Solutions of Question Paper Code: 30/4/1

Section A

Multiple Choice Questions

1. (*a*) [The graph of $y = f(x)$ cuts/meets x-axis at three points.
So, $y = f(x)$ has three zeros.]

2. (*d*) [Let there be n terms in A.P. Then,

$$a = 5 \text{ and } a_n = 45$$

Also, $$S = \frac{n}{2}[a + a_n] = 400$$

$\Rightarrow$ $$\frac{n}{2}[5 + 45] = 400$$

$\Rightarrow$ $$n = 16$$

Thus, the given A.P. has 16 terms.]

OR

(*c*) [Here, $a = -15$, $d = 4$
So, $a_9 = a + 8d$
$= -15 + 32 = 17.$]

3. (*b*) [Since A, B and C are collinear,

$$1(0 - b) + 0(b - 2) + a(2 - 0) = 0$$

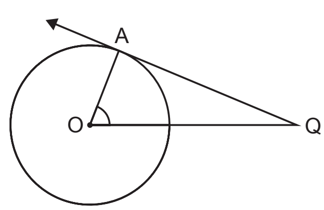
$\Rightarrow$ $$-b + 2a = 0$$

or $$2a = b.$$]

4. (*d*) [PTQO is a cyclic quadrilateral.
So, $\angle PTQ = 180° - \angle POQ$
$= 180° - 115°$
$= 65°.$]

OR

(*c*) [Here, QA = 5 cm, OQ = 8 cm
In right ΔOAQ, we have



$$OA^2 = OQ^2 - QA^2$$
$$= 8^2 - 5^2$$
$$= 64 - 25 = 39$$

$\therefore$ Radius of the circle = OA = $\sqrt{39}$ cm.]

5. (*a*) [$\cos(10° + \theta) = \sin 30°$ is equivalent to $\cos(10° + \theta) = \cos 60°$.
$\Rightarrow$ $10° + \theta = 60°$
$\Rightarrow$ $\theta = 50°.$]

6. (*d*) [Of the (3 + 5 + 7 =) 15 balls in the bag, 10 balls are not black.
So, required probability is $\frac{10}{15}$ or $\frac{2}{3}$.]

7. (*b*) [Since $y = 0$ and $y = -6$ represent parallel lines, the pair of equations has no solution.]

8. **(*b*)** The empirical formula.

$$\begin{aligned} 3\text{ Median} &= \text{Mode} + 2\text{ Mean} \\ \Rightarrow \quad \text{Mode} &= 3\text{ Median} - 2\text{ Mean} \\ &= 3(15) - 2(14) \\ &= 45 - 28 \\ &= 17. \end{aligned}$$

9. **(*d*)** Equation $x^2 - 4x + k = 0$ will have distinct real roots if

$D = (-4)^2 - 4(k)(1) > 0$

i.e., $16 - 4k > 0$

or $k < 4$.

10. **(*a*)** Mid-point of AB is $P\left(\frac{-5+4}{2}, \frac{2+6}{2}\right)$, *i.e.*, $P\left(-\frac{1}{2}, 4\right)$

Equating it with $P\left(\frac{a}{8}, 4\right)$, we have

$$\frac{a}{8} = -\frac{1}{2}$$

$\Rightarrow$ $a = -4$.

Fill in the blanks.

11. irrational — As $\sqrt{5}$ is irrational, $2+\sqrt{5}$ is irrational.

$\Rightarrow$ $\frac{2+\sqrt{5}}{3}$ is irrational.

12. 18 — Since $\Delta ABC \sim \Delta DEF$,

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \quad \frac{81}{144} = \frac{BC^2}{24^2}$$

$$\Rightarrow \quad BC^2 = \frac{81}{144} \times 24^2 = 324$$

$\Rightarrow$ BC = 18 cm.

13. $2\sqrt{a^2+b^2}$ — Distance between (a, b) and $(-a, -b)$ is

$$= \sqrt{(a+a)^2 + (b+b)^2} = \sqrt{(2a)^2 + (2b)^2} = 2\sqrt{a^2+b^2}.$$

14. 1 — Since $\tan A = 1$, $A = 45°$

$$\begin{aligned} \therefore \quad 2\sin A\cos A &= 2 \times \sin 45° \times \cos 45° \\ &= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \\ &= 1. \end{aligned}$$

15. 4 — Let r cm be the radius of the smaller ball. Then,

$$8 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(8)^3$$

$\Rightarrow$ $r^3 = 64$

$\Rightarrow$ $r = 4$.

Very Short Answer Questions

16. We know that

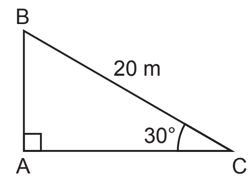
$$\text{LCM}(135, 225) = \frac{\text{Product of 135 and 225}}{\text{HCF}(135, 225)}$$

$$= \frac{135 \times 225}{45}$$

$$= 675$$

So, LCM(135, 225) = 675.

17. Here, in the figure, AB represents the pole.



In right ΔBAC, $\frac{AB}{CB} = \sin 30°$

$\Rightarrow \quad \frac{AB}{20} = \frac{1}{2}$

$\Rightarrow \quad AB = 10$ m

Thus, the height of the pole is 10 m.

18. No. of possible outcomes = 36

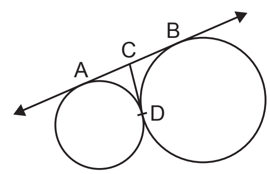
No. of favourable outcomes is 0. [Because, sum of two numbers may at most be 12.]

So, required probability is $\frac{0}{36}$, *i.e.*, 0.

19. Here, $\frac{229}{2^2 \times 5^7} = \frac{229 \times 2^5}{2^7 \times 5^7} = \frac{229 \times 2^5}{(10)^7}$

Hence, the given rational number will terminate after 7 decimal places.

20. From the given figure, CA = CD and CD = CB



$\Rightarrow \quad 2CD = CA + CB$

$\Rightarrow \quad 2CD = AB = 8$

$\Rightarrow \quad CD = 4$ cm.

Section B

21. $6x^2 + 11x + 3 = 0$

$\Rightarrow \quad 6x^2 + 9x + 2x + 3 = 0$

$\Rightarrow \quad 3x(2x + 3) + 1(2x + 3) = 0$

$\Rightarrow \quad (2x + 3)(3x + 1) = 0$

$\Rightarrow \quad 2x + 3 = 0 \quad \text{or} \quad 3x + 1 = 0$

i.e., $\quad x = -\frac{3}{2} \quad \text{or} \quad x = -\frac{1}{3}$.

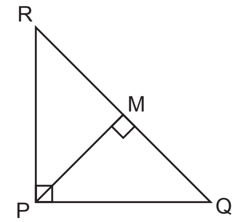
22. We know that if $\Delta ABC \sim \Delta PQR$, then

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR}$$

$\Rightarrow \quad \frac{9}{PQ} = \frac{30}{20}$

$\Rightarrow \quad PQ = 6$ cm.

OR



Here, $\Delta PMQ \sim \Delta RPQ$

$$\Rightarrow \quad \frac{PQ}{MQ} = \frac{RQ}{PQ}$$

$$\text{or} \quad \frac{PQ}{QM} = \frac{QR}{PQ}$$

$$\Rightarrow \quad PQ^2 = QM \cdot QR.$$

23. $\left(\frac{\sin 47°}{\cos 43°}\right)^2 + \left(\frac{\cos 30°}{\cot 30°}\right)^2 - (\sin 60°)^2$

$$= \left[\frac{\sin(90° - 43°)}{\cos 43°}\right]^2 + \left(\frac{\frac{\sqrt{3}}{2}}{\sqrt{3}}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \left(\frac{\cos 43°}{\cos 43°}\right)^2 + \frac{1}{4} - \frac{3}{4}$$

$$= 1 + \frac{1}{4} - \frac{3}{4}$$

$$= \frac{1}{2}.$$

24. Here, the modal class is 60–80.

For this class, $l = 60$, $h = 20$, $f_1 = 16$, $f_0 = 12$ and $f_2 = 4$.

So, $$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 60 + \frac{16 - 12}{32 - 12 - 4} \times 20$$

$$= 60 + \frac{80}{16} = 65.$$

OR

Classes	500–600	600–700	700–800	800–900	900–1000
Frequency	36	32	32	20	30
Cum. Frequency	36	68	100	120	150

Here, $$\frac{N}{2} = 75$$

So, the median class is 700–800

For this class, $l = 700$, $h = 100$, $\frac{N}{2} = 75$, $c = 68$, and $f = 32$.

So, $$\text{Median} = l + \frac{\frac{N}{2} - c}{f} \times h$$

$$= 700 + \frac{75 - 68}{32} \times 100$$

$$= 700 + \frac{700}{32}$$

$$= 721.875.$$

25. Area of the canvas used in making the tent

= Curved surface area of the cylindrical part + Curved surface area of the conical part

$= [2\pi(2)(2.1) + \pi(2)(2.8)]$ sq m

$= (26.4 + 17.6)$ sq m

= 44 sq m.

26. (*i*) Of the 600 plants, there are 260 fruit plants or flowering plants.

So, required probability $= \frac{260}{600}$, *i.e.*, $\frac{26}{60}$.

(*ii*) Of the 600 plants, there are 290 plants which are either Neem plants or Peepal plants.

So, required probability $= \frac{290}{600}$, *i.e.*, $\frac{29}{60}$.

Section C

27. Refer to solution of Q. No. 35 of Set 30/2/1.

28. Let a be the first term and d, the common difference of the A.P.

Then, $S_{30} = \frac{30}{2}[2a + 29d] = 1920$ or $2a + 29d = 128$ ...(1)

Also, $a_4 = a + 3d = 18$ ...(2)

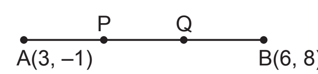
Solving Eqs. (1) and (2), we get

$d = 4$ and $a = 6$

Thus, $a_{11} = a + 10d = 6 + 40 = 46$

Hence, 11th term of the A.P. is 46.

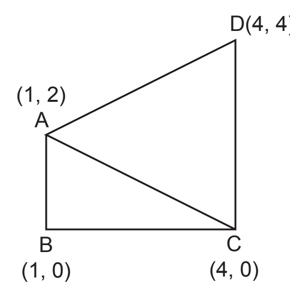
29. Let the given points be A and B and P and Q be the points of trisection of AB, as shown in the figure. Here, P divides AB in the ratio 1 : 2 and Q divides AB in the ratio 2 : 1.



So, $P\left(\frac{6+6}{3}, \frac{8-2}{3}\right)$ and $Q\left(\frac{12+3}{3}, \frac{16-1}{3}\right)$

i.e., P(4, 2) and Q(5, 5).

OR



Area of quad. ABCD = Area of ΔABC + Area of ΔACD

$$= \left|\frac{1}{2}[(1)(0-0) + 1(0-2) + 4(2-0)]\right|$$

$$+ \left|\frac{1}{2}[(1)(0-4) + 4(4-2) + 4(2-0)]\right|$$

$= (3 + 6)$

= 9 sq units.

Alternatively

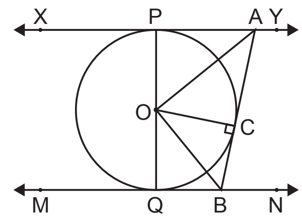
Since BA || CD, ABCD is a trapezium.

So, its area $= \frac{1}{2} \times (BA + CD) \times BC$

$= \frac{1}{2} \times (2 + 4) \times 3$

= 9 sq units.

30. Join OC. In ΔAPO and ΔACO, we have



$$AP = AC,\ OP = OC,\ OA = OA$$

So, by SSS Congruence Criterion,

$$\Delta APO \cong \Delta ACO$$

$\Rightarrow$ $\angle PAO = \angle CAO$

$\Rightarrow$ $\angle PAC = 2\angle OAC$ ...(1)

Similarly, $\angle QBC = 2\angle OBC$ ...(2)

Adding (1) and (2), we have

$$\angle PAC + \angle QBC = 2(\angle OAC + \angle OBC)$$

$\Rightarrow$ $180° = 2(\angle OAC + \angle OBC)$

$\Rightarrow$ $\angle OAC + \angle OBC = 90°$ ...(3)

Now, in ΔAOB, we have

$$\angle AOB = 180° - (\angle OAC + \angle OBC)$$

$= 180° - 90°$ [From (3)]

$= 90°$

Thus, proved.

31. Given equations are

$$\frac{2}{x} + \frac{3}{y} = 11 \quad ...(1)$$

and $\frac{5}{x} - \frac{4}{y} = -7$ ...(2)

Eq. (1) × 5 and Eq. (2) × 2 give

$$\frac{10}{x} + \frac{15}{y} = 55 \quad ...(3)$$

and $\frac{10}{x} - \frac{8}{y} = -14$ ...(4)

Subtracting Eq. (4) from Eq. (3), we have

$$\frac{23}{y} = 69$$

$\Rightarrow$ $y = \frac{1}{3}$

Substituting $y = \frac{1}{3}$ in Eq. (1), we have

$$\frac{2}{x} + 9 = 11$$

i.e., $\frac{2}{x} = 2$

or $x = 1$

Thus, $x = 1$, $y = \frac{1}{3}$ is the required solution.

Hence, $5x - 3y = 5(1) - 3\left(\frac{1}{3}\right) = 5 - 1 = 4.$

OR

Let the fixed charge be ₹ x and charges per km be ₹ y.

Then, $x + 10y = 75$ and $x + 15y = 110$

Solving the two equations, we get

$$x = 5, y = 7$$

Thus, the fixed charge is ₹ 5 and the charge per km is ₹ 7.

Hence, charge for 35 km is ₹ [5 + 35(7)], *i.e.*, ₹ 250.

32. Here, L.H.S. $= \dfrac{\sin\theta - \cos\theta + 1}{\cos\theta + \sin\theta - 1}$

$$= \frac{\tan\theta - 1 + \sec\theta}{1 + \tan\theta - \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{1 + \tan\theta - \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta)(1 - \sec\theta + \tan\theta)}{1 + \tan\theta - \sec\theta}$$

$$= \tan\theta + \sec\theta$$

$$= (\tan\theta + \sec\theta) \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta}$$

$$= \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta}$$

$$= \frac{1}{\sec\theta - \tan\theta} = \text{R.H.S.}$$

33. Area of the shaded region

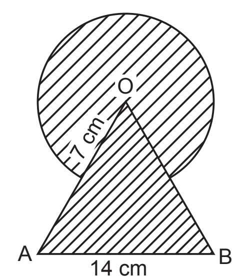
= Area of the triangle OAB + Area of the circle, centred at O

– Area of the sector of the central angle 60° and radius 7 cm

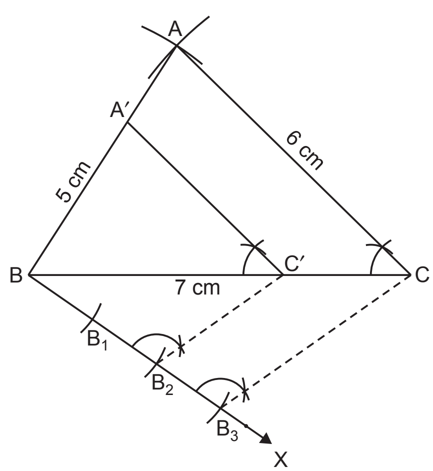
$$= \left[\sqrt{3}(7)^2 + \pi(7)^2 - \frac{60}{360} \times \pi \times (7)^2\right] \text{ sq cm}$$

= (84.77 + 154 – 25.67) sq cm

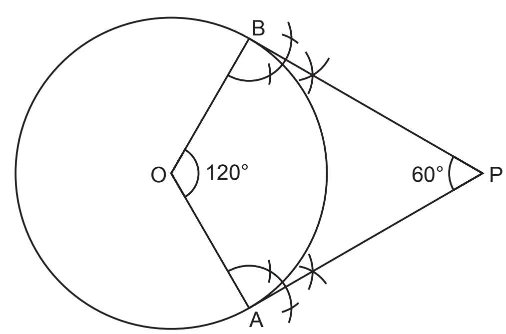
= 213.10 sq cm.



34.



OR



Δ A′BC′ is the required triangle.

Section D

35. Let the speed of the aircraft originally be x km/h.

Time taken for a distance of 600 km is $\frac{600}{x}$ hours.

Time taken for same distance with speed $(x - 200)$ km/h is $\frac{600}{x-200}$ hours.

As per the question,

$$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2} \qquad \left[\because \text{ 30 minutes} = \frac{1}{2}\text{hour}\right]$$

$$\Rightarrow \quad 600\left[\frac{x - x + 200}{x(x-200)}\right] = \frac{1}{2}$$

$$\Rightarrow \quad x(x - 200) = 240000$$

$$\Rightarrow \quad x^2 - 200x - 240000 = 0$$

$$\Rightarrow \quad (x - 600)(x + 400) = 0$$

$$\Rightarrow \quad x - 600 = 0 \qquad [\because x + 400 \neq 0.]$$

$$\Rightarrow \quad x = 600$$

Thus, the speed of the aircraft originally was 600 km/h.

OR

Let the original number of persons be 'x'. Then, each person gets ₹$\left(\frac{9000}{x}\right)$.

When the number of persons is '$x + 20$', then, each person gets ₹ $\left(\frac{9000}{x+20}\right)$.

As per the question,

$$\frac{9000}{x} - \frac{9000}{x+20} = 160$$

$$\Rightarrow \quad 9000\left[\frac{x+20-x}{x(x+20)}\right] = 160$$

$$\Rightarrow \quad x(x + 20) - 1125 = 0$$

$$\Rightarrow \quad x^2 + 20x - 1125 = 0$$

$$\Rightarrow \quad (x + 45)(x - 25) = 0$$

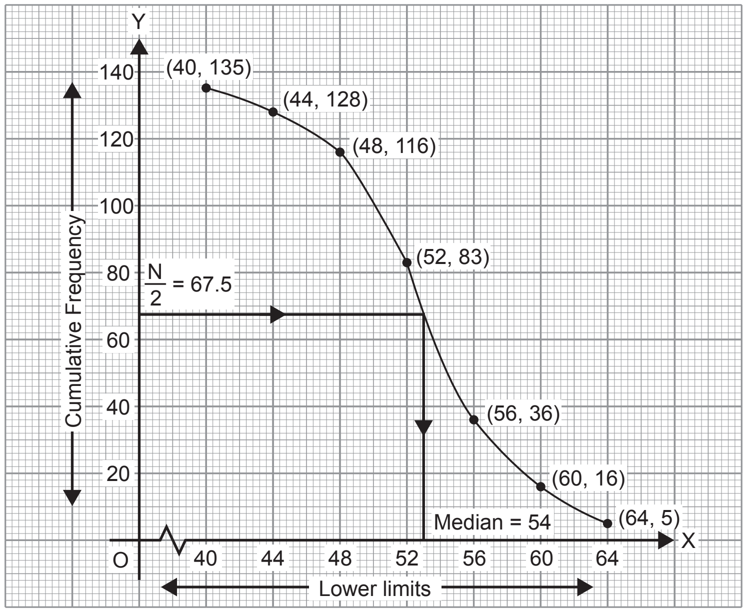
$$\Rightarrow \quad x - 25 = 0 \qquad [\because x + 45 \neq 0.]$$

$$\Rightarrow \quad x = 25$$

Thus, originally the number of persons was 25.

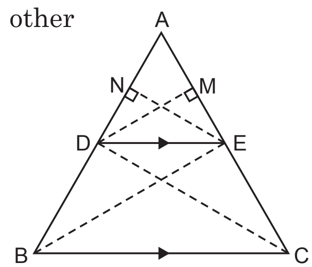
36. 'More than' type cumulative frequency distribution is

Weight (in kg)	*Cumulative Frequency*
More than or equal to 40	135
More than or equal to 44	128
More than or equal to 48	116
More than or equal to 52	83
More than or equal to 56	36
More than or equal to 60	16
More than or equal to 64	5



From the above cumulative frequency curve, median weight of the students is 54 kg.

37. We are given a triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.



We need to prove that $\frac{AD}{DB} = \frac{AE}{EC}$.

Let us join BE and CD and then draw DM ⊥ AC and EN ⊥ AB.

Now, $ar(\Delta ADE) = \frac{1}{2} \times AD \times EN$

and $ar(\Delta BDE) = \frac{1}{2} \times DB \times EN$

Also, $ar(\Delta ADE) = \frac{1}{2} \times AE \times DM$

and $ar(\Delta DEC) = \frac{1}{2} \times EC \times DM$

Therefore, $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{DB}$ and $\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{AE}{EC}$ ...(1)

Further, ΔBDE and ΔDEC are on the same base DE and between the same parallels BC and DE. So, $ar(\Delta BDE) = ar(\Delta DEC)$.

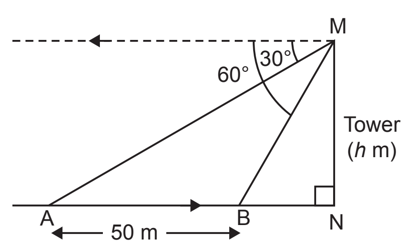
Thus, from (1), we have

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

OR

Refer to solution of Q. No. 32 of Set 30/3/1.

38. Let 'h' metres be the height of the tower. A and B are the two positions of the car, where AB = 50 m.



In right ΔANM, we have

$$\frac{NM}{AN} = \tan 30° = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $$AN = \sqrt{3}\,NM \quad ...(1)$$

In right ΔBNM, we have

$$\frac{\text{NM}}{\text{BN}} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \qquad \text{BN} = \frac{\text{NM}}{\sqrt{3}} \qquad \text{...(2)}$$

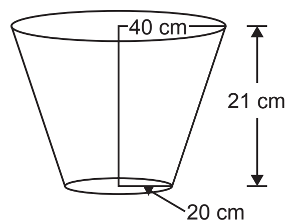
From (1) and (2), we have

$$\begin{aligned} \text{AB} &= \text{AN} - \text{BN} \\ &= \sqrt{3}\,\text{NM} - \frac{\text{NM}}{\sqrt{3}} \\ &= \text{NM}\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) \\ &= \frac{2}{\sqrt{3}}\,\text{NM} \end{aligned}$$

$$\Rightarrow \qquad \frac{2}{\sqrt{3}}\,\text{NM} = 50$$

$$\Rightarrow \qquad \text{NM} = 25\sqrt{3} \text{ metres}$$

Thus, the height of the tower is $25\sqrt{3}$ metres.



39. Volume of the bucket $= \frac{1}{3}\pi \times 21(20^2 + 40^2 + 20 \times 40)$

$= 22(400 + 1600 + 800)$

$= 22 \times 2800 = 61600$ cu cm

Area of the sheet required to make the bucket

= Area of the base + Curved surface area

$= \pi(20)^2 + \pi(40 + 20)\sqrt{21^2 + (40 - 20)^2}$

$= 400\pi + 1740\pi$

$= 2140\pi$ sq cm or 6725.8 sq cm (approx).

40. Since $\sqrt{3}$ and $-\sqrt{3}$ are zeros of $f(x)$, $(x - \sqrt{3})(x + \sqrt{3})$, *i.e.*, $(x^2 - 3)$ is a factor of $f(x)$.

To obtain other two zeros, we shall determine the quotient by dividing $f(x)$ with $(x^2 - 3)$.

$$\begin{array}{r|l} & 2x^2 + 3x + 1 \\ \hline x^2 - 3 & 2x^4 + 3x^3 - 5x^2 - 9x - 3 \\ & 2x^4 \qquad\quad - 6x^2 \\ & - \qquad\qquad + \\ \hline & \qquad 3x^3 + x^2 - 9x - 3 \\ & \qquad 3x^3 \qquad - 9x \\ & \qquad - \qquad\quad + \\ \hline & \qquad\qquad x^2 \qquad - 3 \\ & \qquad\qquad x^2 \qquad - 3 \\ & \qquad\qquad - \qquad\ + \\ \hline & \qquad\qquad\qquad\quad 0 \\ \hline \end{array}$$

Here, quotient $= 2x^2 + 3x + 1$

$= (2x + 1)(x + 1)$

So, the other two zeros are -1 and $-\frac{1}{2}$.

OR

Let α and β be the zeros of $5x^2 + 2x - 3$.

Then,

$$\alpha + \beta = -\frac{2}{5} \quad \text{and} \quad \alpha\beta = -\frac{3}{5}.$$

Now, $$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-\frac{2}{5}}{-\frac{3}{5}} = \frac{2}{3}$$

and $$\frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = -\frac{5}{3}$$

Thus, a quadratic polynomial whose zeros are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is

$$x^2 - \frac{2}{3}x - \frac{5}{3}, \ i.e., \ 3x^2 - 2x - 5.$$