SOLUTIONS OF ICSE SPECIMEN QUESTIONS 2020

Section A

Solution 1

(*a*) Let $f(x) = 4x^3 - 2x^2 + kx + 5$

On dividing by $(2x + 1)$, the polynomial $f(x)$ leaves remainder – 10.

$$\therefore \quad f\left(-\frac{1}{2}\right) = -10 \qquad \left(\because 2x+1=0 \Rightarrow x=-\frac{1}{2}\right)$$

$$\Rightarrow \quad 4\left(-\frac{1}{2}\right)^3 - 2\left(-\frac{1}{2}\right)^2 + k\left(-\frac{1}{2}\right)+5 = -10$$

$$\Rightarrow \quad -\frac{1}{2}-\frac{1}{2}-\frac{k}{2}+5 = -10$$

$$\Rightarrow \quad -1-\frac{k}{2}+5 = -10$$

$$\Rightarrow \quad -\frac{k}{2}+4 = -10$$

$$\Rightarrow \quad -\frac{k}{2} = -14$$

$$\Rightarrow \quad k = 28.$$

(*b*) Here, P = ₹ 1,600, n = 18 months and M.V. (Maturity value) = ₹ 31,080.

Let r be the rate of interest per annum.

Using, M.V. (Maturity value) = $\left\{1+\frac{(n+1)\cdot r}{2400}\right\}\cdot \text{P}\cdot n$, we have

$$31080 = \left\{1+\frac{(18+1)r}{2400}\right\}(1600)\cdot(18)$$

$$\Rightarrow \quad 31080 = \left\{\frac{2400+19r}{2400}\right\}\cdot(1600)\cdot(18)$$

$$\Rightarrow \quad 31080 = (2400 + 19r)(12)$$

$$\Rightarrow \quad 2590 = 2400 + 19r$$

$$\Rightarrow \quad 19r = 190$$

$$\Rightarrow \quad r = 10.$$

Hence, the rate of interest is 10% per annum.

(*c*) Here, the marked price of article is ₹ 1,200.

∴ Sale value for wholesaler (Purchase value for shopkeeper)

$$= ₹\ 1200\left(1-\frac{10}{100}\right)$$

$$= ₹\ 1200\left(\frac{90}{100}\right) = ₹\ 1080.$$

(*i*) Since the rate of GST is 6%

∴ the amount of GST paid by the shopkeeper to the wholesaler.

= 6% of ₹ 1080 = ₹ 64.80

This amount (₹ 60.80) is paid by the wholesaler as GST.

(*ii*) The amount paid by the customer for the item

$= ₹\ 1200 + 6\%$ of $₹\ 1200$

$= ₹\ 1200 + ₹\ 72$

$= ₹\ 1272.$

Solution 2

(*a*) Here, $-5\frac{1}{2} - x \le \frac{1}{2} - 3x \le 3\frac{1}{2} - x$ is equivalent to

$$-5\frac{1}{2} - x \le \frac{1}{2} - 3x \quad \text{and} \quad \frac{1}{2} - 3x \le 3\frac{1}{2} - x$$

$$\Rightarrow \quad -\frac{11}{2} - x \le \frac{1}{2} - 3x \quad \text{and} \quad \frac{1}{2} - 3x \le \frac{7}{2} - x$$

$$\Rightarrow \quad 3x - x \le \frac{1}{2} + \frac{11}{2} \quad \text{and} \quad -3x + x \le \frac{7}{2} - \frac{1}{2}$$

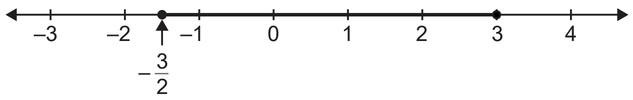
$$\Rightarrow \quad 2x \le 6 \quad \text{and} \quad -2x \le 3$$

$$\Rightarrow \quad x \le 3 \quad \text{and} \quad x \ge -\frac{3}{2}$$

$$\Rightarrow \quad -\frac{3}{2} \le x \le 3$$

The solution set is $\left\{x : -\frac{3}{2} \le x \le 3, x \in \text{R}\right\}$

The solution on the real number line is represented as



(*b*) The given A.P. is 7, 11, 15, 19, ...

Here, $a = 7$ and $d = 11 - 7 = 4$

Now, $a_{16} = a + 15d$

$= 7 + 15(4)$

$= 67$

and $S_6 = \frac{6}{2}\{2a + (6 - 1)d\}$

$= 3\{2(7) + 5(4)\}$

$= 3(34)$

$= 102.$

(*c*) Join AC.

(*i*) Since ABCD is a cyclic quadrilateral

$\therefore \quad \angle ADC + \angle ABC = 180°$ [Opposite angles of a cyclic quadrilateral are supplementary.]

$\Rightarrow \quad \angle ADC = 180° - \angle ABC$

$\Rightarrow \quad \angle ADC = 180° - 93°$

$= 87°.$

(*ii*) Now, $\angle CAD = \angle DCE$ [Angles in the alternate segments]

$= 35°$

(*iii*) In ΔACD, $\angle ACD = 180° - (\angle ADC + \angle CAD)$

$= 180° - (87° + 35°)$

$= 180° - 122°$

$= 58°.$

Solution 3

(*a*) Here, $\frac{\sec A}{\sec A - 1} + \frac{\sec A}{\sec A + 1}$

$$= \sec A \left\{ \frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} \right\}$$

$$= \sec A \left\{ \frac{(\sec A + 1) + (\sec A - 1)}{\sec^2 A - 1} \right\}$$

$$= \sec A \cdot \left\{ \frac{2 \sec A}{\tan^2 A} \right\}$$

$$= 2 \sec^2 A \cdot \cot^2 A$$

$$= 2 \cdot \frac{1}{\cos^2 A} \cdot \frac{\cos^2 A}{\sin^2 A}$$

$$= \frac{2}{\sin^2 A}$$

$$= 2 \operatorname{cosec}^2 A.$$

(*b*) Here, $3\begin{bmatrix} 5 & -6 \\ 4 & x \end{bmatrix} - \begin{bmatrix} 6 & y \\ 0 & 6 \end{bmatrix} = 3\begin{bmatrix} 3 & -2 \\ 4 & 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 15 & -18 \\ 12 & 3x \end{bmatrix} - \begin{bmatrix} 6 & y \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & -6 \\ 12 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 15-6 & -18-y \\ 12-0 & 3x-6 \end{bmatrix} = \begin{bmatrix} 9 & -6 \\ 12 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9 & -18-y \\ 12 & 3x-6 \end{bmatrix} = \begin{bmatrix} 9 & -6 \\ 12 & 0 \end{bmatrix}$$

Corresponding elements of above matrices must be equal.

∴ We have, $-18 - y = -6 \Rightarrow y = -12$ and $3x - 6 = 0 \Rightarrow x = 2$.

Hence, $x = 2$ and $y = -12$.

(*c*) Here, $(k + 1)x^2 - 4kx + 9 = 0$ ...(1)

Comparing with $ax^2 + bx + c = 0$, we get

$a = k + 1, \quad b = -4k \quad$ and $\quad c = 9$

For the equation to have real and equal roots,

$$b^2 - 4ac = 0$$

$$\Rightarrow (-4k)^2 - 4(k + 1) \cdot (9) = 0$$

$$\Rightarrow 16k^2 - 36(k + 1) = 0$$

$$\Rightarrow 4k^2 - 9(k + 1) = 0$$

$$\Rightarrow 4k^2 - 9k - 9 = 0$$

$$\Rightarrow (k - 3)(4k + 3) = 0$$

$$\Rightarrow k = 3 \text{ or } k = -\frac{3}{4}.$$

When, $k = 3$, equation (1) reduces to

$$4x^2 - 12x + 9 = 0$$

$$\Rightarrow (2k - 3)^2 = 0$$

$$\Rightarrow x = \frac{3}{2}, \frac{3}{2}.$$

When, $k = -\frac{3}{4}$, equation (1) reduces to

$$\frac{1}{4}x^2 + 3x + 9 = 0$$

$$\Rightarrow \quad \left(\frac{1}{2}x + 3\right)^2 = 0$$

$$\Rightarrow \quad x = -6, -6.$$

Solution 4

(*a*) Here, the total number of balls in the box are 15 (4 red, 5 black and 6 white).

So the total number of possible outcomes are 15.

(*i*) The number of favourable outcomes to the event that the 'drawn ball is black' are 5 (since there are 5 black balls in the box).

$\therefore$ P(ball drawn is black) = $\frac{5}{15} = \frac{1}{3}$.

(*ii*) The number of favourable outcomes to the event that the 'drawn ball is red or white' are 10 (since there are 4 red and 6 white balls in the box).

$\therefore$ P(drawn ball is red or white) = $\frac{10}{15} = \frac{2}{3}$.

(*b*) The given variates (weights of students) are already in ascending order. We construct the cumulative frequency table.

Variate (Weight)	*Frequency (Number of Students)*	*Cumulative Frequency*
35	4	4
47	3	7
52	5	12
56	3	15
60	2	17

Now, $\Sigma f = 17$ and $\frac{\Sigma f}{2} = 8.5$

The cumulative frequency just greater than 8.5 is 12 and the value of x corresponding to 12 is 52.

Thus, the median of given distribution is 52.

In the given distribution, the variate 52 has the maximum frequency.

So, mode = 52.

(*c*) Here, r (radius of cylinder) = 7 cm

h (height of cylinder) = 14 cm

r_1 (radius of spheres) = 3.5 cm

Let n be the number of spheres formed.

$\therefore$ n (Volume of sphere) = Volume of cylinder

$$\Rightarrow \quad n\left(\frac{4}{3}\pi r_1^3\right) = \pi r^2 h$$

$$\Rightarrow \quad n = \frac{3}{4} \cdot \frac{r^2 h}{r_1^3}$$

$$\Rightarrow \quad n = \frac{3}{4} \cdot \frac{(7)^2(14)}{(3.5)^3} = 12$$

Hence, the number of spheres formed is 12.

Section B

Solution 5

(*a*) Let a be the first term and d the common difference of A.P.

Here, $a_2 = 10 \quad \Rightarrow \quad a + d = 10$ ...(1)

and $a_{45} = 96 \quad \Rightarrow \quad a + 44d = 96$ ...(2)

Solving (1) and (2), we get $a = 8$ and $d = 2$.

Now,
$$S_{15} = \frac{15}{2}\{2a + (15-1)d\}$$
$$= \frac{15}{2}\{2(8) + 14(2)\}$$
$$= 15.(22)$$
$$= 330.$$

Hence, the first term is 8, common difference is 2 and the sum of first 15 terms is 330.

(*b*) Here,
$$A^2 = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} (3)(3)+(-1)(0) & (3)(-1)+(-1)(2) \\ (0)(3)+(2)(0) & (0)(-1)+(2)(2) \end{bmatrix}$$
$$= \begin{bmatrix} 9 & -5 \\ 0 & 4 \end{bmatrix}$$

Now, $A^2 - 2B = 3A + 5I$

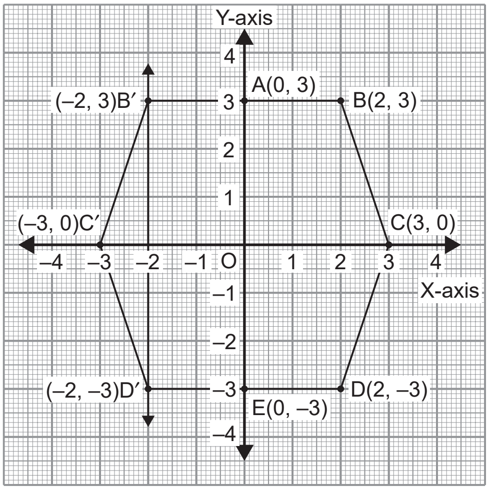
$$\Rightarrow \quad 2B = A^2 - 3A - 5I$$
$$= \begin{bmatrix} 9 & -5 \\ 0 & 4 \end{bmatrix} - 3\begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 9 & -5 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 9 & -3 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} 9 & -5 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 14 & -3 \\ 0 & 11 \end{bmatrix}$$
$$\Rightarrow \quad 2B = \begin{bmatrix} -5 & -2 \\ 0 & -7 \end{bmatrix}$$

or
$$B = \frac{1}{2}\begin{bmatrix} -5 & -2 \\ 0 & -7 \end{bmatrix}.$$

(*c*) (*iii*) Here, $R_x[B(2, 3)] \rightarrow B'(-2, 3)$

$R_y[C(3, 0)] \rightarrow C'(-3, 0)$

$R_y[D(2, -3)] \rightarrow D'(-2, -3)$.



(*iv*) From figure, we see that line B′D′ is parallel to y-axis at a distance of –2 units to the left of y-axis. So its equation is

$$x = -2 \quad \text{or} \quad x + 2 = 0.$$

(*v*) The figure BCDD′C′B′B is a hexagon.

Solution 6

(*a*) (*i*) In ΔABC and ΔEDC

As AB is parallel to ED

$\angle ABC = \angle EDC$ [Corresponding Angles]

and $\angle C = \angle C$ [Common Angles]

$\therefore$ $\Delta ABC \sim \Delta EDC.$ [By AA Similarity Criterion]

(*ii*) Since $\Delta ABC \sim \Delta EDC$

$$\therefore \quad \frac{AB}{DE} = \frac{BC}{DC}$$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{BC}{BC - BD}$$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{BC}{BC - \left(\frac{BC}{3}\right)} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{12.3}{DE} = \frac{3}{2} \quad \Rightarrow \quad DE = (12.3) \cdot \frac{2}{3}$$

$$\Rightarrow \quad DE = 8.2 \text{ cm}$$

(*iii*) Since $\Delta ABC \sim \Delta EDC$

$$\therefore \quad \frac{ar(\Delta EDC)}{ar(\Delta ABC)} = \frac{DE^2}{AB^2}$$

$$\Rightarrow \quad \frac{ar(\Delta EDC)}{ar(\Delta ABC)} = \frac{(8.2)^2}{(12.3)^2} = \frac{4}{9}.$$

(*b*) Let line $y = -3$ divide the line joining the points A(–2, 5) and B(–5, –6) in the ratio $m_1 : m_2$ at the point P, *i.e.*, AP : PB = $m_1 : m_2$.

$$\therefore \quad \text{Coordinates of P are } \left(\frac{-5m_1 - 2m_2}{m_1 + m_2}, \frac{-6m_1 + 5m_2}{m_1 + m_2}\right)$$

As the point P lies on the line $y = -3$, we have

$$\frac{-6m_1 + 5m_2}{m_1 + m_2} = -3$$

$$\Rightarrow \quad -6m_1 + 5m_2 = -3m_1 - 3m_2$$

$$\Rightarrow \quad 3m_1 = 8m_2$$

$$\Rightarrow \quad m_1 : m_2 = 8 : 3$$

Hence, the coordinates of point of intersection (P) are

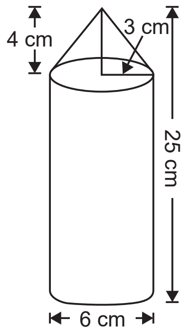
$$\left(\frac{-5\left(\frac{m_1}{m_2}\right) - 2}{\frac{m_1}{m_2} + 1}, -3\right)$$

i.e.,
$$\left(\frac{-5\left(\frac{8}{3}\right) - 2}{\frac{8}{3} + 1}, -3\right) \text{ or } \left(-\frac{46}{11}, -3\right).$$

(*c*) Here, h (height of cylinder) = 25 cm – 4 cm = 21 cm

h_1(height of cone) = 4 cm

r (radius of base of cylinder and also of cone) = 3 cm.



(*i*) Volume of solid = Volume of cylinder + Volume of cone

$$= \pi r^2 h + \frac{1}{3}\pi r^2 h_1$$

$$= \pi r^2\left(h + \frac{1}{3}h_1\right)$$

$$= \frac{22}{7}\cdot(3)^2\left\{21 + \frac{1}{3}\cdot(4)\right\} \text{ cu cm}$$

$$= \frac{22}{7}\cdot(9)\left\{\frac{67}{3}\right\} \text{ cu cm}$$

= 631.71 cu cm

= 632 cu cm. (to the nearest whole number)

(*ii*) Curved surface area of the solid

= Curved surface area of cylinder + Curved surface area of cone

= $2\pi rh + \pi rl$, where l is the slant height of cone.

$$= \pi r\left(2h + \sqrt{h_1^2 + r^2}\right) \qquad \left(\because\ l = \sqrt{h_1^2 + r^2}\right)$$

$$= \frac{22}{7}\cdot(3)\left\{2(21) + \sqrt{(4)^2 + (3)^2}\right\} \text{ sq cm}$$

$$= \frac{22}{7}\cdot(3)\{42 + 5\} \text{ sq cm}$$

$$= \frac{22}{7}\cdot(3)\cdot(47) \text{ sq cm}$$

= 443.14 sq cm = 443 sq cm. (to the nearest whole number)

Solution 7

(*a*) (*i*) $\angle BAT = 90°$ [Angle in a semicircle]

Now, $\angle TAP + \angle BAT = 180°$

$\Rightarrow \quad \angle TAP = 180° - \angle BAT$

$= 180° - 90°$

$= 90°$

In Δ APT, $\angle APT = 180° - (\angle ATP + \angle TAP)$

$= 180° - (40° + 90°)$

$= 180° - 130°$

$= 50°$.

(*ii*) We know that if a chord and a tangent intersect externally, then the product of lengths of segments is equal to the square of the length of the tangent.

$\therefore \quad PT^2 = PA \cdot PB$

$= PA \cdot (PA + AB)$

$= 9(9 + 7)$

$= 144$

$\Rightarrow \quad = 12$ cm.

(*b*) Let 'a' be the first term and r the common ratio of G.P.

(*i*) Here, $a = 4$

and $a_8 = 512 \Rightarrow ar^7 = 512$

$\Rightarrow 4(r)^7 = 512$

$\Rightarrow r^7 = 128$

$\Rightarrow r = 2$

$\therefore$ The common ratio of G.P. is 2.

(*ii*) Now, $$S_5 = \frac{a(r^5 - 1)}{r - 1}$$

$$= \frac{4\{(2)^5 - 1\}}{2 - 1}$$

$$= 4(31) = 124.$$

(*c*) Let us construct the table as under.

Class	*Mid-value (x)*	*Frequency (f)*	*Product (fx)*
0–20	10	15	150
20–40	30	20	600
40–60	50	30	1500
60–80	70	a	$70a$
80–100	90	10	900

Here, $\Sigma fx = 70a + 3150,\ \Sigma f = a + 75$

Now, $$\text{mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\Rightarrow 49 = \frac{70a + 3150}{a + 75}$$

$\Rightarrow 49a + 3675 = 70a + 3150$

$\Rightarrow 21a = 525$

$\Rightarrow a = 25$

Hence, the missing frequency 'a' is 25.

Solution 8

(*a*) Here, $(\sin A + \text{cosec} A^2) + (\cos A + \sec A)^2$

$= (\sin^2 A + \text{cosec}^2 A + 2 \sin A \,\text{cosec} A) + (\cos^2 A + \sec^2 A + 2 \cos A \sec A)$

$= (\sin^2 A + \text{cosec}^2 A + 2) + (\cos^2 A + \sec^2 A + 2)$

$= (\sin^2 A + \cos^2 A) + \text{cosec}^2 A + \sec^2 A + 4$

$= 1 + \text{cosec}^2 A + \sec^2 A + 4$

$= 5 + \text{cosec}^2 A + \sec^2 A$

$$= 5 + \left(\frac{1}{\sin^2 A} + \frac{1}{\cos^2 A}\right)$$

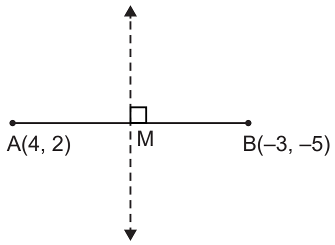
$$= 5 + \left(\frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A}\right)$$

$$= 5 + \frac{1}{\sin^2 A \cos^2 A}$$

$= 5 + \text{cosec}^2 A \cdot \sec^2 A.$

(*b*) The given points are A(4, 2) and B(–3, –5).

Let M be the mid-point of the segment AB, then the coordinates of M are



$\left(\frac{4+(-3)}{2}, \frac{2+(-5)}{2}\right)$, *i.e.*, $\left(\frac{1}{2}, -\frac{3}{2}\right)$

Slope of line AB = $\frac{-5-2}{-3-4} = 1$

$\therefore$ The slope of a line perpendicular to line AB = –1. The equation of the line through M$\left(\frac{1}{2}, -\frac{3}{2}\right)$ and having slope –1 is

$$y-\left(-\frac{3}{2}\right) = -1\left(x-\frac{1}{2}\right)$$

$$\Rightarrow \quad \frac{2y+3}{2} = -1\left(\frac{2x-1}{2}\right)$$

$$\Rightarrow \quad 2y + 3 = -2x + 1$$

$$\Rightarrow \quad 2y + 2x + 2 = 0 \quad \text{or} \quad x + y + 1 = 0.$$

(*c*) Here, $$\frac{x^3+12x}{6x^2+8} = \frac{y^3+27y}{9y^2+27}$$

Applying componendo and dividendo, we get

$$\frac{(x^3+12x)+(6x^2+8)}{(x^3+12x)-(6x^2+8)} = \frac{(y^3+27y)+(9y^2+27)}{(y^3+27y)-(9y^2+27)}$$

$$\Rightarrow \quad \frac{x^3+12x+6x^2+8}{x^3+12x-6x^2-8} = \frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$$

$$\Rightarrow \quad \frac{(x+2)^3}{(x-2)^3} = \frac{(y+3)^3}{(y-3)^3}$$

Taking cube root both sides, we get

$$\frac{x+2}{x-2} = \frac{y+3}{y-3}$$

Applying componendo and dividendo, we get

$$\frac{(x+2)+(x-2)}{(x+2)-(x-2)} = \frac{(y+3)+(y-3)}{(y+3)-(y-3)}$$

$$\Rightarrow \quad \frac{2x}{4} = \frac{2y}{6}$$

$$\Rightarrow \quad \frac{x}{y} = \frac{4}{6} \quad \Rightarrow \quad \frac{x}{y} = \frac{2}{3} \quad \text{or} \quad x : y = 2 : 3.$$

Solution 9

(*a*) Let x and y be two natural numbers ($y > x$).

According to the question,

$$y^2 - x^2 = 84 \quad \Rightarrow \quad y^2 = 84 + x^2 \qquad ...(1)$$

and $$y^2 = 25x \qquad ...(2)$$

From (1) and (2), we get

$$84 + x^2 = 25x$$

$$\Rightarrow \quad x^2 - 25x + 84 = 0$$

$$\Rightarrow \quad (x - 21)(x - 4) = 0$$

$\Rightarrow \qquad x = 21 \quad$ or $\quad x = 4$

When $x = 21$, $\quad y^2 = 25(21) \Rightarrow y^2 = 525$

$\Rightarrow \quad y = 5\sqrt{21}$ which is not a natural number.

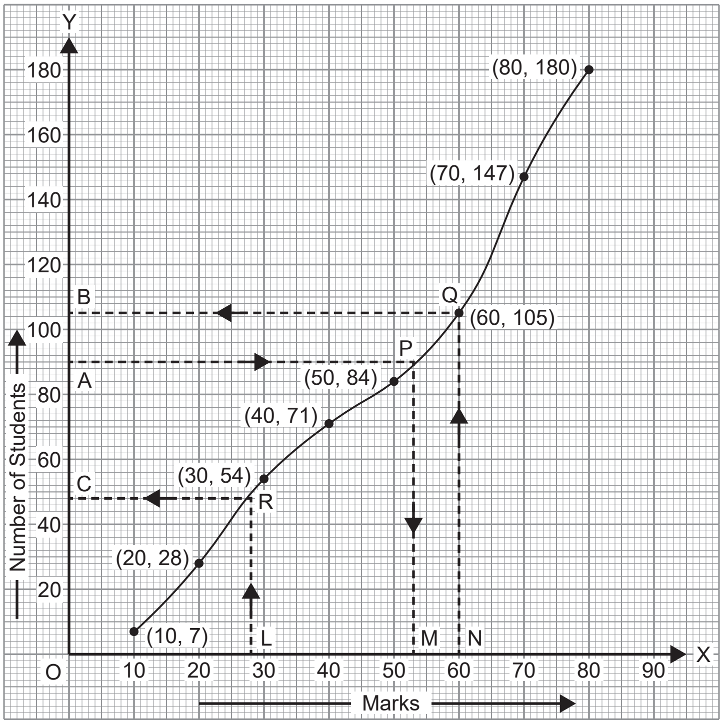
When $x = 4$, $\quad y^2 = 25(4) \Rightarrow y^2 = 100$

$\Rightarrow \quad y = 10$ which is a natural number.

Hence, the numbers are 4 and 10.

(*b*) Here, the total number of students (Σf) = 180.

Plot the points (10, 7), (20, 28), (30, 54), (40, 71), (50, 84), (60, 105), (70, 147) and (80, 180) on graph and join these points by freehand to get the required ogive.



(*i*) To read off the estimated value of median, draw a horizontal line from point A representing cumulative frequency $\frac{\Sigma f}{2}$ (= 90). Let this line meet the ogive at P. From P, draw a perpendicular PM on x-axis. M gives the value of median.

Here estimated value of median is 53 marks.

(*ii*) Here, 75% marks = (75% of 80) marks

= 60 marks.

In order to find the number of students who scored distinction (75% and above), draw a vertical line from point N (on x-axis) representing 60 marks. Let this line meet ogive at Q. From Q, draw a horizontal line to meet the y-axis at B. The ordinate of point B represents 105 students on y-axis.

$\therefore$ Number of students who scored distinctions.

= Total number of students – Number of students who scored distinction.

= 180 – 105 = 75.

(*iii*) Here, 35% marks = (35% of 80) marks

= 28 marks.

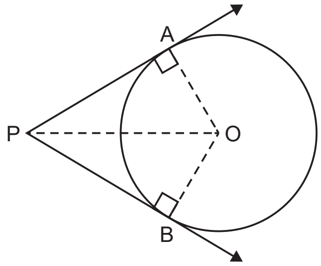
In order to find the number of students who passed the examination, draw a vertical line from point L (on x-axis) representing 28 marks. Let this line meet, ogive at R. From R draw a horizontal line to meet the y-axis at C. The ordinate of point C represents 48 students on y-axis.

∴ Number of students who passed the examination

= Total number of students – Number of students who scored < 35 % marks.

= 180 – 47 = 133.

Solution 10



(*a*) **Given:** P is an external point to a circle with centre O. PA and PB are two tangents drawn from P to the circle, A and B being points of contact.

To prove: PA = PB

Construction: Join OA, OB and OP.

Proof: In ΔOAP and ΔOBP

$OA = OB$ [Radii of same circle]

$OP = OP$ [Common]

$\angle OAP = \angle OBP$ [Each 90°, because angle between tangent and radius is 90°.]

∴ $\Delta OAP \cong \Delta OBP$ [By RHS Congruence Criterion]

∴ $PA = PB$. [Corresponding parts of congruent triangles are congruent.]

(*b*) (*i*) The coordinates of points P, Q and R are (4, –3), (–2, –2), and (2, 0) respectively.

(*ii*) Slope of QR = $\dfrac{0-(-2)}{2-(-2)} = \dfrac{2}{4} = \dfrac{1}{2}$

∴ Slope of line parallel to QR = $\dfrac{1}{2}$

Equation of line through P(4, –3) and having slope $\dfrac{1}{2}$ is

$$y-(-3) = \frac{1}{2}(x-4)$$

$$\Rightarrow \quad 2y + 6 = x - 4 \quad \text{or} \quad 2y - x + 10 = 0.$$

(*c*) Here, sales are intrastate, so GST comprises SGST and CGST.

The purchase value for walnut is ₹ 650 and the rate of GST is 5%, so GST comprises 2.5 % as SGST and 2.5% as CGST.

SGST = 2.5% of ₹ 650 = ₹ 16.25

CGST = 2.5% of ₹ 650 = ₹ 16.25

Amount to be paid for walnut = ₹ 650 + SGST + CGST

= ₹ 650 + ₹ 16.25 + ₹ 16.25

= ₹ 682.50

The purchase value for potato chips is (₹ 50 × 2), *i.e.*, ₹ 100 and the rate of GST is 0%

SGST = NIL

CGST = NIL

Amount to be paid for potato chips = ₹ 100.

The purchase value for coffee is (₹ 80 × 2), *i.e.*, ₹ 160 and the rate of GST is 18%, so GST comprises 9% as SGST and 9% as CGST.

SGST = 9% of ₹ 160 = ₹ 14.40

CGST = 9% of ₹ 160 = ₹ 14.40

Amount to be paid for coffee = ₹ 160 + SGST + CGST

= ₹ 160 + ₹ 14.40 + ₹ 14.40

= ₹ 188.80

(*i*) Total amount of SGST paid = ₹ 16.25 + ₹ 14.40
= ₹ 30.65.

(*ii*) Total amount of the bill = ₹ 682.50 + ₹ 100 + ₹ 188.80
= ₹ 971.30.

Solution 11

(*a*) Here, N.V. of a share = ₹ 50
M.V. of a share = ₹ 80
Rate of dividend = 20%

(*i*) Annual income = (20% of ₹ 50)(Number of shares)

$\Rightarrow$ ₹ 900 = ₹ 10 · (Number of shares)

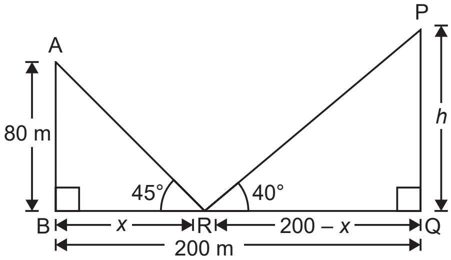
$\Rightarrow$ Number of shares = $\frac{₹ 900}{₹ 10} = 90$

$\therefore$ Amount invested by Mr. Sharma = (₹ 80)(90) = ₹ 7200.

(*ii*) Percentage return on his investment = $\frac{₹ 900}{₹ 7200} \times 100$
= 12.5.

(*b*) Let PQ = h metres and BR = x metres. Then,
RQ = $(200 - x)$ metres.

In right-angled ΔABR,



$$\tan 45° = \frac{AB}{BR} = \frac{80}{x}$$

$$\Rightarrow \quad 1 = \frac{80}{x}$$

$$\Rightarrow \quad x = 80 \text{ metres}$$

Since $x = 80$ metres, therefore RQ = $200 - x$
$= 200 - 80 = 120$ metres

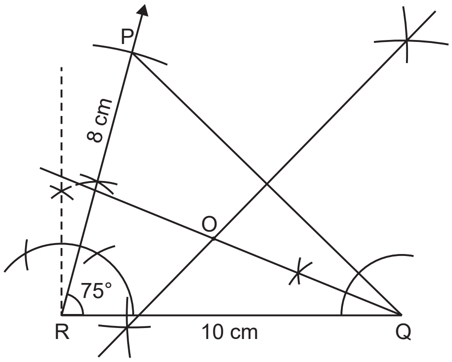
In right-angled ΔPQR, $\tan 40° = \frac{PQ}{RQ} = \frac{h}{120}$

$\Rightarrow \quad 0.83 = \frac{h}{120} \quad \Rightarrow \quad h = 120(0.83)$

$\Rightarrow \quad h = 99.6$ metres

or $h = 100$ metres. (to the nearest metre)

(*c*) **Steps of Construction**



(*i*) Construct ΔPQR with the given data.

(*ii*) The locus of points equidistant from QR and QP is the bisector of $\angle$PQR.
Draw bisector of $\angle$PQR.

(*iii*) The locus of points which are equidistant from P and Q is the perpendicular bisector of PQ.
Draw the perpendicular bisector of PQ.

(*iv*) The point of intersection of bisector of $\angle$PQR and perpendicular bisector of PQ is point O which satisfies conditions (*i*) and (*ii*) given in question.