SOLUTIONS TO COMPETENCY BASED PRACTICE QUESTIONS

Unit I: Pure Arithmetic (Chapter 1)

I: Multiple Choice Questions (1 mark each)

1. (*c*) We have, $\left(\sqrt{x^3}\right)^{\frac{2}{3}} = \left\{\left(x^3\right)^{\frac{1}{2}}\right\}^{\frac{2}{3}}$

$$= \left\{(x)^{\frac{3}{2}}\right\}^{\frac{2}{3}} = (x)^{\frac{3}{2}\times\frac{2}{3}} = x$$

2. (*b*) The surds $\sqrt[3]{2}$, $\sqrt[4]{2}$ and $\sqrt[12]{32}$ have orders respectively 3, 4 and 12; whose L.C.M. is 12.

Now, $\sqrt[3]{2} = \sqrt[3\times 4]{2^4} = \sqrt[12]{16}$, and

$\sqrt[4]{2} = \sqrt[4\times 3]{2^3} = \sqrt[12]{8}$.

Thus, the given product $= \sqrt[3]{2}\times\sqrt[4]{2}\times\sqrt[12]{32} = \sqrt[12]{16}\times\sqrt[12]{8}\times\sqrt[12]{32}$

$$= \sqrt[12]{16\times 8\times 32} = \sqrt[12]{2^4\times 2^3\times 2^5}$$

$$= \sqrt[12]{2^{4+3+5}} = \sqrt[12]{2^{12}} = 2.$$

3. (*c*) Statement 1 is true:

We have, $\sqrt[3]{4} = \sqrt[3\times 4]{4^4} = \sqrt[12]{256}$

and $\sqrt[4]{5} = \sqrt[4\times 3]{5^3} = \sqrt[12]{125}$

Since $\sqrt[12]{125} < \sqrt[12]{256} \Rightarrow \sqrt[4]{5} < \sqrt[3]{4}$

$\therefore$ Of the surds $\sqrt[3]{4}$ and $\sqrt[4]{5}$, $\sqrt[4]{5}$ is the smaller one.
Therefore, Statement 1 is true.

Statement 2 is true:

L.C.M. of 2, 3 and 5 is 30.

Now, $\sqrt[3]{2} = (2)^{\frac{1}{3}} = (2^{10})^{\frac{1}{30}} = \sqrt[30]{2^{10}}$;

$\sqrt[5]{4} = (4)^{\frac{1}{5}} = (4^6)^{\frac{1}{30}} = \sqrt[30]{4^6}$, and

$\sqrt{3} = (3)^{\frac{1}{2}} = (3^{15})^{\frac{1}{30}} = \sqrt[30]{3^{15}}$

Clearly, $3^{15} > 4^6 > 2^{10}$

$\Rightarrow$ $\sqrt{3} > \sqrt[5]{4} > \sqrt[3]{2}$

Therefore, Statement 2 is true.
Hence, option (*c*) is the correct answer.

4. (*a*) Given, $a = 2+\sqrt{3} \Rightarrow \dfrac{1}{a} = \dfrac{1}{2+\sqrt{3}} = \dfrac{(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$

$$= \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$\therefore$ $a - \dfrac{1}{a} = (2+\sqrt{3}) - (2-\sqrt{3}) = 2\sqrt{3}.$

5. (*d*) Let $x = 0.12\bar{3}$

or $x = 0.12333\ldots$

$\Rightarrow$ $100x = 12.333\ldots$ ...(1)

and $1000x = 123.333\ldots$ ...(2)

$\therefore$ $1000x - 100x = 111$

$\Rightarrow$ $900x = 111$ or $x = \frac{111}{900}$.

II: Short Answer Questions-1 (3 marks each)

6. We write 13 as the sum of the squares of two natural numbers:

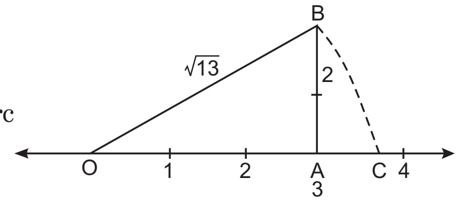
$$13 = 9 + 4 = 3^2 + 2^2$$

On the number line, take OA = 3 units.

Draw BA = 2 units, perpendicular to OA. Join OB.

By Pythagoras' Theorem, $OB = \sqrt{13}$.

Using a compass with centre O and radius OB, draw an arc which intersects the number line at the point C.



Then, C corresponds to $\sqrt{13}$.

Remark: We can also take OA = 2 units and AB = 3 units.

7. We have, $a = \frac{3+\sqrt{5}}{2}$

$\Rightarrow$ $\frac{1}{a} = \frac{2}{3+\sqrt{5}}$

$= \frac{2}{(3+\sqrt{5})} \times \frac{(3-\sqrt{5})}{(3-\sqrt{5})}$ [Rationalising the denominator]

$= \frac{6-2\sqrt{5}}{9-5} = \frac{6-2\sqrt{5}}{4} = \frac{3-\sqrt{5}}{2}$

Now, $a + \frac{1}{a} = \frac{3+\sqrt{5}}{2} + \frac{3-\sqrt{5}}{2}$

$= \frac{6}{2} = 3$

$\therefore$ $\left(a + \frac{1}{a}\right)^2 = (3)^2$

$\Rightarrow$ $a^2 + \frac{1}{a^2} + 2 = 9$

or $a^2 + \frac{1}{a^2} = 9 - 2 = 7$

Hence, $a^2 + \frac{1}{a^2} = 7$.

8. $\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5\left((2^3)^{\frac{1}{3}} + (3^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$

$$= [5(2 + 3)^3]^{\frac{1}{4}}$$

$$= [5(5)^3]^{\frac{1}{4}}$$

$$= [5^4]^{\frac{1}{4}} = 5.$$

III: Short Answer Questions-2 (4 marks each)

9. We have, $\dfrac{4}{3\sqrt{3} - 2\sqrt{2}} + \dfrac{3}{3\sqrt{3} + 2\sqrt{2}}$

$$= \frac{4(3\sqrt{3} + 2\sqrt{2}) + 3(3\sqrt{3} - 2\sqrt{2})}{(3\sqrt{3} - 2\sqrt{2})(3\sqrt{3} + 2\sqrt{2})}$$

$$= \frac{12\sqrt{3} + 8\sqrt{2} + 9\sqrt{3} - 6\sqrt{2}}{27 - 8}$$

$$= \frac{21\sqrt{3} + 2\sqrt{2}}{19} = \frac{21 \times 1.732 + 2 \times 1.414}{19}$$

$$= \frac{36.372 + 2.828}{19} = \frac{39.2}{19} = 2.063.$$

10. Here, $\dfrac{4}{(216)^{-\frac{2}{3}}} + \dfrac{1}{(256)^{-\frac{3}{4}}} + \dfrac{2}{(243)^{-\frac{1}{5}}}$

$$= \frac{4}{(6^3)^{-\frac{2}{3}}} + \frac{1}{(4^4)^{-\frac{3}{4}}} + \frac{2}{(3^5)^{-\frac{1}{5}}}$$

$$= \frac{4}{(6)^{-2}} + \frac{1}{(4)^{-3}} + \frac{2}{(3)^{-1}}$$

$$= 4 \times (6)^2 + 1 \times (4)^3 + 2 \times 3$$

$$= 4 \times 36 + 64 + 6$$

$$= 144 + 70 = 214.$$

IV: Long Answer Questions (5 marks each)

11. To express $0.6 + 0.\overline{7} + 0.4\overline{7}$ in the form $\dfrac{p}{q}$, we first find the values of recurring decimals $0.\overline{7}$ and $0.4\overline{7}$.

Let $x = 0.\overline{7}$

$\Rightarrow$ $x = 0.777...$

$\Rightarrow$ $10x = 7.777...$

$\therefore$ $10x - x = 7$

$\Rightarrow$ $9x = 7$ or $x = \dfrac{7}{9}$

Also, let $y = 0.4\overline{7}$

$\Rightarrow$ $y = 0.4777...$

$\Rightarrow$ $10y = 4.777...$

$\Rightarrow$ $100y = 47.777...$

$\therefore$ $100y - 10y = 43$

$\Rightarrow$ $90y = 43$ or $y = \dfrac{43}{90}$

Now, $0.6 + 0.\overline{7} + 0.4\overline{7} = 0.6 + \frac{7}{9} + \frac{43}{90}$

$$= \frac{6}{10} + \frac{7}{9} + \frac{43}{90}$$

$$= \frac{54 + 70 + 43}{90} = \frac{167}{90}.$$

12. Given, $a = 5 + 2\sqrt{6}$ and $b = \frac{1}{a}$. Then,

$$b = \frac{1}{a} = \frac{1}{5 + 2\sqrt{6}} = \frac{1}{5 + 2\sqrt{6}} \times \frac{5 - 2\sqrt{6}}{5 - 2\sqrt{6}}$$

$$= \frac{5 - 2\sqrt{6}}{5^2 - (2\sqrt{6})^2} = \frac{5 - 2\sqrt{6}}{25 - 24} = 5 - 2\sqrt{6}$$

Therefore, $a^2 + b^2 = (a + b)^2 - 2ab$

Here, $a + b = (5 + 2\sqrt{6}) + (5 - 2\sqrt{6}) = 10$

and $ab = (5 + 2\sqrt{6})(5 - 2\sqrt{6})$

$$= 5^2 - (2\sqrt{6})^2 = 25 - 24 = 1$$

Therefore, $a^2 + b^2 = 10^2 - 2 \times 1 = 100 - 2 = 98.$

Unit II: Commercial Mathematics (Chapter 2)

I: Multiple Choice Questions (1 mark each)

1. (*a*) Using the formula, $A = P\left(1+\frac{r}{100}\right)^n$, we have

$$5600 = 5000\left(1+\frac{r}{100}\right) \qquad [\because \text{For 1 year, } n = 1]$$

$$\Rightarrow \quad \frac{56}{50} = 1+\frac{r}{100} \quad \Rightarrow \quad \frac{28}{25}-1 = \frac{r}{100}$$

$$\Rightarrow \quad \frac{3}{25} = \frac{r}{100} \quad \Rightarrow \quad r = 12$$

Alternatively:

We know that for first year simple interest and compound interest are same. So, rate of interest can also be find out by using simple interest formula.

Here, simple interest = ₹ 5600 – ₹ 5000 = ₹ 600

$$\therefore \quad 600 = \frac{5000\times r\times 1}{100} \qquad \left(\because \text{ S.I.} = \frac{P\times r\times t}{100}\right)$$

$$\Rightarrow \quad 600 = 50r \quad \text{or} \quad r = 12$$

Thus, rate of interest is 12% p.a.

2. (*b*) Compound interest will be more than double if it is more than principal.

Let n be the least number of years in which a sum of money ₹ P (say) become more than double.

According to condition,

$$\text{C.I.} > P$$

$$\Rightarrow \quad A - P > P \qquad [\because \quad \text{C.I.} = A - P]$$

$$\Rightarrow \quad A > 2P$$

$$\Rightarrow \quad P\left(1+\frac{20}{100}\right)^n > 2P \qquad \left[\because \ A = P\left(1+\frac{r}{100}\right)^n\right]$$

$$\Rightarrow \quad \left(1+\frac{1}{5}\right)^n > 2$$

$$\Rightarrow \quad \left(\frac{6}{5}\right)^n > 2$$

The expression $\left(\frac{6}{5}\right)^n$ will be greater than 2, if $n = 4$.

Hence, the required least number of complete years is 4.

Note: $\left(\frac{6}{5}\right)^4$, *i.e.*, $\frac{1296}{625} = 2.0736$, which is greater than 2.

3. (*c*) In case of depreciation, we use the formula, $A = P\left(1-\frac{r}{100}\right)^n$.

Therefore,

After one year, value of the car = ₹ 8,40,000 $\left(1-\frac{10}{100}\right)$

= ₹ 7,56,000

Therefore, Statements (*i*) and (*ii*) are false.

Now,

After 2 years, value of the car $= ₹\ 8{,}40{,}000\left(1-\frac{10}{100}\right)^2$

$$= ₹\ 8{,}40{,}000 \times \left(\frac{90}{100}\right)^2$$

$$= ₹\ 8{,}40{,}000 \times \frac{81}{100} = ₹\ 6{,}80{,}400$$

Therefore, Statement (*iii*) is true.

Hence, out of the given statements, only (*iii*) is true.

II: Short Answer Questions-1 (3 marks each)

4. Here, C.I. = ₹ 331, $r = 10\%$ p.a. and $n = 3$ years. **P = ?**

Since Amount, A = P + C.I.

$$A = P + 331$$

Using the formula, $A = P\left(1+\frac{r}{100}\right)^n$, we have

$$P + 331 = P\left(1+\frac{10}{100}\right)^3 = P\left(\frac{11}{10}\right)^3$$

$$\Rightarrow \quad 1+\frac{331}{P} = \left(\frac{11}{10}\right)^3 = 1.331$$

$$\Rightarrow \quad \frac{331}{P} = 1.331 - 1 = 0.331 \quad \Rightarrow \quad P = \frac{331}{0.331} = 1000$$

Thus, the principal is ₹ 1,000.

5. Let 2 years ago, Smriti bought the laptop for ₹ P. Then,

Present value of the laptop = ₹ 81,000

Annual rate of deprecation = 10%

In case of depreciation, we use the formula,

Present value = value n years ago $\times \left(1-\frac{r}{100}\right)^n$

$$\therefore \quad ₹\ 81000 = P\left(1-\frac{10}{100}\right)^2$$

$$\Rightarrow \quad ₹\ 81000 = P\left(\frac{9}{10}\right)^2$$

$$\Rightarrow \quad P = ₹\ \frac{81000 \times 100}{81} = ₹\ 100000$$

Hence, value of the laptop 2 years ago was ₹ 1,00,000.

6. We have,

Principal, P = ₹ 8,000; rate of interest, $r = 10\%$ p.a.

We know that simple interest for first year = compound interest for first year

$\therefore$ Compound interest on ₹ 8000 for first year at 10% p.a.

$$= ₹\left(\frac{8000 \times 10 \times 1}{100}\right) = ₹\ 800$$

Now, Principal for second year = ₹ 8000 + ₹ 800 = ₹ 8800

$\therefore$ Compound interest for second year $= ₹\left(\frac{8800 \times 10 \times 1}{100}\right) = ₹\ 880$.

Hence, the compound interest on ₹ 8,000 for second year is ₹ 880.

III: Short Answer Questions-2 (4 marks each)

7. Let the given sum be ₹ P and $r\%$ be the rate of interest per annum.

According to the given condition, we have

$$₹\ 9680 = P\left(1+\frac{r}{100}\right)^2 \quad ...(1)$$

and $$₹\ 10648 = P\left(1+\frac{r}{100}\right)^3 \quad ...(2)$$

Dividing (2) by (1), we get

$$\frac{10648}{9680} = \left(1+\frac{r}{100}\right)$$

$$\Rightarrow \quad \frac{1331}{1210} = 1+\frac{r}{100} \qquad [\because \text{Dividing Nr. and Dr. on L.H.S. by 8}]$$

$$\Rightarrow \quad \frac{11}{10} = 1+\frac{r}{100}$$

$$\Rightarrow \quad \frac{r}{100} = \frac{1}{10} \quad \text{or} \quad r = 10$$

Thus, the rate of interest per annum is 10%.

Now, substituting $r = 10$ in (1), we get

$$₹\ 9680 = P\left(1+\frac{10}{100}\right)^2$$

$$\Rightarrow \quad ₹\ 9680 = P\left(\frac{11}{10}\right)^2 = \frac{121}{100}P$$

$$\Rightarrow \quad P = ₹\ \frac{9680\times 100}{121} = ₹\ 8000$$

Hence, the required sum is ₹ 8,000.

8. Let the certain sum of money be ₹ P.

Then, compound interest (C.I.) on ₹ P at 5% per annum for 2 years is given by

$$246 = P\left[\left(1+\frac{5}{100}\right)^2 - 1\right] \qquad [\because \text{C.I. is given ₹ 246.}]$$

$$\Rightarrow \quad 246 = P\left[\left(\frac{21}{20}\right)^2 - 1\right]$$

$$\Rightarrow \quad 246 = \frac{41P}{400}$$

$$\Rightarrow \quad P = \frac{246\times 400}{41} \quad \Rightarrow \quad P = ₹\ 2400$$

Now, simple interest (S.I.) on ₹ 2400 for 3 years at 6% per annum

$$= ₹\left(\frac{2400\times 6\times 3}{100}\right) = ₹\ 432$$

Thus, the simple interest on the given sum is ₹ 432.

IV: Long Answer Questions (5 marks each)

9. Let the sum borrowed be ₹ P.

Then, amount at the end of first year $= ₹\left(P+\frac{P\times 10\times 1}{100}\right) = ₹\frac{11}{10}P$ [∵ Amount = Principal + Interest]

∴ Principal for the second year $= ₹\left(\frac{11}{10}P - 4800\right)$

$$\text{Amount at the end of second year} = ₹\left(\frac{11}{10}P - 4800\right) + \frac{₹\left(\frac{11}{10}P-4800\right)\times 10\times 1}{100}$$

$$= ₹\left[\frac{11}{10}P - 4800 + \frac{1}{10}\left(\frac{11}{10}P - 4800\right)\right]$$

$$= ₹\left[\frac{11}{10}P - 4800 + \frac{11}{100}P - 480\right]$$

$$= ₹\left[\frac{121}{100}P - 5280\right]$$

$$\therefore \quad ₹\left[\frac{121}{100}P - 5280\right] = ₹\ 6820$$

$$\Rightarrow \quad \frac{121}{100}P = 6820 + 5280 = 12100$$

$$\Rightarrow \quad P = \frac{12100\times 100}{121} = 10000$$

Hence, the sum borrowed was ₹ 10,000.

10. Let the sum of money lent out be ₹ P.

<u>Calculating Compound Interest (C.I.)</u>

Here, the interest is payable half yearly. So, we use the formula,

$$A = P\left(1+\frac{r/2}{100}\right)^{2n}$$, where r is rate of interest p.a. and n is the number of years.

$$\therefore \quad A = P\left(1+\frac{10/2}{100}\right)^{2\times 1} \qquad [\because r = 10\% \text{ p.a. and } n = 1]$$

$$\Rightarrow \quad A = P\left(1+\frac{5}{100}\right)^{2}$$

$$\Rightarrow \quad P + C.I. = P\left(1+\frac{5}{100}\right)^{2} \qquad [\because A = P + C.I.]$$

$$\Rightarrow \quad C.I. = \left[P\left(1+\frac{5}{100}\right)^{2} - P\right] = 0.1025\ P$$

<u>Calculating Simple Interest (S.I.)</u>

Using the simple interest formula, S.I. $= \frac{P\times r\times t}{100}$, we have

$$S.I. = \frac{P\times 10\times 1}{100} = 0.1\ P$$

Given that $\quad C.I. - S.I. = ₹\ 15$

$$\Rightarrow \quad 0.1025\ P - 0.1\ P = 15$$

$$\Rightarrow \quad 0.0025\ P = 15$$

$$\Rightarrow \quad P = \frac{15}{0.0025}$$

$$\Rightarrow \quad P = ₹\ 6{,}000.$$

Unit III: Algebra (Chapters 3–6)

I: Multiple Choice Questions (1 mark each)

1. (*b*) Here, $\frac{x}{y}+\frac{y}{x}=1 \Rightarrow x^2+y^2=xy$

or $x^2+y^2-xy=0$

$\therefore x^3+y^3=(x+y)(x^2+y^2-xy)$

$=(x+y)\times 0$ $[\because x^2+y^2-xy=0]$

$=0.$

2. (*b*) We have, $(x+3)^3 = x^3+(3)^3+3(x)^2(3)+3x(3)^2$

$= x^3+27+9x^2+27x$

$\therefore$ The coefficient of x^2 in the expansion of $(x+3)^3$ is 9.

3. (*c*) We have, $(x+y)^3-(x^3+y^3)$

$= x^3+y^3+3xy(x+y)-x^3-y^3$

$= 3xy(x+y)$

Thus, $3xy$ is a factor of the given expression.

4. (*c*) Putting $x = 1-2m$ and $y = 3m$ in the equation $5x+3y-7=0$, we get

$$5(1-2m)+3(3m)-7 = 0$$

$$\Rightarrow \quad 5-10m+9m-7 = 0$$

$$\Rightarrow \quad -m-2 = 0 \quad \text{or} \quad m = -2.$$

5. (*a*) In rectangle ABCD, we have

$$AB = DC \Rightarrow x+3y=13 \quad \ldots(1)$$

and $$AD = BC \Rightarrow 3x+y=7 \quad \ldots(2)$$

From eq. (1), $x = 13-3y$

Substituting $x = 13-3y$ in eq. (2) we get

$$3(13-3y)+y = 7$$

$$\Rightarrow \quad 39-9y+y = 7 \Rightarrow 39-8y=7$$

$$\Rightarrow \quad 8y = 39-7 = 32$$

$$\Rightarrow \quad y = 4$$

Putting $y = 4$ in eq. (1), we get

$$x+3\times 4 = 13 \Rightarrow x+12=13 \quad \text{or} \quad x = 13-12 = 1$$

Thus, the values of x and y respectively are 1 and 4.

6. (*d*) We have,

$$\sqrt{x^{-1}y}\cdot\sqrt{y^{-1}z}\cdot\sqrt{z^{-1}x}$$

$$=\sqrt{\frac{y}{x}}\cdot\sqrt{\frac{z}{y}}\cdot\sqrt{\frac{x}{z}} = 1.$$

7. (*b*) Given, $2^{x-1}+2^{x+1} = 160$

$$\Rightarrow \quad \frac{2^x}{2}+2\cdot 2^x = 160$$

$$\Rightarrow \quad 2^x+4\cdot 2^x = 320$$

$$\Rightarrow \quad 2^x(1+4) = 320 \Rightarrow 5\cdot 2^x = 300$$

$$\Rightarrow \quad 2^x = \frac{320}{5} = 64$$

$$\therefore \quad 2^x = 64 \Rightarrow 2^x = 2^6 \Rightarrow x = 6.$$

8. (*c*) Assertion (A) is true:

Given, $\log_8 m + \log_8 \frac{1}{2} = \frac{2}{3}$

$\Rightarrow \quad \log_8 \left(\frac{m}{2}\right) = \frac{2}{3}$ $\quad [\because \log_a m + \log_a n = \log_a (mn)]$

$\Rightarrow \quad \frac{m}{2} = (8)^{\frac{2}{3}}$

$\Rightarrow \quad \frac{m}{2} = (2^3)^{\frac{2}{3}}$

$\Rightarrow \quad \frac{m}{2} = (2)^2 \quad \Rightarrow \quad \frac{m}{2} = 4 \quad$ or $\quad m = 8$

Therefore, Assertion (A) is true.

Reason (R) is true:

By the laws of logarithms, we have

$$\log_a m + \log_a n = \log_a (mn)$$

Therefore, statement given in Reason (R) is true and applied in Assertion (A) to simplify the expression.

Hence, option (*c*) is the correct answer.

II: Short Answer Question (3 marks each)

9. Here, area of the given rectangle is $4p^2 + 4p - 3$.

Factorising the expression, we have

$$\begin{aligned} 4p^2 + 4p - 3 &= 4p^2 + 6p - 2p - 3 \\ &= 2p(2p + 3) - 1(2p + 3) \\ &= (2p + 3)(2p - 1) \end{aligned}$$

Thus, possible length and breadth of the rectangle can be $(2p + 3)$ and $(2p - 1)$.

10. Here, the expression $1 - 64a^3 - 12a + 48a^2$ can be rewritten as:

$$(1)^3 - (4a)^3 - 3(1)^2(4a) + 3(1)(4a)^2 \quad \ldots(1)$$

To factorise the above expression, we use the identity

$$a^3 - b^3 - 3a^2b + 3ab^2 = (a - b)^3 \quad \ldots(2)$$

Comparing (1) with L.H.S. of (2), we get

$$a = 1 \text{ and } b = 4a$$

Hence, $\quad 1 - 64a^3 - 12a + 48a^2 = (1 - 4a)^3$

$$= (1 - 4a)(1 - 4a)(1 - 4a).$$

11. The given pair of equations is:

$$\frac{x}{10} + \frac{y}{5} - 1 = 0 \quad \ldots(1)$$

and $$\frac{x}{8} + \frac{y}{6} = 15 \quad \ldots(2)$$

Simplifying eq. (1), we get

$$x + 2y = 10 \quad \ldots(3)$$

Simplifying eq. (2), we get

$$3x + 4y = 360 \quad \ldots(4)$$

From eq. (3), we have $x = 10 - 2y$

Substituting $x = 10 - 2y$ in eq. (4), we get

$$3(10-2y)+4y = 360$$
$$\Rightarrow \quad 30-6y+4y = 360$$
$$\Rightarrow \quad 30-2y = 360$$
$$\Rightarrow \quad -2y = 360-30$$
$$\Rightarrow \quad -2y = 330 \quad \text{or} \quad y = -165$$

Putting $y = -165$ in eq. (3), we get

$$x + 2(-165) = 10$$
$$\Rightarrow \quad x - 330 = 10 \Rightarrow x = 330 + 10$$
$$\text{or} \quad x = 340$$

Thus, $x = 340$ and $y = -165$ is the solution of the given pair of equation.

Now, given $\quad y = \lambda x + 5$

$$\Rightarrow \quad -165 = 340\lambda + 5 \qquad [\because x = 340 \text{ and } y = -165]$$
$$\Rightarrow \quad -165-5 = 340\lambda$$
$$\Rightarrow \quad -170 = 340\lambda \quad \text{or} \quad \lambda = \frac{-170}{340} = \frac{-1}{2}$$

Hence, the value of λ is $\frac{-1}{2}$.

12. Let $2^x = 3^y = 6^{-z} = k$. Then,

$$2^x = k \Rightarrow 2 = k^{\frac{1}{x}} \qquad \ldots(1)$$
$$3^y = k \Rightarrow 3 = k^{\frac{1}{y}} \qquad \ldots(2)$$
and
$$6^{-z} = k \Rightarrow 6 = k^{-\frac{1}{z}} \qquad \ldots(3)$$

From eq. (1) and (2), we have

$$2 \times 3 = k^{\frac{1}{x}} \times k^{\frac{1}{y}}$$
$$6 = k^{\frac{1}{x}+\frac{1}{y}}$$
$$\Rightarrow \quad k^{-\frac{1}{z}} = k^{\frac{1}{x}+\frac{1}{y}} \qquad \left[\because 6 = k^{-\frac{1}{z}} \text{ from eq. (3)}\right]$$
$$\Rightarrow \quad \frac{1}{x}+\frac{1}{y} = -\frac{1}{z}$$
$$\Rightarrow \quad \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0 \qquad \textbf{Proved.}$$

13. Given, $\quad \log \frac{a-b}{2} = \frac{1}{2}(\log a + \log b)$

$$\Rightarrow \quad \log \frac{a-b}{2} = \frac{1}{2}\log(ab) \qquad [\because \log m + \log n = \log (mn)]$$
$$\Rightarrow \quad \log \frac{a-b}{2} = \log \sqrt{ab} \qquad [\because m \log n = \log n^m]$$
$$\Rightarrow \quad \frac{a-b}{2} = \sqrt{ab}$$
$$\Rightarrow \quad \frac{(a-b)^2}{4} = ab \qquad \text{[Squaring both sides]}$$
$$\Rightarrow \quad (a-b)^2 = 4ab$$
$$\Rightarrow \quad a^2+b^2-2ab = 4ab \quad \text{or} \quad a^2+b^2-6ab = 0 \qquad \textbf{Proved.}$$

III: Short Answer Questions-2 (4 marks each)

14. The given expression is:

$$(x-2y)^3+(2y-3z)^3+(3z-x)^3$$

Here, $(x-2y)+(2y-3z)+(3z-x)=0$

Now, the identity

$$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca) \text{ reduces to}$$

$$a^3+b^3+c^3=3abc \quad \text{if} \quad a+b+c=0$$

Hence, $(x-2y)^3+(2y-3z)^3+(3z-x)^3=3(x-2y)(2y-3z)(3z-x)$.

15. Let the present ages of Sarita and her daughter be x years and y years respectively.

Then, using the first condition, we have

$$x=4y \quad \text{...(1)}$$

Using the second condition, we have

$$x+5=3(y+5)$$

$$\Rightarrow \quad x+5=3y+15$$

$$\Rightarrow \quad x-3y=15-5$$

$$\Rightarrow \quad x-3y=10 \quad \text{...(2)}$$

Substituting $x=4y$ from eq. (1) in eq. (2), we get

$$4y-3y=10$$

$$\Rightarrow \quad y=10$$

Putting $y=10$ in eq. (1), we get

$$x=4\times 10 \quad \Rightarrow \quad x=40$$

$\therefore$ Present age of Sarita = 40 years, and

Present age of the daughter = 10 years

Hence, the present age of the daughter is 10 years.

16. Equations of the two scratches represented by straight lines are:

$$8x+5y=3 \quad \text{...(1)}$$

and $$5x-8y=13 \quad \text{...(2)}$$

To find the point of intersection, we solve the two equations simultaneously.
Multiplying eq. (1) by 8 and eq. (2) by 5 the two equations becomes

$$64x+40y=24 \quad \text{...(3)}$$

and $$25x-40y=65 \quad \text{...(4)}$$

On adding eq. (3) and eq. (4), we get

$$89x=89 \quad \Rightarrow \quad x=1$$

Substituting the value of x in eq. (1), we get

$$8\times 1+5y=3$$

$$\Rightarrow \quad 8+5y=3$$

$$\Rightarrow \quad 5y=3-8=-5$$

or $$y=-1$$

Thus, the point of intersection of the two scratches is $(1, -1)$.

17. We have, $x = 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$

$\Rightarrow$ $x^3 = \left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)^3$

$= \left(2^{\frac{1}{3}}\right)^3 + \left(2^{\frac{2}{3}}\right)^3 + 3\left(2^{\frac{1}{3}}\right)\left(2^{\frac{2}{3}}\right)\left[2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right]$ [Using $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$]

$= 2 + (2^2) + 3x\left(2^{\frac{1}{3}+\frac{2}{3}}\right)$ $\left[\because\ 2^{\frac{1}{3}} + 2^{\frac{2}{3}} = x\right]$

$= 2 + 4 + 3x\,(2) = 6 + 6x$ $\left[\because\ 2^{\frac{1}{3}+\frac{2}{3}} = 2\right]$

$\therefore$ $x^3 = 6 + 6x$

or $x^3 - 6x = 6$ **Proved.**

18. Given, $m = \log_{10} 20$ and $n = \log_{10} 25$. Then,

$2 \log_{10} (p + 1) = 2m - n$

$\Rightarrow$ $2 \log_{10} (p + 1) = 2 \log_{10} 20 - \log_{10} 25$

$= \log_{10} (20)^2 - \log_{10} 25$

$= \log_{10} 400 - \log_{10} 25$

$= \log_{10} \frac{400}{25} = \log_{10} 16$

$\therefore$ $2 \log_{10} (p + 1) = \log_{10} 16$

$\Rightarrow$ $\log_{10} (p + 1)^2 = \log_{10} 16$

$\Rightarrow$ $(p + 1)^2 = 16$

$\Rightarrow$ $p + 1 = 4$ [Taking only +ve value]

$\Rightarrow$ $p = 3.$

IV: Long Answer Questions (5 marks each)

19. Given, $a + b + c = 5$ ...(1)

and $ab + bc + ca = 10$ ...(2)

From (1), we have

$(a + b + c)^2 = (5)^2$

$\Rightarrow$ $a^2 + b^2 + c^2 + 2(ab + bc + ca) = 25$

$\Rightarrow$ $a^2 + b^2 + c^2 + 2 \times 10 = 25$ [Using (2)]

$\Rightarrow$ $a^2 + b^2 + c^2 + 20 = 25$

$\Rightarrow$ $a^2 + b^2 + c^2 = 25 - 20$

or $a^2 + b^2 + c^2 = 5$ ...(3)

Now, $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= (a + b + c)[(a^2 + b^2 + c^2 - (ab + bc + ca)]$

$= 5(5 - 10)$ [Using (1), (2) and (3)]

$= 5 \times (-5) = -25.$

20. Let ₹ x be the fixed charge for the first two days and ₹ y be the additional charge for each extra day thereafter. Than,

Charges for the book kept for 6 days

$x + 4y = 22$ ...(1)

Charges for the book kept for 4 days

$x + 2y = 16$ ...(2)

Subtracting eq. (2) from eq. (1), we get

$$2y = 6 \qquad \text{or} \qquad y = 3$$

Substituting the value of y in eq. (1), we get

$$x + 4 \times 3 = 22$$

$\Rightarrow$ $x + 12 = 22 \;\Rightarrow\; x = 22 - 12 = 10$

Thus, the fixed charge is ₹ 10 and charge for each extra day is ₹ 3.

21. We have, $\left(\dfrac{a^{x+1}}{a^{y+1}}\right)^{x+y}\left(\dfrac{a^{y+2}}{a^{z+2}}\right)^{y+z}\left(\dfrac{a^{z+3}}{a^{x+3}}\right)^{z+x}$

$$= (a^{x+1-y-1})^{x+y}\,(a^{y+2-z-2})^{y+z}\,(a^{z+3-x-3})^{z+x}$$

$$= (a^{x-y})^{x+y}\,(a^{y-z})^{y+z}\,(a^{z-x})^{z+x}$$

$$= a^{x^2-y^2}\cdot a^{y^2-z^2}\cdot a^{z^2-x^2}$$

$$= a^{x^2-y^2+y^2-z^2+z^2-x^2} = a^0 = 1$$ **Proved.**

22. Given, $\log x = m + n$ and $\log y = m - n$. Then,

$$\log\left(\frac{10x}{y^2}\right) = \log(10x) - \log y^2 \qquad \left[\because \log\frac{a}{b} = \log a - \log b\right]$$

$$= \log 10 + \log x - 2\log y \qquad [\because \log a^b = b\log a]$$

$$= 1 + (m + n) - 2(m - n) \qquad [\because \log 10 = 1]$$

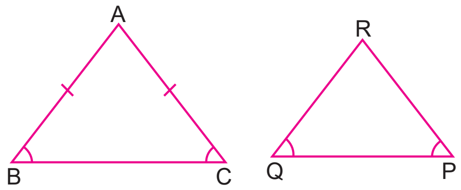
$$= 1 + m + n - 2m + 2n$$

$$= 1 - m + 3n$$

Thus, $\log\left(\dfrac{10x}{y^2}\right) = 1 - m + 3n.$

Unit IV: Geometry (Chapters 7–13)

I: Multiple Choice Questions (1 mark each)



1. (*b*) ΔABC, AB = AC

$\Rightarrow$ ΔABC is isosceles

Also, AB = AC $\Rightarrow \angle B = \angle C$

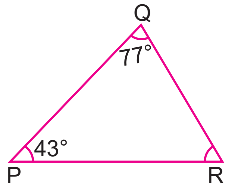
Now, in ΔPQR, $\angle P = \angle C$ and $\angle Q = \angle B$

But $\angle B = \angle C \Rightarrow \angle P = \angle Q$

$\Rightarrow$ QR = PR $\Rightarrow$ ΔPQR is also isosceles.

No relation is given between the sides BC and PQ. Therefore, ASA congruence cannot be established.

So, Δs ABC and PQR are isosceles but not congruent.



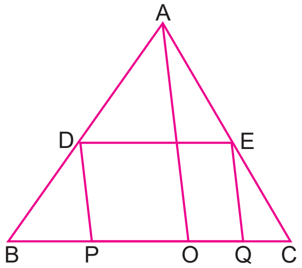
2. (*d*) In ΔPQR, we have

$\angle P = 43°, \angle Q = 77° \Rightarrow \angle R = 180° - (43° + 77°)$

$= 180° - 120° = 60°$

$\therefore$ $\angle Q$ is the greatest angle.

$\Rightarrow$ PR > PQ [$\because$ Side opposite to the greatest angle is the largest.]



3. (*d*) In ΔABC, D and E are mid-points of AB and AC respectively.

Therefore, by Mid-point Theorem, DE || BC or DE || PQ.

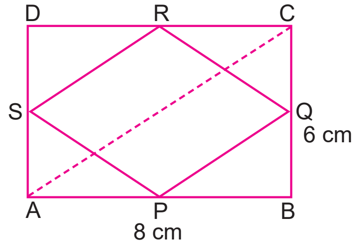
Similarly, in ΔAOB, DP || AO ...(*i*)

and in ΔAOC, EQ || AO ...(*ii*)

From (*i*) and (*ii*) DP || EQ

Thus, in quad. DEQP, DE || PQ and DE || EQ

$\Rightarrow$ DEQP is a parallelogram.



4. (*c*) Let PQRS be the figure obtained by joining the mid-points of the adjacent sides of the rectangle ABCD.

Then, by Pythagoras' Theorem, AC = BD = 10 cm.

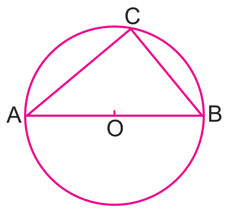
Now, by Mid-point Theorem, in ΔABC,

PQ || AC and $PQ = \frac{1}{2} AC$

$\therefore$ PQ = 5 cm

Similarly, we can prove PQ = QR = RS = SP = 5 cm.

Since diagonals PR ≠ SQ, PQRS is a rhombus.



5. (*a*) <u>Statement 1 is true:</u>

Since AOB is the diameter of the circle, $\angle ACB = 90°$.

$\therefore$ ΔACB is a right-angled triangle.

So, by Pythagoras' Theorem, $AC^2 + BC^2 = AB^2$

Therefore, Statement 1 is true.

<u>Statement 2 is false:</u>

If ABCD is a cyclic quadrilateral, then

$\angle A + \angle C = 180°$ [$\because$ Opposite angles of a cyclic quad. are supplementary.]

and $\angle B + \angle D = 180°$

Here, $\angle A + \angle C = 90° + 95° = 185° \neq 180°$

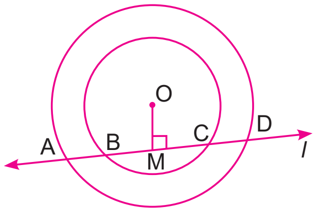
and, $\angle B + \angle D = 70° + 105° = 175° \neq 180°$

Thus, ABCD is not a cyclic quadrilateral.

Therefore, Statement 2 is false.

Hence, option (*a*) is the correct answer.

6. (*b*) Assertion (A) is false:



In the figure, we have AD = 12 cm, and BC = 8 cm.

From O, draw OM ⊥ AD. Then,

AM = DM = 6 cm [∵ Perpendicular from the centre bisects the chord.]

Similarly, BM = CM = 4 cm

Now, AB = AM – BM

⇒ AB = 6 cm – 4 cm = 2 cm

Therefore, Assertion (A) is false.

Reason (R) is true:

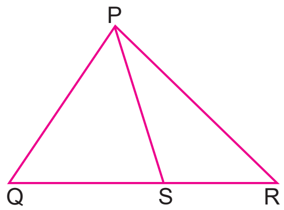
By the chord properties of a circle, the statement given in Reason (R) is true.

Therefore, Reason (R) is true.

Hence, option (*b*) is the correct answer.

II: Short Answer Questions (3 marks each)

7. In ΔPQS by triangular inequalities



PQ + QS > PS ...(1)

Similarly, in ΔPRS

SR + RP > PS ...(2)

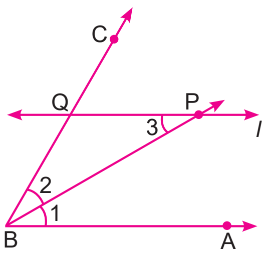
On adding (1) and (2), we get

(PQ + QS) + (SR + RP) > 2PS

⇒ PQ + (QS + SR) + RP > 2PS

⇒ PQ + QR + RP > 2PS **Proved.**

8. Given: P is a point on the bisector of ∠ABC. Line *l* is drawn parallel to BA to meet BC at Q.



To prove: ΔBPQ is isosceles.

Proof: BP is the bisector of ∠ABC

⇒ ∠1 = ∠2 ...(1)

l || BA and transversal BP intersects them, therefore

∠1 = ∠3 ...(2) [Alternate angles]

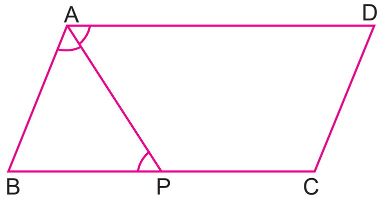
From (1) and (2), we have

∠2 = ∠3

⇒ PQ = BQ

⇒ ΔBPQ is isosceles. **Proved.**

9. Given: P is the mid-point of side BC of parallelogram ABCD such that ∠BAP = ∠DAP.



To prove: AD = 2CD

Proof: We have,

∠BAP = ∠DAP [Given]

∠APB = ∠DAP [Alternate angles]

∴ ∠BAP = ∠APB

$\Rightarrow$ $BP = AB$ [Sides opp. to equal angles are also equal.]

$\Rightarrow$ $2BP = 2AB$ ...(1)

Since P is the mid-point of BC

$$BP = \frac{BC}{2} \quad \text{or} \quad BC = 2BP$$

$\therefore$ From (1), we have

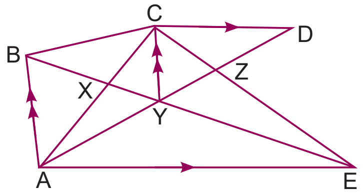
$$BC = 2AB$$

or $AD = 2CD$. **Proved**. [Opp. sides of a || gm are equal.]

10. In the figure, we have

CD || AE and CY || BA

Therefore, Δs ABC and ABY are on the same base AB and between same parallels BA and CY.

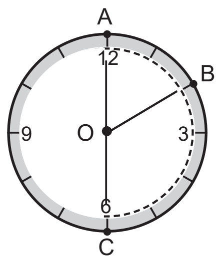


$\Rightarrow$ $ar(\Delta ABC) = ar(\Delta ABY)$

$\Rightarrow$ $ar(\Delta ABC) - ar(\Delta ABX) = ar(\Delta ABY) - ar(\Delta ABX)$

$\Rightarrow$ $ar(\Delta CBX) = ar(\Delta AXY)$. **Proved**.

11. Given, arc length traced by the second hand of the clock from A to B is half the arc length traced by it from B to C.



$\therefore$ $\widehat{AB} = \text{arc } \frac{1}{2}(\widehat{BC})$

$\Rightarrow$ $\angle AOB = \frac{1}{2}\angle BOC$

Also, $\angle AOB + \angle BOC = 180°$

Therefore, $\frac{1}{2}\angle BOC + \angle BOC = 180°$

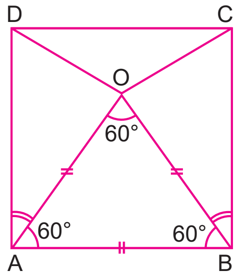
$\Rightarrow$ $\frac{3}{2}\angle BOC = 180°$

or $\angle BOC = \frac{2}{3} \times 180° = 120°$

Thus, the second hand subtends an angle of 120° when it moves from B to C.

III: Short Answer Questions-2 (4 marks each)

12. Given: A square ABCD in which ΔOAB is equilateral.



To prove: ΔOCD is isosceles.

Proof: Since ΔOAB is equilateral,

$$\angle OAB = \angle OBA = 60°$$

Also, ABCD is a square,

$\Rightarrow$ $\angle DAB = \angle CBA = 90°$

$\therefore$ $\angle DAB - \angle OAB = \angle CBA - \angle OBA = 30°$

$\Rightarrow$ $\angle OAD = \angle OBC = 30°$ ...(1)

Now, in Δs OAD and OBC

$OA = OB$ [Sides of equilateral triangle]

$\angle OAD = \angle OBC$ [From (1)]

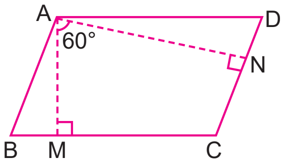
$AD = BC$ [Sides of a square]

$\therefore$ By SAS Congruence Criterion, $\Delta OAD \cong \Delta OBC$

$\Rightarrow$ $OD = OC$ [CPCT]

Hence, ΔOCD is isosceles.

13. In the figure, AM and BN are the perpendiculars drawn through the obtuse angle A of the parallelogram ABCD. Then,



In quad. AMCN, We have

$\angle MAN = 60°$ [Given]

$\angle AMC = 90°$ [∵ AM is an altitude on BC.]

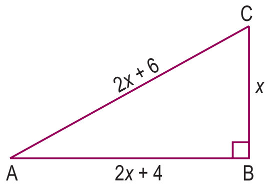
$\angle ANC = 90°$ [∵ AN is an altitude on CD.]

∴ $\angle MCN = 360° - [\angle MAN + \angle AMC + \angle ANC]$
$= 360° - [60° + 90° + 90°] = 360° - 240° = 120°$

So, in parallelogram ABCD,

$\angle BCD = 120°$ [∵ ∠BCD = ∠MCN]
⇒ $\angle BAD = 120°$ [∵ ABCD is a ||gm, ∠BAD = ∠BCD]
Also, $\angle ABC + \angle BCD = 180°$ [∵ Adjacent angles of a ||gm are supplementary.]
⇒ $\angle ABC + 120° = 180°$
⇒ $\angle ABC = 180° - 120° = 60°$
∴ $\angle ADC = \angle ABC = 60°$
Thus, the angles of the parallelogram are 120°, 60°, 120°, 60°.

14. Let ABC be the given triangle, right-angled at B such that perpendicular BC is its shortest side.



Let BC = x cm.
Then, hypotenuse AC = $(2x + 6)$ cm
and base AB = $(2x + 6) - 2 = (2x + 4)$ cm
Now, in right triangle ABC,

$$AC^2 = AB^2 + BC^2$$
⇒ $(2x + 6)^2 = (2x + 4)^2 + x^2$
⇒ $4x^2 + 24x + 36 = 4x^2 + 16x + 16 + x^2$
⇒ $x^2 - 8x - 20 = 0$
⇒ $x^2 - 10x + 2x - 20 = 0$
⇒ $x(x - 10) + 2(x - 10) = 0$
⇒ $(x - 10)(x + 2) = 0$
⇒ $x = 10$ or $x = -2$
⇒ $x = 10$ [$x = -2$ is rejected, since the length of a side cannot be negative.]

Therefore, the sides of the triangle are

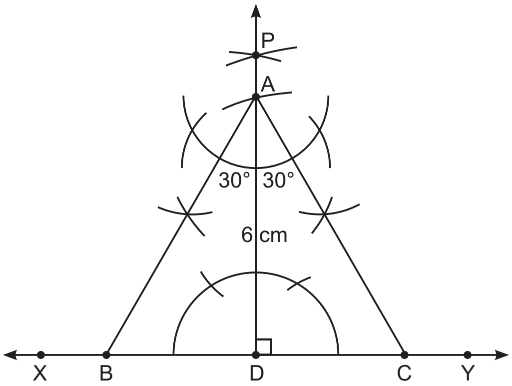
$BC = x = 10$ cm,
$AB = 2x + 4 = 2 \times 10 + 4 = 24$ cm
and $AC = 2x + 6 = 2 \times 10 + 6 = 26$ cm

Now, area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 24 \times 10$ sq cm $= 120$ sq cm

Thus, the area of the given triangle is 120 sq cm.

15. Draw a line XY. Take any point D on this line. Construct perpendicular PD on XY. Cut a line segment AD from D equal to 6 cm. Make angles equal to 30° at A on both sides of AD, say ∠CAD and ∠BAD where B and C lie on XY. Then ABC is the required triangle.



Justification

Since ∠A = 30° + 30° = 60° and AD ⊥ BC.

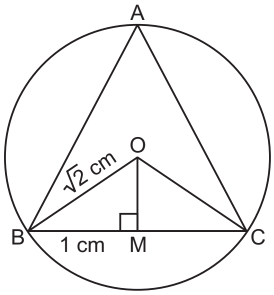
By ASA Congruence Criterion, ΔABD ≅ ΔACD

Therefore, ∠B = ∠C = 60° [By CPCT]

Hence, ΔABC is an equilateral triangle with altitude AD = 6 cm.

16. Let the chord BC divide the circle into two segments and A be the point in the major segment. We have to prove that ∠BAC = 45°.

Draw OM ⊥ BC, then OM bisects the chord BC.



$\therefore \quad BM = \frac{BC}{2} = 1 \text{ cm} \quad [\because BC = 2 \text{ cm}]$

Now, in right ΔBMO, we have

$$OM^2 = OB^2 - BM^2$$
$$= (\sqrt{2})^2 - (1)^2 = 2 - 1 = 1 \text{ cm}$$

$\therefore$ In ΔBMO, $\quad BM = OM = 1 \text{ cm}$

$\Rightarrow \quad \angle BOM = \angle OBM = 45° \quad [\because \angle BMO = 90°]$

Thus, $\quad \angle BOM = \angle COM = 45° \quad [\because \Delta BMO \cong \Delta CMO]$

$\therefore \quad \angle BOM + \angle COM = 45° + 45°$

$\Rightarrow \quad \angle BOC = 90°$

Now, arc BC subtends ∠BOC at the centre and ∠BAC in the remaining part of the circle.

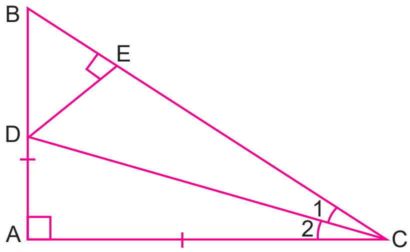
Therefore, $\quad \angle BOC = 2\angle BAC$ [∵ Angle subtended by an arc at the centre is double the angle subtended by it in the remaining part of the circle.]

$\Rightarrow \quad \angle BAC = \frac{1}{2}\angle BOC$

$$= \frac{1}{2} \times 90° = 45°$$

Hence, the chord BC subtends an angle of 45° at a point A in the major segment.

IV: Long Answer Questions (5 marks each)



17. **Given:** ABC is a right triangle such that AB = AC and bisector of angle C intersects the side AB at D.

To Prove: AC + AD = BC

Construction: From D, draw DE ⊥ BC.

Prove: In Δs DAC and DEC, we have

$\angle CAD = \angle CED$ [Each 90°]

$\angle 1 = \angle 2$ [$\because$ CD bisects $\angle ACB$]

$CD = CD$ [Common]

$\therefore$ By AAS Congruence Criterion, $\Delta DAC \cong \Delta DEC$.

Therefore, by CPCT

$AD = DE$ ...(1)

and $AC = EC$ ...(2)

In ΔABC, $AB = AC$ [Given]

$\Rightarrow$ $\angle C = \angle B$ [$\because$ Angles opp. to equal sides are equal.]

$\Rightarrow$ $\angle C = \angle B = 45°$ [$\because \angle A = 90°$]

Also, in ΔBED, $\angle BED = 90°$ [By construction]

$\Rightarrow$ $\angle BDE = 45°$ [$\because \angle B = 45°$]

$\therefore$ $\angle BDE = \angle EBD$

$\Rightarrow$ $BE = DE$ ...(3)

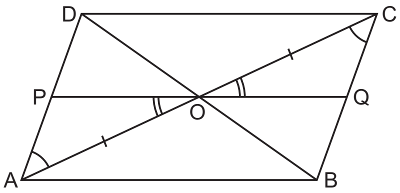
From, (1) and (3), we get $BE = DE = AD$

Now, from the figure, $BC = BE + EC$

$\Rightarrow$ $BC = AD + AC$ [$\because$ From (2), EC = AC]

Proved.

18. In parallelogram ABCD,



AD || BC and transversal AC intersects them, therefore

$\angle DAO = \angle BCO$ [Alternate angles]

or $\angle PAO = \angle QCO$ ...(1)

Now, in Δs AOP and COQ, we have

$\angle AOP = \angle COQ$ [Vertically opp. angles]

$AO = CO$ [Diagonals of a || gm bisect each other.]

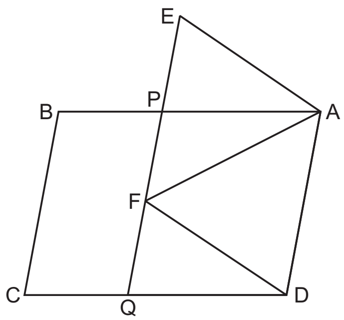
$\angle PAO = \angle QCO$ [Proved in (1)]

$\therefore$ By ASA Congruence Criterion, $\Delta AOP \cong \Delta COQ$

$\Rightarrow$ $PO = QO$, *i.e.*, O bisects PQ.

Hence, PQ is bisected at O.

19. In the figure, we have



AD || EF [$\because$ AEFD is a || gm.]

$\Rightarrow$ AD || EQ

or AD || PQ ...(1)

Also, AB || DC [$\because$ ABCD is a || gm.]

$\Rightarrow$ AP || DQ ...(2)

Thus, APQD is also a || gm. [Using (1) and (2)]

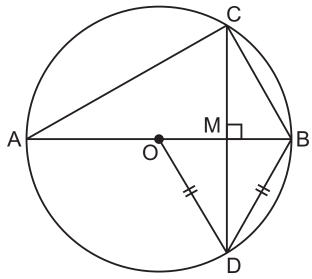
Since || gms AEFD and APQD are on the same base AD and between same parallels AD and EQ, we have

$ar(\text{|| gm AEFD}) = ar(\text{|| gm APQD})$

$\Rightarrow$ $ar(\Delta PEA) + ar(\Delta APF) + ar(\Delta AFD) = ar(\Delta APF) + ar(\Delta AFD) + ar(\Delta QFD)$

$\Rightarrow$ $ar(\Delta PEA) = ar(\Delta QFD)$.

20. In ΔBOD, we have $BD = OD$



But, $OD = OB$ [Radii of the circle]

$\therefore$ $BD = OD = OB$

Thus, ΔBOD is equilateral.

$\therefore$ $\angle BOD = 60° \Rightarrow \angle BCD = 30°$

[$\because$ Angle subtended by an arc at the centre is double the angle subtended by it in the remaining part of the circle.]

or $\angle BCM = 30°$

Now, AB is the diameter of the circle.

$\therefore$ $\angle ACB = 90°$ [Angle in the semicircle is a right angle.]

$\Rightarrow$ $\angle ACM = \angle ACB - \angle BCM = 90° - 30° = 60°$

Also, in right ΔACM,

$\angle CAM = 90° - \angle ACM$

$\Rightarrow$ $\angle CAB = 90° - 60° = 30°.$ [$\because \angle CAM = \angle CAB$]

Unit V: Statistics (Chapters 14–15)

I: Multiple Choice Questions (1 mark each)

1. (*b*) $\overline{x}_1$ is the mean of n_1 observations

$\therefore$ Sum of all observations in x_1 group = $\Sigma n_1\overline{x}_1$

$\overline{x}_2$ is the mean of n_2 observations

$\therefore$ Sum of all observations in x_2 group = $\Sigma n_2\overline{x}_2$

Continuing in this way,

$\overline{x}_n$ is the mean of n_n observations

$\therefore$ Sum of all observations in x_n group = $\Sigma n_n\,\overline{x}_n$

Thus, mean ($\overline{x}$) of all the groups taken together

$$= \frac{\Sigma n_1\overline{x}_1 + \Sigma n_2\overline{x}_2 + \ldots \Sigma n_n\overline{x}_n}{n_1 + n_2 + \ldots n_n}$$

$$= \frac{\sum_{i=1}^{n} n_i\overline{x}_i}{\sum_{i=1}^{n} n_i}$$

2. (*b*) For a continuous frequency distribution,

$$\text{mid-point} = \frac{\text{lower class limit + upper class limit}}{2}$$

$$\Rightarrow \quad m = \frac{l + \text{ upper class limit}}{2}$$

$$\Rightarrow \quad \text{upper class limit} = 2m - l$$

3. (*c*) In a frequency polygon abscissae of the points are the class marks of the respective classes.

4. (*b*) The correct mean $= \dfrac{50 \times 80 + 72 - 27}{50}$

$$= \frac{4045}{50} = 80.9$$

II: Short Answer Questions-1 (3 marks each)

5. According to the given histogram, we have the following frequency distribution table.

Wages (in ₹)	Number of Workers
100–150	15
150–200	50
200–250	30
250–300	35
300–350	20
350–400	10

6. First we modify the given distribution with proper class intervals as shown below:

Rainfall (in mm)	Number of Days
0–2	27
2–4	12
4–6	10
6–8	5
8–10	4
10–12	2

According to the above distribution—

(*i*) The rainfall was recorded between 4 mm and 6 mm on 10 days.

(*ii*) The rainfall was less than 4 mm on (27 + 12 =) 39 days.

(*iii*) The rainfall was 6 mm or more on (5 + 4 + 2 =) 11 days.

7. The given distribution is non-overlapping. So we first change it into an overlapping distribution as:

Class Interval	10.5–20.5	20.5–30.5	30.5–40.5	40.5–50.5	50.5–60.5
Frequency	25	12	10	15	8

We also update this frequency distribution table by adding the two classes of the same width at the two extremes and inserting a row of class marks, as shown below:

Class Interval	0.5–10.5	10.5–20.5	20.5–30.5	30.5–40.5	40.5–50.5	50.5–60.5	60.5–70.5
Class Mark	5.5	15.5	25.5	35.5	45.5	55.5	65.5
Frequency	0	25	12	10	15	8	0

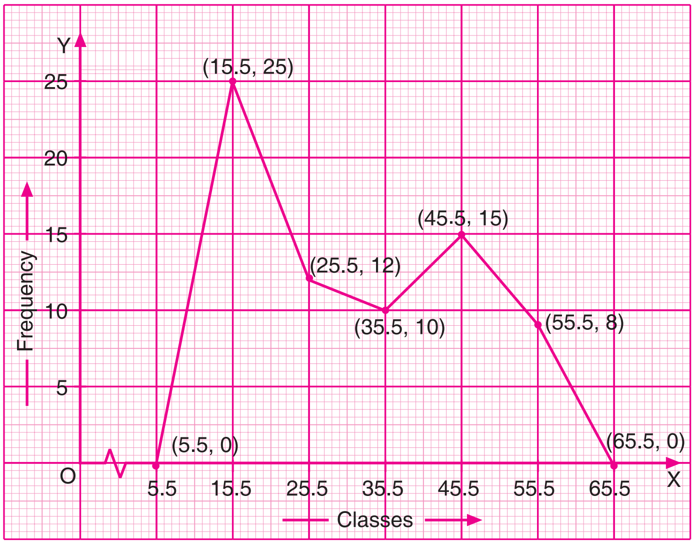
Drawing Frequency Polygon:

Step 1: We draw and label the x and y axes.

Step 2: Using the frequencies: 0, 25, 12, 10, 15, 8, 0 as y-values and the class marks: 5.5, 15.5, 25.5, 35.5, 45.5, 55.5 and 65.5 as the x-values, we plot the points (5.5, 0), (15.5, 25), (25.5, 12), (35.5, 10), (45.5, 15), (55.5, 8) and (65.5, 0).

Step 3: Connect adjacent points with line segments.

The required frequency polygon is drawn in the following graph.

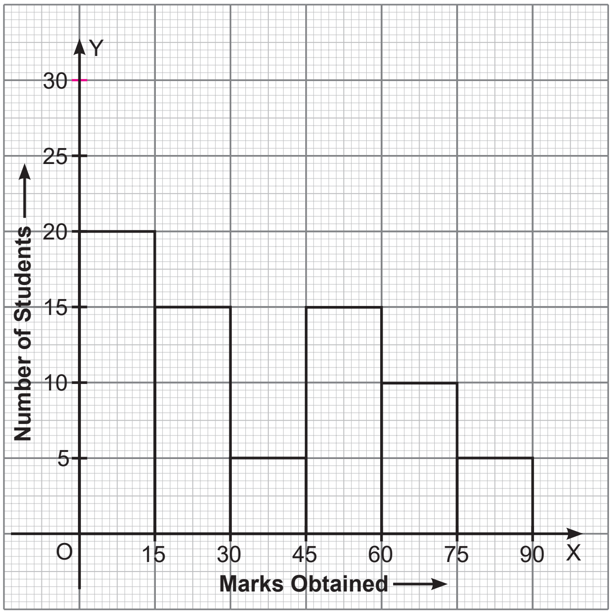


III: Short Answer Questions-2 (4 marks each)

8. From the given distribution, we have the following overtapping (or continuous) frequency distribution.

Marks Obtained	Number of Students
0–15	20
15–30	15
30–45	5
45–60	15
60–75	10
75–90	5

The required histogram for the above distribution is drawn below.



9. To compute the value of a and frequencies of 30 and 70, we draw the following table:

x	f	$f \times x$
10	17	170
30	$5a + 3$	$150a + 90$
50	32	1600
70	$7a - 11$	$490a - 770$
90	19	1710
Total	$\Sigma f = 12a + 60$	$\Sigma fx = 640a + 2800$

$\therefore$ $\text{Mean} = \dfrac{\Sigma fx}{\Sigma f}$

$\Rightarrow$ $50 = \dfrac{640a + 2800}{12a + 60}$ [$\because$ Mean = 50]

$\Rightarrow$ $600a + 3000 = 640a + 2800$

$\Rightarrow$ $640a - 600a = 3000 - 2800$

$\Rightarrow$ $40a = 200 \Rightarrow a = 5$

Thus, $5a + 3 = 5 \times 5 + 3 = 28$

and $7a - 11 = 7 \times 5 - 11 = 24$

Hence, value of a is 5 and frequencies of 30 and 70 are 28 and 24 respectively.

10. Arranging the data in ascending order, we get

27, 28, 29, 32, 37, 49, 57, 58, 60, 63

Here, number of observations, $n = 10$, which is even.

$\therefore$ Median = mean of $\left(\dfrac{n}{2}\right)$th and $\left(\dfrac{n}{2}+1\right)$th score

= mean of 5th and 6th scores

$= \dfrac{37 + 49}{2} = 43$

On replacing 32 by 50, the ascending order of data becomes,

27, 28, 29, 37, 49, 50, 57, 58, 60, 63

∴ New median = mean of 5th and 6th scores

$$= \frac{49+50}{2} = 49.5.$$

IV: Long Answer Questions (5 marks each)

11. The frequency distribution table for the given data is as under:

Marks	Tally Marks	Frequency
0–10	||||	4
10–20	~~||||~~ |	6
20–30	~~||||~~ ~~||||~~ ||||	14
30–40	~~||||~~ ||||	9
40–50	~~||||~~ |	6
50–60	|	1

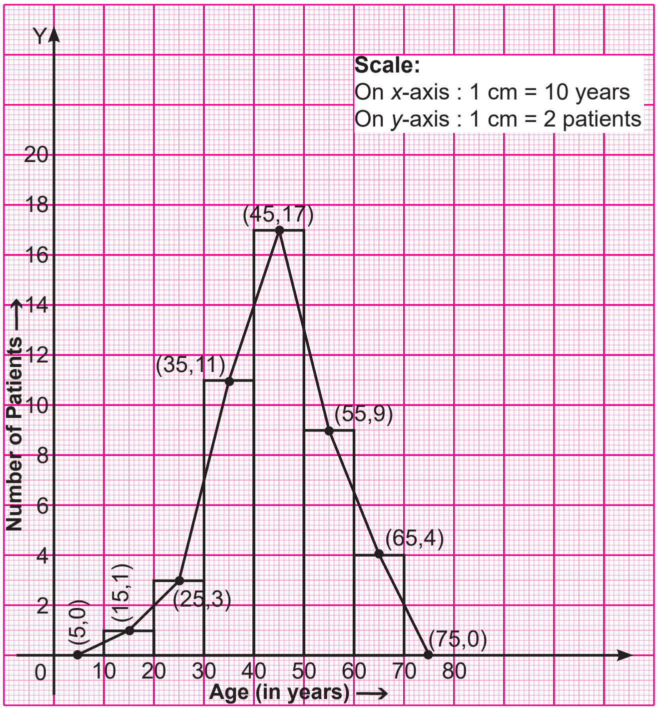
From the above table, we can prepare a *'less than'* cumulative frequency table as below:

Marks	Cumulative Frequency
Less than 10	4
Less than 20	10
Less than 30	24
Less than 40	33
Less than 50	39
Less than 60	40

12. First, we modify the given table by inserting two more class intervals namely 70–80 (in the beginning) and 0–10 (at the end) each with zero frequency. Also, we add a column of class marks to get the following table.

Age (in years)	Class Mark	Number of Patients
70–80	75	0
60–70	65	4
50–60	55	9
40–50	45	17
30–40	35	11
20–30	25	3
10–20	15	1
0–10	5	0

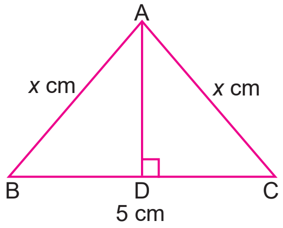
Now, taking age (in years) on x-axis and number of patients on y-axis, we plot the points (75, 0), (65, 4) ... and so on. After joining these points by a ruler, we obtain the frequency polygon as shown in the following graph.



Unit VI: Mensuration (Chapters 16–17)

I: Multiple Choice Questions (1 mark each)

1. (*a*) Let ABC be the isosceles triangle in which AB = AC = x cm and base BC = 5 cm. Then,



Perimeter of ΔABC = AB + AC + BC

$\Rightarrow \quad 11 = 2x + 5$

$\Rightarrow \quad 2x = 6 \quad \text{or} \quad x = 3$

$\Rightarrow \quad AB = AC = 3 \text{ cm}$

Draw AD $\perp$ BC. Then, AD bisects BC

$\Rightarrow \quad BD = CD = \frac{5}{2}$ cm

In right ΔADB, AD = $\sqrt{AB^2 - BD^2}$

$$= \sqrt{(3)^2 - \left(\frac{5}{2}\right)^2} = \sqrt{9 - \frac{25}{4}} = \frac{\sqrt{11}}{2}$$

Thus, area of ΔABC = $\frac{1}{2} \times BC \times AD$

$$= \frac{1}{2} \times 5 \times \frac{\sqrt{11}}{2} \text{ cm}^2 = \frac{5}{4}\sqrt{11} \text{ cm}^2.$$

2. (*b*) If 'a' be the side of the square and 'r' be the radius of the circle, then

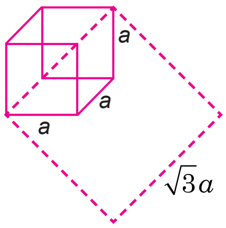
$4a = 2\pi r$ [$\because$ Their perimeters are equal.]

$\Rightarrow \quad \frac{a}{r} = \frac{\pi}{2}$

$$\therefore \quad \frac{\text{area of square}}{\text{area of circle}} = \frac{a^2}{\pi r^2} = \frac{1}{\pi}\left(\frac{a}{r}\right)^2$$

$$= \frac{1}{\pi}\left(\frac{\pi}{2}\right)^2 = \frac{\pi}{4} = \frac{22}{7 \times 4} = \frac{11}{14}, \textit{ i.e., } 11 : 14.$$

3. (*c*) We know that the length of the diagonal of a cube of edge 'a' is $\sqrt{3}a$.



$\therefore \quad \left(\sqrt{3}a\right)^2 = 192$

$3a^2 = 192 \quad \Rightarrow \quad a^2 = 64$

$\Rightarrow \quad a = 8$

So, edge of the cube is 8 cm.

4. (*d*) Volume of rain water collected on the terrace = 8 m × 8 m × $\frac{5}{100}$ m

= 3.2 cu m

= 3200 litres or 3.2 kilolitres.

II: Short Answer Questions-1 (3 marks each)

5. Here, area of the playground = 4800 sq m

and depth of gravel needed = 1 cm = $\frac{1}{100}$ m

∴ Volume of the gravel to cover the playground

$$= 4800 \times \frac{1}{100} \text{ cu m} = 48 \text{ cu m}$$

Now, cost of 1 cu m of gravel = ₹ 5.25

∴ Cost of 48 cu m of gravel = ₹ 48 × 5.25 = ₹ 252.

6. We have, length of the reservoir = 20 m,

breadth of the reservoir = 12 m; and

Depth of the water used, as level went down = 1 m

∴ Volume of water used on the day = 20 m × 12 m × 1 m

= 240 cu m

Also, 1 cu m = 1000 litres

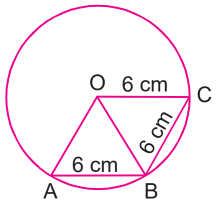
∴ 240 cu m = 240 × 1000 litres

$$= \frac{240 \times 1000}{1000} \text{ kilolitres} \qquad \left[\because 1 \text{ litre} = \frac{1}{1000} \text{kilolitres}\right]$$

= 240 kilolitres

Hence, 240 kilolitres water was used on that day.

7. Obviously, OA = OB = OC = AB = BC = 6 cm



Therefore, Δs OAB and OBC are equilateral triangles each of sides 6 cm.

$$\therefore \text{ Area of equilateral } \Delta \text{ OAB} = \frac{\sqrt{3}}{4}(6)^2 \text{ sq cm}$$

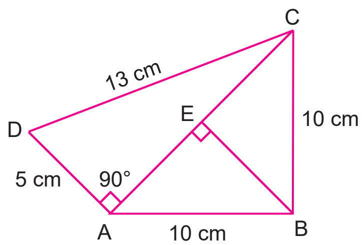
$$= 9\sqrt{3} \text{ sq cm}$$

∴ Area of rhombus OABC = 2 × area of Δ OAB

$$= 2 \times 9\sqrt{3} \text{ sq cm} = 18\sqrt{3} \text{ sq cm.}$$

III: Short Answer Questions-2 (4 marks each)

8. Draw BE ⊥ AC.



In Δ ADC, $AC^2 = CD^2 - AD^2$

$= (13)^2 - (5)^2$

$= 169 - 25 = 144$

⇒ AC = 12 cm

In Δ ABC, AB = BC and BE ⊥ AC

∴ E is the mid-point of AC.

Hence, $CE = \frac{1}{2}(12 \text{ cm}) = 6 \text{ cm}$

Now, $BE^2 = BC^2 - CE^2 = (10)^2 - (6)^2$

$= 100 - 36 = 64$

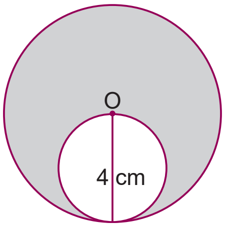
⇒ BE = 8 cm

$\therefore$ Area of quad. ABCD = Area of Δ ABC + Area of Δ ACD

$$= \frac{1}{2} \times AC \times BE + \frac{1}{2} \times AC \times AD$$

$$= \left(\frac{1}{2} \times 12 \times 8\right) + \left(\frac{1}{2} \times 12 \times 5\right) = 48 + 30 = 78 \text{ sq cm.}$$

9. (*a*) Here, radius of bigger circle is 4 cm so the radius of smaller circle will be 2 cm.



Thus, area of shaded region = area of bigger circle – area of smaller circle

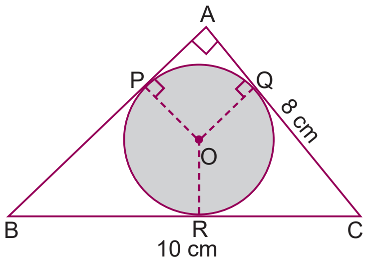
$$= \pi(4)^2 \text{ sq cm} - \pi(2)^2 \text{ sq cm}$$

$$= (16\pi - 14\pi) \text{ sq cm} = 12\pi \text{ sq cm}$$

$$= 12 \times \frac{22}{7} \text{ sq cm}$$

$$= 37\frac{5}{7} \text{ sq cm.}$$

(*b*) Let the circle touch ΔABC at P, Q and R respectively as shown in the figure.



Draw OP $\perp$ AB, OQ $\perp$ AC and OR $\perp$ BC.

Then, AP = AQ = OP = OQ = r (say)

Thus, OPAQ is a square of side r cm and radius of the circle is also r cm.

Now, in right triangle ABC,

$AB^2 + AC^2 = BC^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad AB^2 + (8)^2 = (10)^2$

$\Rightarrow \quad AB^2 = (10)^2 - (8)^2 = 100 - 64 = 36$

Thus, $\quad AB = 6$ cm

From the figure, BP = BR and CQ = CR

$\Rightarrow \quad AB - AP = BR$ and $AC - AQ = CR$

$\Rightarrow \quad 6 - r = BR$ and $8 - r = CR$

On adding the two relations, we get

$6 - r + 8 - r = BR + CR$

$\Rightarrow \quad 14 - 2r = BC$

$\Rightarrow \quad 14 - 2r = 10$ [$\because$ BC = 10 cm]

$\Rightarrow \quad 2r = 4$ or $r = 2$ cm.

Thus, radius of the circle is 2 cm.

Hence, area of the shaded region = $\pi(2)^2$ sq cm

$$= \frac{22}{7}(2)^2 \text{ sq cm} = \frac{88}{7} \text{ sq cm} \quad \text{or} \quad 12\frac{4}{7} \text{ sq cm.}$$

10. Let a cm be the side of the solid iron cube. Then,

Volume of the iron cube = a^3 cu cm

Dimensions of the base of the rectangular vessel = 60 cm × 45 cm

Height of the vessel = 20 cm

Since vessel is half-filled with water, therefore

$$\text{Volume of water in the vessel} = \frac{1}{2} \times \text{Volume of vessel}$$
$$= \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 60 \times 45 \times 20 \text{ cu cm}$$
$$= 27000 \text{ cu cm}$$

On dropping the solid iron cube into the vessel the water level comes up to brim.

$\therefore$ Volume of the solid cube = Volume of water in the vessel

$\Rightarrow$ $a^3 = 27000$ cu cm

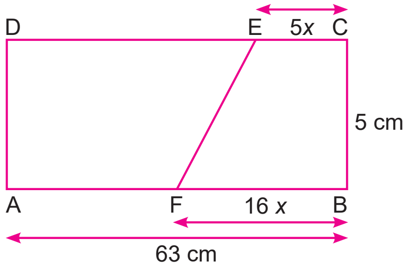
$\Rightarrow$ $a^3 = (30 \times 30 \times 30)$ cu cm

or $a^3 = 30 \text{ cm} \times 30 \text{ cm} \times 30 \text{ cm}$

Hence, size of the solid iron cube is 30 cm × 30 cm × 30 cm.

IV: Long Answer Questions (5 marks each)

11. From rectangle ABCD, let a trapezium BCEF be cut off so that BF = $16x$ and CE = $5x$.



Now, in rectangle ABCD, we have

AB = 63 cm and BC = 5 cm.

$\therefore$ Area of rec. ABCD = AB × BC = 315 sq cm

Also, we have

$$\text{Area of trap. BCEF} = \frac{4}{15} (\text{Area of rec. ABCD})$$

$\Rightarrow$ $\frac{1}{2}(\text{BF} + \text{CE}) \times \text{BC} = \frac{4}{15} \times 315$

$\Rightarrow$ $\frac{1}{2}(16x + 5x) \times 5 = 84$

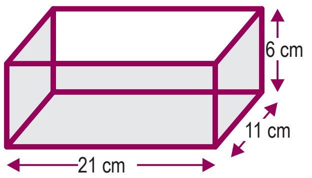
$\Rightarrow$ $21x = \frac{84 \times 2}{5} \quad \Rightarrow \quad x = \frac{8}{5}$ cm

$\therefore$ $\text{BF} = 16 \times \frac{8}{5} \text{ cm} = 25.6 \text{ cm}$

and $\text{CE} = 5 \times \frac{8}{5} \text{ cm} = 8 \text{ cm}$

Hence, the lengths of the parallel sides of the trapezium are 25.6 cm and 8 cm.

12. External dimensions of the open wooden box are 21 cm, 11 cm and 6 cm.



Thickness of the wood = 0.5 cm

So, internal dimensions of the box are 20 cm, 10 cm and 5.5 cm

$\therefore$ Internal surface area of the box = $[2(20 + 10) \times 5.5 + 20 \times 10]$ sq cm

[$\because$ Surface area of open box = lateral surface area + area of the base]

$= (11 \times 30 + 200)$ sq cm

$= (330 + 200)$ sq cm

$= 530$ sq cm

$\because$ Cost of painting 530 sq cm of the box = ₹ 106

$\therefore$ Cost of painting 1 sq cm of the box = ₹$\frac{106}{530}$ = ₹$\frac{1}{5}$, *i.e.*, 20 paise

Hence, the rate of painting per sq cm is 20 paise.

Unit VII: Trigonometry (Chapters 18)

I: Multiple Choice Questions (1 mark each)

1. (*b*) Given, $3 \tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{3}$

$$\therefore \quad \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \frac{\tan\theta - 1}{\tan\theta + 1} \quad \text{(Dividing Nr. and Dr. by } \cos\theta)$$

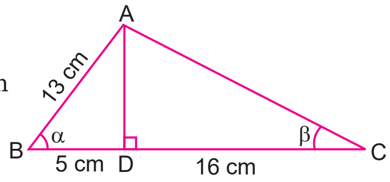
$$= \frac{\frac{4}{3} - 1}{\frac{4}{3} + 1} = \frac{\frac{1}{3}}{\frac{7}{3}} = \frac{1}{7}.$$

2. (*a*) We have, $\operatorname{cosec}\theta = \sqrt{10} \Rightarrow \operatorname{cosec}^2\theta = 10$

$$\Rightarrow 1 + \cot^2\theta = 10$$
$$\Rightarrow \cot^2\theta = 9$$
$$\Rightarrow \cot\theta = 3 \text{ and } \tan\theta = \frac{1}{3}$$

$$\therefore \tan\theta + \cot\theta = \frac{1}{3} + 3 = \frac{10}{3}.$$

3. (*d*) Statement 1 is false:



In right ΔADB, $AD = \sqrt{AB^2 - BD^2}$

$$= \sqrt{(13)^2 - (5)^2} = \sqrt{169 - 25} = \sqrt{144} = 12 \text{ cm}$$

$$\therefore \quad \sec\alpha = \frac{AB}{BD} \Rightarrow \sec\alpha = \frac{13}{5}$$

Therefore, Statement 1 is false.

Statement 2 is false:

In right ΔADC, $AC = \sqrt{CD^2 + AD^2}$

$$= \sqrt{(16)^2 + (12)^2} = \sqrt{256 + 144}$$
$$= \sqrt{400} = 20 \text{ cm}$$

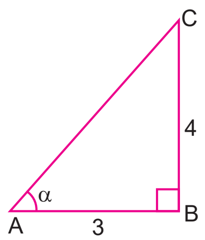
$$\therefore \operatorname{cosec}\beta = \frac{AC}{AD} \Rightarrow \operatorname{cosec}\beta = \frac{20}{12} \quad \text{or} \quad \operatorname{cosec}\beta = \frac{5}{3}.$$

Therefore, Statement 2 is false.

Hence, option (*d*) is the correct answer.

II: Short Answer Questions-1 (3 marks each)

4. Given, $\cot\alpha = \frac{3}{4}$



In right triangle ABC, $AC = \sqrt{AB^2 + BC^2}$

$$= \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\therefore \cos\alpha = \frac{AB}{AC} \Rightarrow \cos\alpha = \frac{3}{5}$$

Now, $$\sqrt{\frac{1 + \cos\alpha}{1 - \cos\alpha}} = \sqrt{\frac{1 + \frac{3}{5}}{1 - \frac{3}{5}}}$$

$$= \sqrt{\frac{\frac{8}{5}}{\frac{2}{5}}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2 \qquad \textbf{Proved.}$$

5. Given that sin(A + B) = sin A cos B + cos A sin B

Let A = 45° and B = 30°. Then,

$$\sin(A + B) = \sin(45° + 30°) = \sin 75° \qquad ...(1)$$

$$\sin A \cos B + \cos A \sin B = \sin 45° \cos 30° + \cos 45° \sin 30°$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} \qquad ...(2)$$

According to the question, left hand sides of (1) and (2) are equal.

$$\therefore \quad \sin 75° = \frac{\sqrt{3}+1}{2\sqrt{2}}.$$

6. We have,

$$\sin\theta = \cos\theta$$

$$\Rightarrow \quad \tan\theta = 1$$

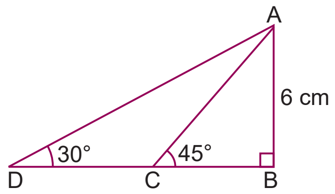
$$\Rightarrow \quad \theta = 45°$$

$$\therefore \quad 3\tan^2\theta + 2\sin^2\theta - 4 = 3\tan^2 45° + 2\sin^2 45° - 4$$

$$= 3(1)^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 - 4 = 3 + 1 - 4 = 0.$$

III: Short Answer Questions-2 (4 marks each)

7. In right triangle ABC,



$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow \quad 1 = \frac{6}{BC} \qquad [\because \tan 45° = 1]$$

$$\Rightarrow \quad BC = 6 \text{ cm}$$

Also, in right triangle ABD,

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{6}{BD}$$

$$\Rightarrow \quad BD = 6\sqrt{3} \text{ cm}$$

$$\therefore \quad CD = BD - BC$$

$$= (6\sqrt{3} - 6) \text{ cm} = 6(\sqrt{3} - 1) \text{ cm}.$$

8. We have, $\dfrac{12\cos^2 30° - 2\tan^2 60° + 4\sin^2 90°}{4\,\text{cosec}^2 45°}$

$$= \frac{12\left(\frac{\sqrt{3}}{2}\right)^2 - 2(\sqrt{3})^2 + 4(1)^2}{4(\sqrt{2})^2}$$

$$= \frac{12 \times \frac{3}{4} - 2 \times 3 + 4}{4 \times 2} = \frac{9 - 6 + 4}{8} = \frac{7}{8}.$$

9. (*i*) Given, $2 \sin \theta - 1 = 0$

$\Rightarrow \quad 2 \sin \theta = 1$

$\Rightarrow \quad \sin \theta = \frac{1}{2}$ $\quad \left[\because \sin 30^\circ = \frac{1}{2}\right]$

$\Rightarrow \quad \sin \theta = \sin 30^\circ$

$\Rightarrow \quad \theta = 30^\circ$ $\quad [\because \theta$ is an acute angle.]

Now, $3 \tan^2 \theta + 8 \cos 2\theta - 5 = 3 \tan^2 30^\circ + 8 \cos 2 \times 30^\circ - 5$ $\quad [\because \theta = 30^\circ]$

$$= 3\left(\frac{1}{\sqrt{3}}\right)^2 + 8 \cos 60^\circ - 5$$

$$= 1 + 8 \times \frac{1}{2} - 5 = 1 + 4 - 5 = 5 - 5 = 0.$$

(*ii*) The given formula is

$$\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

Substituting A = 30° on both sides, we get

$$\sin 2 \times 30^\circ = \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$$

$$\Rightarrow \quad \sin 60^\circ = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} \quad \left[\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right]$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2}$$

Hence, $\sin 60^\circ = \frac{\sqrt{3}}{2}$.

10. (*i*) We have, $\frac{\sin 77^\circ}{\cos 13^\circ} + \cos^2 42^\circ - \sin^2 48^\circ - 3 \cot^2 60^\circ$

$$= \frac{\sin (90^\circ - 13^\circ)}{\cos 13^\circ} + \cos^2 42^\circ - \sin^2 (90^\circ - 42^\circ) - 3\left(\frac{1}{\sqrt{3}}\right)^2 \quad \left[\because \cot 60^\circ = \frac{1}{\sqrt{3}}\right]$$

$$= \left(\frac{\cos 13^\circ}{\cos 13^\circ}\right) + \cos^2 42^\circ - \cos^2 42^\circ - 3 \times \frac{1}{3} \quad [\because \sin (90^\circ - \theta) = \cos \theta]$$

$$= 1 - 1 = 0.$$

(*ii*) $\left(\frac{\sin 59^\circ}{\cos 31^\circ}\right)^2 + \frac{\cos^2 23^\circ}{\sin^2 67^\circ} - \frac{1}{4} \sec^2 60^\circ$

$$= \left(\frac{\sin (90^\circ - 31^\circ)}{\cos 31^\circ}\right)^2 + \frac{\cos^2 23^\circ}{\sin^2 (90^\circ - 23^\circ)} - \frac{1}{4} (2)^2 \quad [\because \sec 60^\circ = 2]$$

$$= \left(\frac{\cos 31^\circ}{\cos 31^\circ}\right)^2 + \frac{\cos^2 23^\circ}{\cos^2 23^\circ} - \frac{1}{4} \times 4 \quad [\because \sin (90^\circ - \theta = \cos \theta]$$

$$= 1 + 1 - 1 = 1.$$

Unit VIII: Coordinate Geometry (Chapters 19–20)

I: Multiple Choice Questions (1 mark each)

1. (*d*) The point whose ordinate is 0 lies on the x-axis. Therefore the points Q (7, 0) and O(0, 0) are on the x-axis.

2. (*a*) The point whose abscissa is 0 lies on the y-axis. So, Statement (*i*) is the correct statement. Statement (*ii*) is false as point (3, 0) lies on the x-axis.

Statement (*iii*) is also false as abscissa of a point is positive in I and IV quadrants.

3. (*c*) Putting $x = 0$ in the equation $2x + 3y = 6$, we get

$$2 \times 0 + 3y = 6 \quad \Rightarrow \quad 3y = 6 \quad \Rightarrow \quad y = 2$$

$\therefore$ The graph of the given equation cuts the y-axis at (0, 2).

4. (*d*) If a pair of linear equations is consistent, then the lines will be either intersecting or coincident.

5. (*c*) By distance formula, we have

$$\sqrt{(4-1)^2 + (a-0)^2} = 5$$

$$\Rightarrow \quad \sqrt{(3)^2 + a^2} = 5$$

$$\Rightarrow \quad 9 + a^2 = 25 \qquad [\because \text{ Squaring both sides}]$$

$$\Rightarrow \quad a^2 = 25 - 9 = 16 \quad \Rightarrow \quad a = \pm 4$$

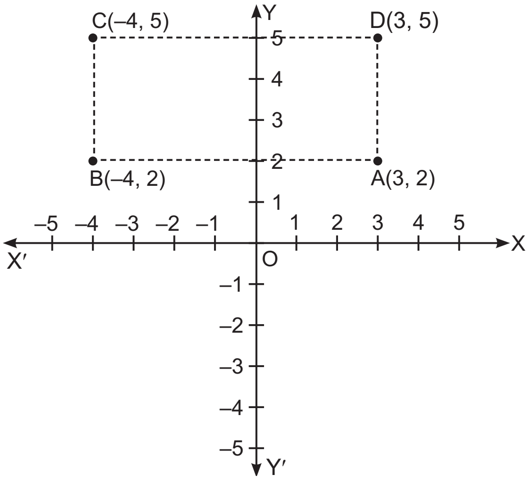
II: Short Answer Questions-1 (3 marks each)

6. Plot the three vertices of the rectangle as A(3, 2), B(–4, 2), C(–4, 5).

We have to find the coordinates of the fourth vertex D so that ABCD is a rectangle.

Since the opposite sides of a rectangle are equal, so the abscissa of D should be equal to abscissa of A, *i.e.*, 3 and the ordinate of D should be equal to the ordinate of C, *i.e.*, 5.

So, the coordinates of D are (3, 5).



7. The given equation is $2x + 5y = 19$

As the ordinate of the point is $1\frac{1}{2}$ times its abscissa, we have

$$y = 1\frac{1}{2}x \quad \text{or} \quad y = \frac{3}{2}x$$

Putting $y = \frac{3}{2}x$ in $2x + 5y = 19$, we get

$$2x + 5 \times \frac{3}{2}x = 19 \quad \Rightarrow \quad 2x + \frac{15}{2}x = 19$$

$$\Rightarrow \quad \frac{19}{2}x = 19 \quad \Rightarrow \quad x = 2$$

Therefore, $y = \frac{3}{2} \times 2 = 3$.

Thus, the required point is (2, 3).

8. Let A(0, 3), B(0, 1) and C($\sqrt{3}$, 2) be the vertices of triangle ABC. Then, by distance formula

$$AB = \sqrt{(0-0)^2+(1-3)^2} = \sqrt{(-2)^2} = \sqrt{4} = 2$$

$$BC = \sqrt{(\sqrt{3}-0)^2+(2-1)^2} = \sqrt{(\sqrt{3})^2+(1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$

and $$AC = \sqrt{(\sqrt{3}-0)^2+(2-3)^2} = \sqrt{(\sqrt{3})^2+(-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$

Since AB = BC = AC, the given points form an equilateral triangle.

III: Short Answer Questions-2 (4 marks each)

9. The given system of simultaneous linear equations is:

$$2x - y - 4 = 0 \quad ...(1)$$

$$x + y + 1 = 0 \quad ...(2)$$

To solve the above equations graphically, we find at least two solutions for each equation.

Table of solution set for eq. (1) is:

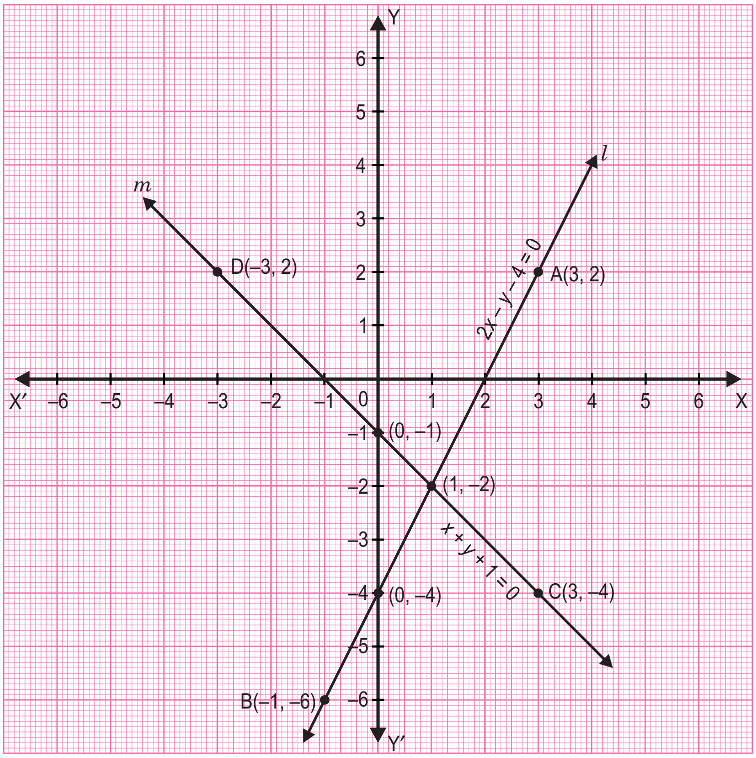
x	3	–1
y	2	–6

To draw the graph of eq. (1), we plot A(3, 2) and B(–1, –6) and join them by a straight line l.

Table of solution set for eq. (2) is:

x	3	–3
y	–4	2

To draw the graph of eq. (2), we plot C(3, –4) and D(–3, 2) and join them by a straight line m.



The two lines intersect each other at (1, –2).

Hence, $x = 1$, $y = -2$ is a solution of the given system of linear equations.

From the graph, we see that

the line $2x - y - 4 = 0$ meets y-axis at $(0, -4)$ and

the line $x + y + 1 = 0$ meets y-axis at $(0, -1)$.

10. The coordinates of the fielders—at *point* and *cover* are P(0, 5) and C(1, 2) respectively and the batsman at B(x, y) is equidistant from both the fielders.

So, we have CB = PB

Using the distance formula, $CB = \sqrt{(x-1)^2 + (y-2)^2}$

and $PB = \sqrt{(x-0)^2 + (y-5)^2}$

Now, CB = PB

$$\Rightarrow \sqrt{(x-1)^2 + (y-2)^2} = \sqrt{(x-0)^2 + (y-5)^2}$$

$$\Rightarrow (x-1)^2 + (y-2)^2 = (x-0)^2 + (y-5)^2$$

$$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 = x^2 + y^2 - 10y + 25$$

$$\Rightarrow -2x + 5 - 4y = -10y + 25$$

$$\Rightarrow 2x - 6y + 20 = 0$$

or $$x - 3y + 10 = 0$$

Thus, the required relation is $x - 3y + 10 = 0$.