST} = 18\% \text{ of ₹ } 700$$
$$= \text{₹ } 126.$$

(*i*) Total amount of GST paid = ₹ 47.50 + ₹ 540 + ₹ 126 = ₹ 713.50

(*ii*) Total bill amount including GST paid by Mr. Bedi

= Purchase value of articles + GST

= (₹ 950 + ₹ 3000 + ₹ 700) + ₹ 713.50 = ₹ 5363.50.

Solution 5

(*a*) Here, the number of shares with company = 500.

$$\text{N.V. of each share} = ₹\ 120$$
$$\text{Rate of dividend} = 15\%$$

(*i*) The total amount of dividend paid by the company

$$= (15\% \text{ of } ₹\ 120)(500)$$
$$= ₹\ 9000.$$

(*ii*) Number of shares hold by Mr. Sharma = 80

$$\therefore \quad \text{Annual income of Mr. Sharma} = (15\% \text{ of } ₹\ 120)(80)$$
$$= ₹\ 1440.$$

Let ₹ x be the M.V. of each share.

$$\therefore \quad 10\% \text{ of } ₹\ x = 15\% \text{ of } ₹\ 120$$
$$\Rightarrow \quad \frac{10x}{100} = 18$$
$$\Rightarrow \quad x = 180.$$

Thus, the M.V. of each share is ₹ 180.

(*b*) Here,
$$\Sigma f = 3 + 7 + f + 9 + 6$$
$$= 25 + f$$

and
$$\Sigma fx = 5(3) + 10(7) + 15f + 20(9) + 25(6)$$
$$= 15 + 70 + 15f + 180 + 150$$
$$= 415 + 15f$$

$$\therefore \quad \text{Mean, } \bar{x} = \frac{\Sigma fx}{\Sigma f}$$
$$\Rightarrow \quad 16 = \frac{415 + 15f}{25 + f}$$
$$\Rightarrow \quad 400 + 16f = 415 + 15f$$
$$\Rightarrow \quad f = 15$$

Hence, the value of f is 15.

(*c*) Let a be the first term, r the common ratio and n the number of terms of G.P.

According to question,

$$a_4 = 10 \quad \Rightarrow \quad ar^3 = 10 \qquad ...(1)$$
$$a_6 = 40 \quad \Rightarrow \quad ar^5 = 40 \qquad ...(2)$$
$$a_n = 640 \quad \Rightarrow \quad ar^{n-1} = 640 \qquad ...(3)$$

Dividing (2) by (1), we get

$$\frac{ar^5}{ar^3} = \frac{40}{10} \quad \Rightarrow \quad r^2 = 4 \quad \Rightarrow \quad r = 2 \qquad [\because r \text{ is positive.}]$$

Substituting $r = 2$ in (1), we get

$$a \cdot (2)^3 = 10 \quad \Rightarrow \quad a = \frac{10}{8} = \frac{5}{4}$$

Substituting $r = 2$ and $a = \frac{5}{4}$ in (3), we get

$$\frac{5}{4}(2)^{n-1} = 640$$
$$\Rightarrow \quad 2^{n-1} = 640 \cdot \left(\frac{4}{5}\right) = 512$$
$$\Rightarrow \quad 2^{n-1} = 2^9$$

$$\Rightarrow \qquad n - 1 = 9 \qquad \Rightarrow \qquad n = 10$$

Hence, the first term of G.P. is $\frac{5}{4}$, common ratio is 2 and the number of terms is 10.

Solution 6

(*a*) Here,

$$A^2 = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (3)(3)+(0)(5) & 3(0)+0(1) \\ (5)(3)+(1)(5) & 5(0)+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (-4)(-4)+(2)(1) & (-4)(2)+(2)(0) \\ (1)(-4)+(0)(1) & (1)(2)+(0)(0) \end{bmatrix}$$

$$= \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

and

$$AB = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (3)(-4)+(0)(1) & (3)(2)+(0)(0) \\ (5)(-4)+(1)(1) & (5)(2)+(1)(0) \end{bmatrix}$$

$$= \begin{bmatrix} -12 & 6 \\ -19 & 10 \end{bmatrix}$$

$$\therefore \quad A^2 - 2AB + B^2 = \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} - 2\begin{bmatrix} -12 & 6 \\ -19 & 10 \end{bmatrix} + \begin{bmatrix} -18 & -8 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} - \begin{bmatrix} -24 & 12 \\ -38 & 20 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 51 & -20 \\ 54 & -17 \end{bmatrix}.$$

(*b*) (*i*) In ΔPAB and ΔPCD

$\angle PAB = \angle PCD$ [Angles in the same segment]

$\angle APB = \angle CPD$ [Vertically opposite angles]

$\therefore \quad \Delta PAB \sim \Delta PCD$ [By AA Similarity Criterion]

(*ii*) Here, PA = 7.5 cm, AB = 9 cm and PC = 5 cm

Since $\Delta PAB \sim \Delta PCD$

$$\therefore \quad \frac{PA}{PC} = \frac{AB}{CD}$$

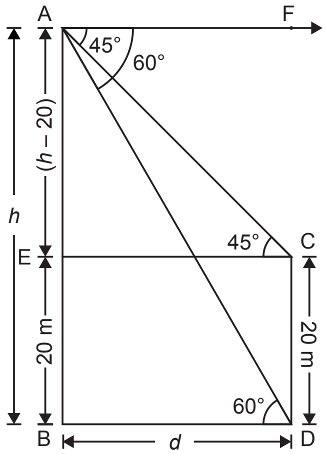
$$\Rightarrow \quad \frac{7.5}{5} = \frac{9}{CD} \quad \Rightarrow \quad CD = \frac{9(5)}{7.5} = 6 \text{ cm}.$$

(*iii*) Since $\Delta PAB \sim \Delta PCD$

$$\therefore \quad \frac{ar(\Delta PAB)}{ar(\Delta PCD)} = \frac{PA^2}{PC^2} = \left(\frac{7.5}{5}\right)^2 = \frac{9}{4}$$

$\therefore \quad ar(\Delta PAB) : ar(\Delta PCD) = 9 : 4.$

(*c*) Let AB be the cliff and CD be the tower of height 20 m. Let h be the height of cliff and d the horizontal distance (both measured in metres) between cliff and tower.



From figure, $\angle ACE = \angle FAC = 45°$

$\angle ADB = \angle FAD = 60°$

In ΔAEC, $\tan 45° = \dfrac{AE}{EC} = \dfrac{h-20}{d}$ [$\because$ EC = BD = d]

$\Rightarrow$ $1 = \dfrac{h-20}{d}$ or $d = h - 20$ ...(1)

In ΔABD, $\tan 60° = \dfrac{AB}{BD} = \dfrac{h}{d}$

$\Rightarrow$ $\sqrt{3} = \dfrac{h}{d}$ or $d = \dfrac{h}{\sqrt{3}}$ ...(2)

From (1) and (2), we get

$$h - 20 = \frac{h}{\sqrt{3}} \Rightarrow h - \frac{h}{\sqrt{3}} = 20$$

$$\Rightarrow \frac{h(\sqrt{3}-1)}{\sqrt{3}} = 20$$

$$\Rightarrow h = \frac{20\sqrt{3}}{\sqrt{3}-1}$$

$$\Rightarrow h = \frac{20\sqrt{3}(\sqrt{3}+1)}{2}$$

$$\Rightarrow h = 10(3 + \sqrt{3})$$

$$\Rightarrow h = 10(3 + 1.732) = 47.32$$

Substituting $h = 47.32$ in (1), we get

$$d = 47.32 - 20 = 27.32$$

Hence, (*i*) the height of cliff is 47.32 m.

(*ii*) the distance between the cliff and tower is 27.32 m.

Solution 7

(*a*) Here, the slope (m_1) of line $5x - 3y + 2 = 0$ is $\dfrac{5}{3}$

and the slope (m_2) of line $6x - py + 7 = 0$ is $\dfrac{6}{p}$.

Since the lines are perpendicular to each other

$\therefore$ $m_1 \times m_2 = -1$

$\Rightarrow$ $\dfrac{5}{3} \cdot \left(\dfrac{6}{p}\right) = -1$

$\Rightarrow$ $p = -10.$

$\therefore$ Slope (m_2) of line $6x - py + 7 = 0$ is $-\dfrac{6}{10}$, *i.e.*, $-\dfrac{3}{5}$ [$\because p = -10$]

$\therefore$ The slope of line parallel to line $6x - py + 7 = 0$ is $-\dfrac{3}{5}$.

Now, equation of line passing through (–2, –1) and having slope $-\dfrac{3}{5}$ is

$$y - (-1) = -\frac{3}{5}(x - (-2))$$

$$\Rightarrow \quad y + 1 = -\frac{3}{5}(x + 2)$$

$$\Rightarrow \quad 5y + 5 = -3x - 6$$

$$\Rightarrow \quad 3x + 5y + 11 = 0.$$

(*b*) Here, $$\frac{x^2 + 2x}{2x + 4} = \frac{y^2 + 3y}{3y + 9}$$

Applying componendo and dividendo, we get

$$\frac{(x^2 + 2x) + (2x + 4)}{(x^2 + 2x) - (2x + 4)} = \frac{(y^2 + 3y) + (3y + 9)}{(y^2 + 3y) - (3y + 9)}$$

$$\Rightarrow \quad \frac{x^2 + 4x + 4}{x^2 - 4} = \frac{y^2 + 6y + 9}{y^2 - 9}$$

$$\Rightarrow \quad \frac{(x + 2)^2}{(x - 2)(x + 2)} = \frac{(y + 3)^2}{(y - 3)(y + 3)}$$

$$\Rightarrow \quad \frac{x + 2}{x - 2} = \frac{y + 3}{y - 3}$$

Applying componendo and dividendo, we get

$$\frac{(x + 2) + (x - 2)}{(x + 2) - (x - 2)} = \frac{(y + 3) + (y - 3)}{(y + 3) - (y - 3)}$$

$$\Rightarrow \quad \frac{2x}{4} = \frac{2y}{6}$$

$$\Rightarrow \quad \frac{x}{y} = \frac{2}{3} \quad \text{or} \quad x : y = 2 : 3.$$

(*c*) Here, $\angle QCO = \angle OCT = 90°$ [Radius through the point of contact is perpendicular to tangent.]

$\Rightarrow \quad \angle BCQ + \angle BCO = 90°$

$\Rightarrow \quad \angle BCO = 90° - \angle BCQ$

$= 90° - 55°$ [$\because \angle BCQ = 55°$.]

$= 35°.$

Also, $\angle PAO = \angle OAT = 90°$ [Radius through the point of contact is perpendicular to tangent.]

$\Rightarrow \quad \angle BAP + \angle BAO = 90°$

$\Rightarrow \quad \angle BAO = 90° - \angle BAP$

$= 90° - 60°$ [$\because \angle BAP = 60°$]

$= 30°.$

(*i*) Now, $\angle OBA = \angle BAO = 30°$ [$\because$ OA = OB]

and $\angle OBC = \angle BCO = 35°$ [$\because$ OB = OC]

(*ii*) Further, $\angle AOC = 2\angle ABC$ [$\because$ Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.]

$= 2\{\angle OBA + \angle OBC\}$

$= 2\{30° + 35°\}$

$= 130°.$

(*iii*) In quadrilateral AOCT,

$\angle OAT + \angle ATC + \angle OCT + \angle AOC = 360°$

$\Rightarrow \quad 90° + \angle ATC + 90° + 130° = 360°$

$\Rightarrow \quad \angle ATC = 360° - (90° + 90° + 130°)$

$= 360° - 310° = 50°.$

Solution 8

(*a*) Let λ be the number to be added to $2x^3 - 3x^2 - 8x$ and the resulting polynomial be $f(x)$, then

$$f(x) = 2x^3 - 3x^2 - 8x + \lambda$$

On dividing by $(2x + 1)$, the polynomial $f(x)$ leaves remainder 10.

$$\therefore \quad f\left(-\frac{1}{2}\right) = 10 \qquad \left[\because\ 2x+1=0 \Rightarrow x = -\frac{1}{2}\right]$$

$$\Rightarrow \quad 2\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 - 8\left(-\frac{1}{2}\right) + \lambda = 10$$

$$\Rightarrow \quad -\frac{2}{8} - \frac{3}{4} + \frac{8}{2} + \lambda = 10$$

$$\Rightarrow \quad -\frac{1}{4} - \frac{3}{4} + 4 + \lambda = 10$$

$$\Rightarrow \quad 3 + \lambda = 10$$

$$\Rightarrow \quad \lambda = 7$$

Hence, the number to be added is 7.

(*b*) Here, P = ₹ 750, n = 2 years (24 months) and M.V. (maturity value) = ₹ 19,125.

Let r be the rate of interest per annum.

Using, M.V. (maturity value) = $\left\{1 + \frac{(n+1)r}{2400}\right\}$. P.$n$, we have

$$19125 = \left\{1 + \frac{(24+1)r}{2400}\right\} \cdot (750)(24)$$

$$\Rightarrow \quad 19125 = \left\{1 + \frac{25r}{2400}\right\} \cdot (750)(24)$$

$$\Rightarrow \quad 19125 = \left\{1 + \frac{r}{96}\right\} \cdot (750)(24)$$

$$\Rightarrow \quad \frac{19125}{(750)(24)} = 1 + \frac{r}{96}$$

$$\Rightarrow \quad \frac{51}{2(24)} = 1 + \frac{r}{96}$$

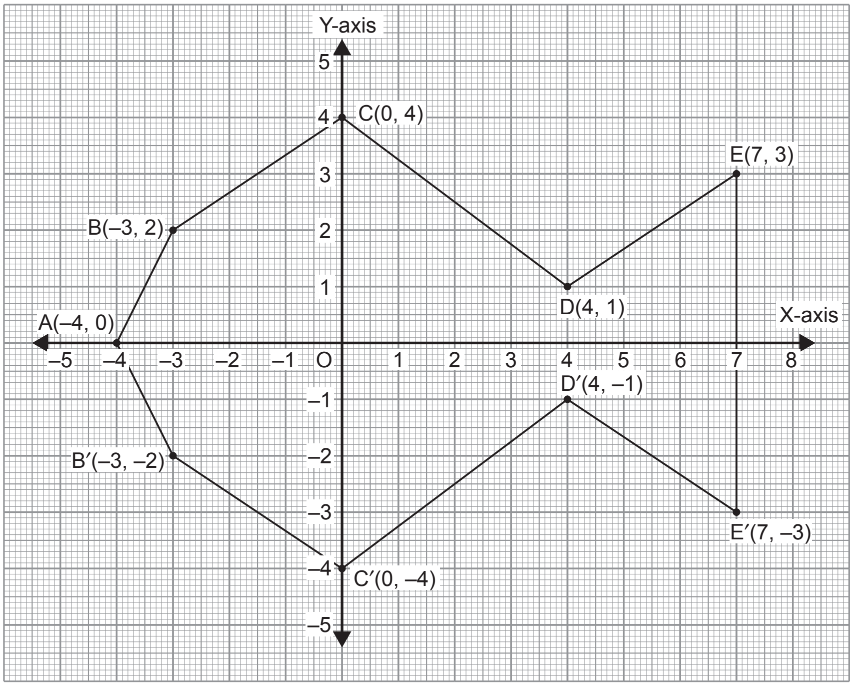
$$\Rightarrow \quad \frac{51}{48} - 1 = \frac{r}{96}$$

$$\Rightarrow \quad \frac{3}{48} = \frac{r}{96}$$

$$\Rightarrow \quad r = 6$$

Hence, the rate of interest is 6% per annum.

(*c*) (*i*)



(*ii*) and (*iii*) $R_x[B(-3, 2)] \rightarrow B'(-3, -2)$ $R_x[C(0, 4)] \rightarrow C'(0, -4)$

$R_x[D(4, 1)] \rightarrow D'(4, -1)$ $R_x[E(7, 3)] \rightarrow E'(7, -3)$.

(*iv*) The closed figure formed is irregular nonagon.

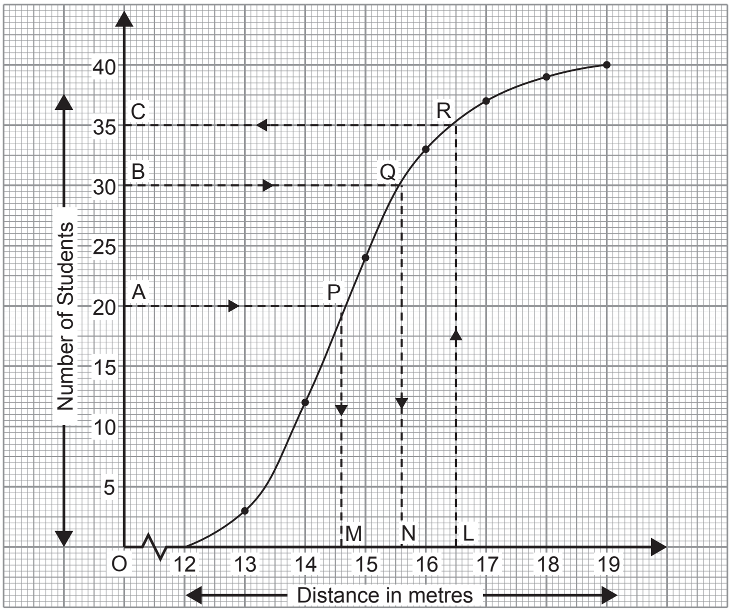
Solution 9

(*a*) The cumulative frequency table for the given data is as follows:

Distance (in m)	12–13	13–14	14–15	15–16	16–17	17–18	18–19
Number of Students (f)	3	9	12	9	4	2	1
Cumulative Frequency (c.f.)	3	12	24	33	37	39	40

Here, total number of students $(\Sigma f) = 40$

Plot the points (13, 3), (14, 12), (15, 24), (16, 33), (17, 37), (18, 39) and (19, 40) on graph and join these points freehand to get the required ogive.



(*i*) To read off the estimated value of median, draw a horizontal line from point A representing cumulative frequency $\frac{\Sigma f}{2}$ (= 20). Let this line meet the ogive at P. From P, draw a perpendicular PM on x-axis. M gives the value of median.
Here, estimated value of median is 14.6 m.

(*ii*) To read off the estimated value of upper quartile, draw a horizontal line from point B representing cumulative frequency $\frac{3\Sigma f}{4}$ (= 30). Let this line meet the ogive at Q. From Q, draw a perpendicular QN on x-axis. N gives the value of upper quartile.
Here, estimated value of upper quartile is 15.6 m.

(*iii*) In order to find the number of students who cover distance which is above $16\frac{1}{2}$ m, draw a vertical line from point L (on x-axis) representing $16\frac{1}{2}$ m. Let this line meet ogive at R. From R, draw a horizontal line to meet the y-axis at C. The ordinate of point C represents 35 students on y-axis.

$\therefore$ Number of students who cover a distance which is above $16\frac{1}{2}$ m

= Total number of students – Number of students who covered distance $\leq 16\frac{1}{2}$ m
= 40 – 35 = 5.

(*b*) Here,
$$x = \frac{\sqrt{2a+1}+\sqrt{2a-1}}{\sqrt{2a+1}-\sqrt{2a-1}}$$

Applying componendo and dividendo, we get
$$\frac{x+1}{x-1} = \frac{(\sqrt{2a+1}+\sqrt{2a-1})+(\sqrt{2a+1}-\sqrt{2a-1})}{(\sqrt{2a+1}+\sqrt{2a-1})-(\sqrt{2a+1}-\sqrt{2a-1})}$$
$$\Rightarrow \quad \frac{x+1}{x-1} = \frac{2\sqrt{2a+1}}{2\sqrt{2a-1}} \quad \Rightarrow \quad \frac{x+1}{x-1} = \frac{\sqrt{2a+1}}{\sqrt{2a-1}}$$

Squaring both sides, we get
$$\frac{(x+1)^2}{(x-1)^2} = \frac{2a+1}{2a-1}$$

Applying componendo and dividendo, we get
$$\frac{(x+1)^2+(x-1)^2}{(x+1)^2-(x-1)^2} = \frac{(2a+1)+(2a-1)}{(2a+1)-(2a-1)} \Rightarrow \frac{2x^2+2}{4x} = \frac{4a}{2}$$
$$\Rightarrow \frac{x^2+1}{4x} = a$$
$$\Rightarrow \quad x^2 + 1 = 4ax \quad \text{or} \quad x^2 - 4ax + 1 = 0.$$

Solution 10

(*a*) Let a be the first term and d the common difference of A.P.
According to question,
$$a_6 = 4a \Rightarrow a + 5d = 4a$$
$$\Rightarrow 3a = 5d \qquad ...(1)$$
and
$$S_6 = 75 \Rightarrow \frac{6}{2}\{2a+(6-1)d\} = 75$$
$$\Rightarrow 2a + 5d = 25 \qquad ...(2)$$

From (1) and (2), we get $a = 5$ and $d = 3$.
Hence, the first term of A.P. is 5 and common difference is 3.

(*b*) As the difference of two natural numbers is 7, let the numbers be x and $x + 7$.

According to question, $x(x + 7) = 450$

$\Rightarrow$ $x^2 + 7x - 450 = 0$

$\Rightarrow$ $(x - 18)(x + 25) = 0$

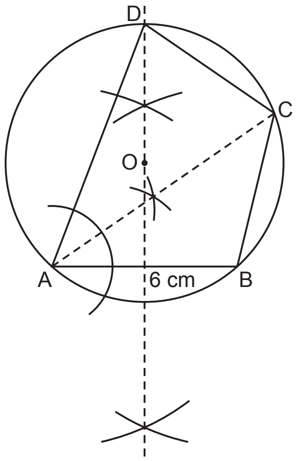
$\Rightarrow$ $x = 18$ or $x = -25$ but $x \in \mathbb{N}$

$\Rightarrow$ $x = 18$

When $x = 18$, then $x + 7 = 18 + 7 = 25$.

Hence, the required numbers are 18 and 25.

(*c*) **Steps of Construction**



1. Taking O as centre and radius 4.5 cm draw a circle.
2. Draw a chord AB of length 6 cm.
3. The locus of points equidistant from A and B is the perpendicular bisector of AB.

   Draw perpendicular bisector of AB which meets the circle at point D.
4. Join AD.

   The locus of points equidistant from AD and AB is the bisector of $\angle$DAB.

   Draw the bisector of $\angle$DAB which meets the circle at point C.
5. Join BC and CD.

   The measure of length of side CD of the quadrilateral is 5.1 cm.

Solution 11

(*a*) Here, scale factor K = 1 : 50

(*i*) Height of actual building $= \frac{1}{\text{K}}$ (Height of the model)

$= 50(0.8)$ m $= 40$ m.

(*ii*) Floor area of the model = K^2 (Floor area of a flat in building)

$$= \left(\frac{1}{50}\right)^2 \cdot (20) \text{ sq m} = \frac{1}{2500} (200000) \text{ sq cm}$$

$= 80$ sq cm.

(*b*) Here, h(height of solid wooden cylinder) = 28 cm

r(radius of solid wooden cylinder and also of cone) = 3 cm

h_1(height of conical cavities) = 10.5 cm

Volume of remaining solid = Volume of solid wooden cylinder – 2(Volume of cone)

$$= \pi r^2 h - 2\left(\frac{1}{3}\pi r^2 h_1\right)$$

$$= \pi r^2\left(h - \frac{2}{3}h_1\right) = \frac{22}{7} \cdot (3)^2 \left\{28 - \frac{2}{3}(10.5)\right\} \text{ cu cm}$$

$$= \frac{22}{7} \cdot (3)\{84 - 21\} \text{ cu cm} = \frac{22}{7} \cdot (3)(63) \text{ cu cm}$$

$= 594$ cu cm.

(*c*) Here, $$\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \left(\frac{1-\tan\theta}{1-\frac{1}{\tan\theta}}\right)^2$$

$$= \left\{-\frac{(\tan\theta-1)}{(\tan\theta-1)} \cdot \tan\theta\right\}^2 = \{-\tan\theta\}^2 = \tan^2\theta.$$