CBSE Sample Question Paper

MATHEMATICS
CLASS X (2017–18)

Time: 3 Hours | ***Max. Marks: 80***

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of 30 questions divided into four sections A, B, C and D.*
3. *Section A contains 6 questions of 1 mark each, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 8 questions of 4 marks each.*
4. *There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculators is not permitted.*

Section A

Question numbers 1 to 6 carry 1 mark each.

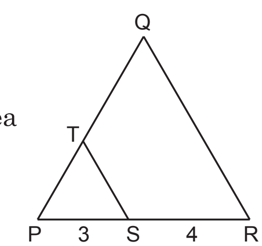
1. Write whether the rational number $\frac{7}{75}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

2. Find the value(s) of k, if the quadratic equation $3x^2 - k\sqrt{3}x + 4 = 0$ has equal roots.

3. Find the eleventh term from the last term of the A.P.: 27, 23, 19, ..., –65.

4. Find the coordinates of the point on y-axis which is nearest to the point (–2, 5).

5. In given figure, ST || RQ, PS = 3 cm and SR = 4 cm. Find the ratio of the area of ΔPST to the area of ΔPRQ.



6. If $\cos A = \frac{2}{5}$, find the value of $4 + 4\tan^2 A$.

Section B

Question numbers 7 to 12 carry 2 marks each.

7. If two positive integers p and q are written as $p = a^2b^3$ and $q = a^3b$; a, b are prime numbers, then verify:

$$\text{LCM}(p, q) \times \text{HCF}(p, q) = pq$$

8. The sum of first n terms of an A.P. is given by $S_n = 2n^2 + 3n$. Find the sixteenth term of the A.P.

9. Find the value(s) of k for which the pair of linear equations $kx + y = k^2$ and $x + ky = 1$ has infinitely many solutions.

10. If $\left(1, \frac{p}{3}\right)$ is the mid-point of the line segment joining the points (2, 0) and $\left(0, \frac{2}{9}\right)$, then show that the line $5x + 3y + 2 = 0$ passes through the point $(-1, 3p)$.

11. A box contains cards numbered 11 to 123. A card is drawn at random from the box. Find the probability that the number on the drawn card is

(*i*) a square number. (*ii*) a multiple of 7.

12. A box contains 12 balls of which some are red in colour. If 6 more red balls are put in the box and a ball is drawn at random, the probability of drawing a red ball doubles than what it was before. Find the number of red balls in the box.

Section C

Question numbers 13 to 22 carry 3 marks each.

13. Show that exactly one of the numbers n, $n + 2$ or $n + 4$ is divisible by 3.

14. Find all the zeros of the polynomial $3x^4 + 6x^3 - 2x^2 - 10x - 5$ if two of its zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

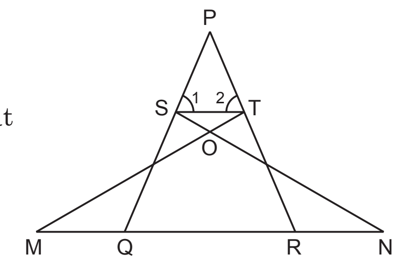
15. Seven times a two-digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3, determine the number.

16. In what ratio does the x-axis divide the line segment joining the points (–4, –6) and (–1, 7)? Find the coordinates of the point of division.

OR

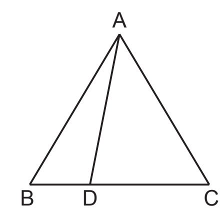
The points A(4, –2), B(7, 2), C(0, 9) and D(–3, 5) form a parallelogram. Find the length of the altitude of the parallelogram on the base AB.

17. In the given figure, $\angle 1 = \angle 2$ and $\Delta NSQ \cong \Delta MTR$, then prove that $\Delta PTS \sim \Delta PRQ$.

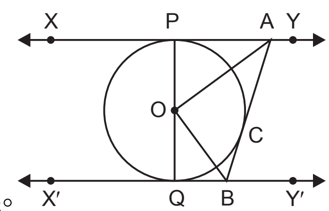


OR

In an equilateral triangle ABC, D is a point on the side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.



18. In the given figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that $\angle AOB = 90°$.

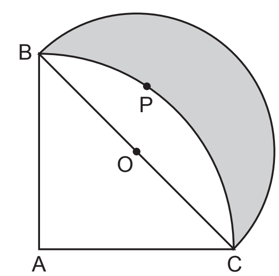


19. Evaluate: $\dfrac{\text{cosec}^2 63° + \tan^2 24°}{\cot^2 66° + \sec^2 27°} + \dfrac{\sin^2 63° + \cos 63° \sin 27° + \sin 27° \sec 63°}{2(\text{cosec}^2 65° - \tan^2 25°)}$

OR

If $\sin\theta + \cos\theta = \sqrt{2}$, then evaluate: $\tan\theta + \cot\theta$

20. In given figure, ABPC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



21. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

OR

A cone of maximum size is carved out from a cube of edge 14 cm. Find the surface area of the remaining solid after the cone is carved out.

22. Find the mode of the following distribution of marks obtained by the students in a examination:

Mark Obtained	0–20	20–40	40–60	60–80	80–100
Number of Students	15	18	21	29	17

Given the mean of the above distribution is 53, using empirical relationship estimate the value of its median.

Section D

Question numbers 23 to 30 carry 4 marks each.

23. A train travelling at a uniform speed for 360 km would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.

OR

Check whether the equation $5x^2 - 6x - 2 = 0$ has real roots and if it has, find them by the method of completing the square. Also verify that the roots obtained satisfy the given equation.

24. An A.P. consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three terms is 429. Find the A.P.

25. Show that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

OR

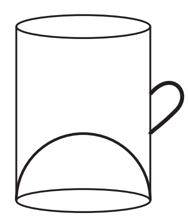
Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

26. Draw a triangle ABC with side BC = 7 cm, $\angle B = 45°$, $\angle A = 105°$. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC.

27. Prove that $\dfrac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \text{cosec}\,\theta + \cot\theta$.

28. The angles of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30° and 60°, respectively. Find the height of the tower and also the horizontal distance between the building and the tower.

29. Two dairy owners A and B sell flavoured milk filled to capacity in mugs of negligible thickness, which are cylindrical in shape with a raised hemispherical bottom. The mugs are 14 cm high and have diameter of 7 cm as shown in given figure. Both A and B sell flavoured milk at the rate of ₹80 per litre. The dairy owner A uses the formula $\pi r^2 h$ to find the volume of milk in the mug and charges ₹43.12 for it. The dairy owner B is of the view that the price of actual quantity of milk should be charged. What according to him should be the price of one mug of milk? Which value is exhibited by the dairy owner B? $\left(\text{use } \pi = \frac{22}{7}\right)$



30. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹18. Find the missing frequency k.

Daily Pocket Allowance (in ₹)	11–13	13–15	15–17	17–19	19–21	21–23	23–25
Number of Children	3	6	9	13	k	5	4

OR

The following frequency distribution shows the distance (in metres) thrown by 68 students in a Javelin throw competition.

Distance (in m)	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Number of Students	4	5	13	20	14	8	4

Draw a less than type ogive for the given data and find the median distance thrown using this curve.

SOLUTIONS TO CBSE SAMPLE QUESTION PAPER

Section A

1. We have, $\frac{7}{75} = \frac{7}{3 \times 5^2}$

Since prime factorisation of denominator, *i.e.*, 75 is other than $2^m \times 5^n$, the rational number $\frac{7}{75}$ will have a non-terminating repeating decimal expansion.

2. The given quadratic equation is

$$3x^2 - k\sqrt{3}x + 4 = 0 \qquad ...(1)$$

Here, $a = 3,\ b = -k\sqrt{3}$ and $c = 4$.

For equal rests, $b^2 - 4ac = 0$

$\therefore \quad (-k\sqrt{3})^2 - 4 \times 3 \times 4 = 0$

$\Rightarrow \quad 3k^2 = 48 \quad \Rightarrow \quad k^2 = 16$

or $\quad k = \pm 4$.

3. The given A.P. is: 27, 23, 19, ..., –65. ...(1)

Let us rewrite this A.P. in reverse order: –65, ..., 19, 23, 27 ...(2)

The 11th term from the last term of the A.P. is same as the 11th term from the beginning of A.P., written in form (2).

Here, $a = -65,\ d = 27 - 23 = 4$

$\therefore$ 11th term from the last term of A.P. (1) = 11th term of the A.P. (2)

$= -65 + (11 - 1)(4)$

$= -65 + 40$

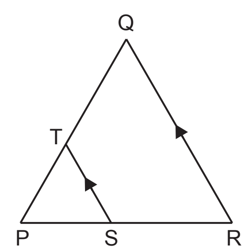
$= -25$.

4. Since the required point is on y-axis, its abscissa = 0

Also, it is nearest to the point (–2, 5), so its ordinate = 5

$\therefore$ Coordinates of the required point are (0, 5).

5. In the given figure,



ST ∥ RQ [Given]

$\therefore \quad \angle PST = \angle PRQ$ [Corresponding angles]

and $\quad \angle PTS = \angle PQR$ [Corresponding angles]

So, by ΔA similarity criterion, $\Delta PST \sim \Delta PRQ$.

$$\therefore \quad \frac{\text{Area of } \Delta PST}{\text{Area of } \Delta PRQ} = \frac{PS^2}{PR^2}$$

$$= \frac{PS^2}{(PS + SR)^2} = \frac{(3)^2}{(3 + 4)^2} = \frac{9}{49}$$

Hence, $\quad ar(\Delta PST) : ar(\Delta PRQ) = 9 : 49$.

6. We have, $\quad \cos A = \frac{2}{5}$

$\Rightarrow \quad \cos^2 A = \frac{4}{25}$

$\Rightarrow \quad 1 - \sin^2 A = \frac{4}{25}$

$\Rightarrow \quad \sin^2 A = 1 - \frac{4}{25} = \frac{21}{25}$

$$\therefore \quad 4 + 4\tan^2 A = 4 + 4\frac{\sin^2 A}{\cos^2 A}$$

$$= 4 + 4 \times \frac{21}{25} \times \frac{25}{4}$$

$$= 4 + 4 \times \frac{21}{4} = 4 + 21 = 25.$$

Section B

7. For the positive integers p and q, we have

$$p = a^2b^3 \text{ and } q = a^3b$$

Since a and b are prime numbers, therefore

$$\text{LCM of } p \text{ and } q = a^3b^3$$

and $$\text{HCF of } p \text{ and } q = a^2b$$

Now, LCM $(p, q) \times$ HCF $(p, q) = a^3b^3 \times a^2b$

$$= a^2b^3 \times a^3b = pq.$$

8. For the given A.P., $S_n = 2n^2 + 3n$ ...(1)

Putting $n = 1$, in (1), we have

$$S_1 = 2(1)^2 + 3(1) = 2 + 3 = 5$$

$\therefore$ First term, $a_1 = 5$

Putting $n = 2$, in (1), we have

$$S_2 = 2(2)^2 + 3(2) = 8 + 6 = 14$$

$$\therefore \quad a_2 = S_2 - S_1 = 14 - 5 = 9$$

So, common difference, $d = a_2 - a_1 = 9 - 5 = 4$

Therefore, $a_{16} = a + 15d$ $[\because \; a_n = a + (n - 1)d]$

$$= 5 + 15 \times 4 = 65$$

Hence, 16th term of the given A.P. is 65.

9. The pair of given linear equations is:

$$\left.\begin{aligned} kx + y &= k^2 \\ \text{and} \quad x + ky &= 1 \end{aligned}\right\} \quad ...(1)$$

The given pair (1) will have infinitely many solutions, if

$$\therefore \quad \frac{k}{1} = \frac{1}{k} = \frac{k^2}{1}$$

Now, $\frac{k}{1} = \frac{1}{k} \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$ ...(1)

and $\frac{1}{k} = \frac{k^2}{1} \Rightarrow k^3 = 1 \Rightarrow k = 1$ ...(2)

From (1) and (2), we have $k = 1$.

10. Since $(1, \frac{p}{3})$ is the mid-point of the line segment joining (2, 0) and $(0, \frac{2}{9})$, therefore

$$\left(1, \frac{p}{3}\right) = \left(\frac{2+0}{2}, \frac{0+\frac{2}{9}}{2}\right)$$

$$\Rightarrow \quad \left(1, \frac{p}{3}\right) = \left(1, \frac{1}{9}\right)$$

$$\Rightarrow \quad \frac{p}{3} = \frac{1}{9} \quad \text{or} \quad p = \frac{1}{3}.$$

The equation of given line is

$$5x + 3y + 2 = 0 \qquad ...(1)$$

and the given point is $(-1, 3p)$, *i.e.*, $(-1, 1)$.

Putting $x = -1$ and $y = 1$ in (1), we have

$$5(-1) + 3(1) + 2 = 0$$

Since the point $(-1, 1)$ satisfies (1), therefore the line $5x + 3y + 2 = 0$ passes through the point $(-1, 3p)$.

11. Total number of cards in the box = 123 – 10 = 113

So, total number of possible outcomes = 113.

(*i*) Number of cards having square numbers = 8

[16, 25, 36, 49, 64, 81, 100 and 121 are square number between 11 and 123.]

So, number of favourable outcomes = 8

$$\therefore \quad \text{Required probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{8}{113}.$$

(*ii*) Number of cards having multiples of 7 = 16

[14, 21, 28, ..., 119, *i.e.*, 16 multiples of 7 are between 11 and 123.]

So, number of favourable outcomes = 16

$$\therefore \quad \text{Required probability} = \frac{\text{Number of faourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{16}{113}.$$

12. Let x be the number of red balls in the box.

Given, total number of balls in the box = 12, and

Number of red balls in the box = x

$$\therefore \quad \text{Prob. (a red ball)} = \frac{x}{12}$$

Now, putting 6 more red balls in the box, we have

Total number of balls in the box = 12 + 6 = 18

Also, Number of red balls in the box = $x + 6$

$$\therefore \quad \text{Prob. (a red ball)} = \frac{x+6}{18}$$

According to the given condition,

$$\frac{x+6}{18} = 2\left(\frac{x}{12}\right)$$

$$\Rightarrow \quad \frac{x+6}{18} = \frac{x}{6}$$

$$\Rightarrow \quad \frac{x+6}{3} = x \quad \Rightarrow \quad x + 6 = 3x$$

$$\Rightarrow \quad 2x = 6 \quad \text{or} \quad x = 3$$

Hence, there are 3 red balls in the box.

Section C

13. Let a be any positive integer and $b = 3$. Then, by Theorem 1.1, we have

$$a = 3q + r,$$

where $q \geq 0$ and $0 \leq r < 3$.

$$\Rightarrow \quad r = 0 \text{ or } 1 \text{ or } 2$$

$$\Rightarrow \quad a = 3q, \quad 3q + 1 \quad \text{or} \quad 3q + 2$$

If $n = 3q$, then n is divisible by 3.

Clearly then $n + 2$ and $n + 4$ are not divisible by 3.

If $n = 3q + 1$, then $\quad n + 2 = 3q + 1 + 2 = 3q + 3$

$$= 3(q + 1) \text{ is divisible by 3.}$$

Clearly then n and $n + 4$ are not divisible by 3

If $n = 3q + 2$, then $\quad n + 4 = 3q + 2 + 4 = 3q + 6$

$$= 3(q + 2) \text{ is divisible by 3}$$

Clearly then n and $n + 2$ are not divisible by 3.

14. Since two zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$, $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ is a factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$.

Now, we apply the Division Algorithm to the given polynomials.

$$\begin{array}{r|l} & 3x^2 + 6x + 3 \\ \hline x^2 - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\ & 3x^4 \qquad\quad - 5x^2 \\ & - \qquad\qquad + \\ \hline & \quad 6x^3 + 3x^2 - 10x - 5 \\ & \quad 6x^3 \qquad\quad - 10x \\ & \quad - \qquad\qquad + \\ \hline & \qquad\qquad 3x^2 - 5 \\ & \qquad\qquad 3x^2 - 5 \\ & \qquad\qquad - \quad + \\ \hline & \qquad\qquad\quad 0 \end{array}$$

So, $$3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$$

$$= 3\left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1)$$

$$= 3\left(x^2 - \frac{5}{3}\right)(x + 1)^2$$

Hence, $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$, –1 and –1 are the four zeros of the given polynomial.

15. Let x be the units digit and y be the tens digit of the two digit number. Then, the given number is $10y + x$.

Also, the number obtained by reversing its digits is $10x + y$.

Now, according to question,

$$7(10y + x) = 4(10x + y)$$

$$\Rightarrow \quad 70y + 7x = 40x + 4y$$

$$\Rightarrow \quad 66y = 33x$$

$$\Rightarrow \quad x - 2y = 0 \qquad ...(1)$$

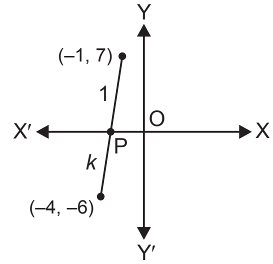
Also, $$x - y = 3 \qquad ...(2)$$

Solving Eqs. (1) and (2) simultaneously, we get

$$x = 6, y = 3$$

Thus, the number is 36.

16. Let x-axis divide the line segment joining (–4, –6) and (–1, 7) at the point P in the ratio $k : 1$.



By section formula, coordinates of P are $\left(\frac{-k-4}{k+1}, \frac{7k-6}{k+1}\right)$.

Since P lies on x-axis, therefore ordinate of P is zero.

i.e., $$\frac{7k-6}{k+1} = 0$$

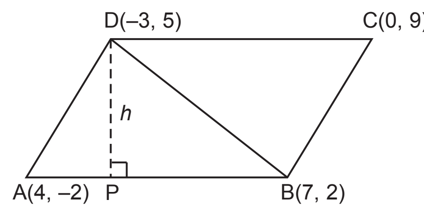
$\Rightarrow$ $$7k-6 = 0 \quad \text{or} \quad k = \frac{6}{7}$$

Hence, the required ratio is $\frac{6}{7} : 1$, *i.e.*, 6 : 7.

Also, coordinates of P are $\left(\frac{-\frac{6}{7}-4}{\frac{6}{7}+1}, 0\right)$, *i.e.*, $\left(\frac{\frac{-34}{7}}{\frac{13}{7}}, 0\right)$ or $\left(\frac{-34}{13}, 0\right)$.

OR

Let 'h' be the length of the altitude DP on the base AB of the parallelogram ABCD, Then,



$$AB = \sqrt{(7-4)^2 + (2+2)^2}$$
$$= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = 5 \text{ units}$$

and $$\text{Area of } \Delta ABD = \frac{1}{2}\left|4(2-5) + 7(5+2) - 3(-2-2)\right|$$
$$= \frac{1}{2}\left|-12 + 49 + 12\right| = \frac{49}{2} \text{ sq units}$$

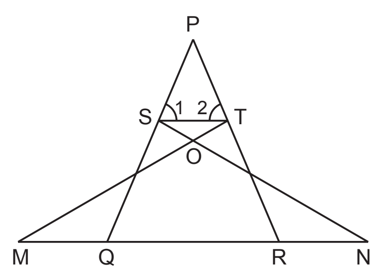
Also, $$\text{Area of } \Delta ABD = \frac{1}{2} \times AB \times DP$$

$\Rightarrow$ $$\frac{49}{2} = \frac{1}{2} \times 5 \times h$$

$\Rightarrow$ $$h = \frac{49}{5} \quad \text{or} \quad h = 9.8 \text{ units}$$

Hence, the length of the altitude of the parallelogram on the base AB is 9.8 units.

17. In the figure, in ΔPST,



$\angle 1 = \angle 2$ [Given]

$\Rightarrow$ PT = PS ...(1)

[Angles opposite to equal sides are equal.]

Also, $\Delta NSQ \cong \Delta MTR$, gives

SQ = TR [CPCT]

or TR = SQ

or TR + PT = SQ + PS [Using (1)]

or PR = PQ ...(2)

$\therefore$ From (1) and (2)

$$\frac{PT}{PR} = \frac{PS}{PQ}$$

Now, in ΔPTS and ΔPRQ

$$\frac{PT}{PR} = \frac{PS}{PQ} \quad \text{[Proved above]}$$

and $\angle P = \angle P$

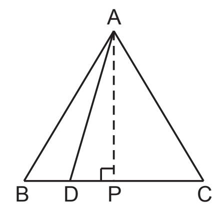
Hence, by SAS similarity criterion $\Delta PTS \sim \Delta PRQ$.

OR

Given: An equilateral ΔABC in which D is a point on the side BC such that $BD = \frac{1}{3}BC$.

To Prove: $9AD^2 = 7AB^2$

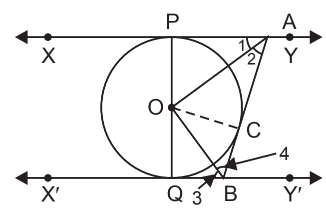
Construction: Draw $AP \perp BC$,



Proof: In right-angled triangle APD, we have

$$AD^2 = AP^2 + DP^2 \quad \text{[By Pythagoras Theorem]}$$
$$= AP^2 + (BP - BD)^2$$
$$= AP^2 + BP^2 + BD^2 - 2\,BP \cdot BD$$
$$= AB^2 + (\tfrac{1}{3}BC)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right) \quad \left[\because\ AP^2 + BP^2 = AB^2,\ BD = \frac{1}{3}BC \text{ and } BP = PC = \frac{BC}{2}\right]$$
$$= AB^2 + \frac{BC^2}{9} - \frac{BC^2}{3}$$
$$= AB^2 + \frac{AB^2}{9} - \frac{AB^2}{3} \quad [\because\ BC = AB]$$
$$= \frac{9AB^2 + AB^2 - 3AB^2}{9}$$
$$\therefore \quad AD^2 = \frac{7AB^2}{9} \quad \text{or} \quad 9AD^2 = 7AB^2.$$

18. Join OC.



In ΔOPA and ΔOCA,

$OP = OC$ [Radii of same circle]

$PA = CA$ [Length of tangents drawn from same point are equal.]

$OA = OA$ [Common]

$\therefore \quad \Delta OPA \cong \Delta OCA$ [By SSS congruency criterion]

Hence, $\angle 1 = \angle 2$ [CPCT]

Similarly, in ΔOQB and ΔOCB,

$\angle 3 = \angle 4$

Now, AB is a transversal between the parallel lines XY and X′Y′, therefore

$\angle PAB + \angle QBA = 180°$ [Cointerior angles are supplementary]

$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

$\Rightarrow \quad 2(\angle 2 + \angle 4) = 180°$ [$\because\ \angle 1 = \angle 2$ and $\angle 3 = \angle 4$]

$\Rightarrow \quad \angle 2 + \angle 4 = 90°$

$\therefore$ In ΔAOB, $\angle 2 + \angle 4 + \angle AOB = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad 90° + \angle AOB = 180°$

$\Rightarrow \quad \angle AOB = 180° - 90° = 90°$

Hence, $\angle AOB = 90°$.

19. $\dfrac{\text{cosec}^2 63° + \tan^2 24°}{\cot^2 66° + \sec^2 27°} + \dfrac{\sin^2 63° + \cos 63° \sin 27° + \sin 27° \sec 63°}{2(\text{cosec}^2 65° - \tan^2 25°)}$

$$= \frac{\text{cosec}^2 (90° - 27°) + \tan^2 24°}{\cot^2 (90° - 24°) + \sec^2 27°} + \frac{\sin^2 (90° - 27°) + \cos(90° - 27°)\sin 27° + \sin 27° \sec(90° - 27°)}{2[\text{cosec}^2 (90° - 25°) - \tan^2 25°]}$$

$$= \frac{\sec^2 27° + \tan^2 24°}{\tan^2 24° + \sec^2 27°} + \frac{\cos^2 27° + \sin^2 27° + \sin 27° \text{ cosec } 27°}{2[\sec^2 25° - \tan^2 25°]}$$

$$= 1 + \frac{1+1}{2(1)} = 1 + \frac{2}{2} = 2.$$

OR

Given, $\sin\theta + \cos\theta = \sqrt{2}$

$$(\sin\theta + \cos\theta)^2 = \left(\sqrt{2}\right)^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

$$1 + 2\sin\theta\cos\theta = 2$$

$$\sin\theta\cos\theta = \frac{1}{2} \qquad ...(1)$$

Now, $\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

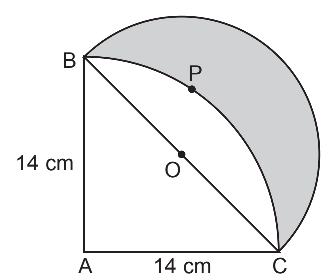
$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} = \frac{1}{\frac{1}{2}} \qquad \text{[Using (1)]}$$

$$= 2$$

Hence, $\tan\theta + \cot\theta = 2$.

20. In quadrant ABPC,



$$AB = AC = r \text{ (say)} = 14 \text{ cm}$$

and $BC = AC\sqrt{2} \qquad [\because \; BC^2 = AC^2 + AB^2]$

$$= 14\sqrt{2} \text{ cm}$$

$\therefore$ Area of quadrant $= \dfrac{1}{4}\pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times (14)^2 = 154 \text{ sq cm}$$

Area of right-angled ΔBAC $= \dfrac{1}{2} \times AC \times AB$

$$= \frac{1}{2} \times 14 \times 14 = 98 \text{ sq cm}$$

Also, area of semicircle on diameter BC

$$= \frac{1}{2}\pi\left(\frac{BC}{2}\right)^2 = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{14\sqrt{2}}{2}\right)^2 = 154 \text{ sq cm}$$

Hence, area of shaded region = Area of ΔBAC + Area of semicircle on diameter BC – Area of quadrant.

= 98 sq cm + 154 sq cm – 154 sq cm = 98 sq cm.

21. Let A sq m be the area irrigated in 30 minutes.

Length-wise water flowing in canal in 30 minutes = $\left(10000 \times \frac{1}{2}\right)$ m = 5000 m

$\therefore$ volume of water flowing in canal in 30 minutes

$= (5000 \times 6 \times 1.5)$ cu m

$= 45000$ cu m ...(1)

Also, volume of water required for irrigation $= A \times \frac{8}{100}$ cu m ...(2)

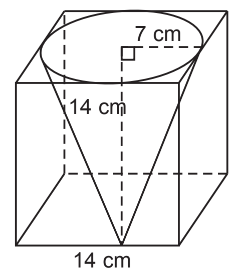
From (1) and (2), we have

$$A \times \frac{8}{100} = 45000 \quad \Rightarrow \quad A = 562500 \text{ sq m}$$

Hence, 5,62,500 sq m area can be irrigated in 30 minutes.

OR

For the maximum size cone carved out from the cube of edge 14 cm, height, $h = 14$ cm, radius, $r = 7$ cm and slant height,



$$l = \sqrt{(14)^2 + (7)^2} = \sqrt{245} = 7\sqrt{5} \text{ cm}$$

$\therefore$ Surface area of the remaining solid

= Total surface area of the cube – Area of the base of the cone + Curved surface area of the cone

$$= 6(\text{edge})^2 - \pi r^2 + \pi r l$$

$$= 6 \times (14)^2 - \frac{22}{7} \times (7)^2 + \frac{22}{7} \times 7 \times 7\sqrt{5}$$

$$= 1176 - 154 + 154\sqrt{5} = \left(1022 + 154\sqrt{5}\right) \text{ sq cm.}$$

22. The given marks distribution is:

Mark Obtained	0–20	20–40	40–60	60–80	80–100
Number of Students	15	18	21	29	17

Here, 60–80 is the modal class as its frequency is the highest.

For modal class 60–80, we have

$l = 60$, $h = 20$, $f_1 = 29$, $f_0 = 21$ and $f_2 = 17$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 60 + \left(\frac{29 - 21}{2 \times 29 - 21 - 17}\right) \times 20$$

$$= 60 + \left(\frac{8}{58 - 38}\right) \times 20$$

$$= 60 + \frac{8}{20} \times 20 = 68$$

Thus, mode of the given distribution is 68.

The mean of the above distribution = 53

The empirical relationship between mean, mode and median is

$$3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$$

$$\Rightarrow \quad 3 \text{ Median} = 68 + 2 \times 53$$

$$\Rightarrow \quad 3 \text{ Median} = 174 \quad \Rightarrow \quad \text{Median} = 58.$$

Section D

23. Let the original speed of the train be x km/h.

Time taken by the train with original speed = $\frac{360}{x}$ hours

Time taken by the train with the increased speed = $\frac{360}{x+5}$ hours

According to the given condition,

$$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$$

$$\Rightarrow \quad 360\left(\frac{1}{x} - \frac{1}{x+5}\right) = \frac{4}{5}$$

$$\Rightarrow \quad \frac{1}{x} - \frac{1}{x+5} = \frac{4}{5} \times \frac{1}{360}$$

$$\Rightarrow \quad \frac{x+5-x}{x(x+5)} = \frac{1}{450}$$

$$\Rightarrow \quad x(x+5) = 2250$$

$$\Rightarrow \quad x^2 + 5x - 2250 = 0$$

$$\Rightarrow \quad x^2 + 50x - 45x - 2250 = 0$$

$$\Rightarrow \quad x(x+50) - 45(x+50) = 0$$

$$\Rightarrow \quad (x+50)(x-45) = 0$$

$$\Rightarrow \quad x = 45 \quad \text{or} \quad x = -50$$

$$\Rightarrow \quad x = 45 \qquad [\because \text{ Speed is not negative.}]$$

Hence, original speed of the train is 45 km/h.

OR

The given quadratic equation is

$$5x^2 - 6x - 2 = 0 \qquad ...(1)$$

Here, $a = 5$, $b = -6$ and $c = -2$

$$\therefore \quad \text{Discriminant, D} = b^2 - 4ac$$

$$= (-6)^2 - 4 \times 5 \times (-2)$$

$$= 36 + 40 = 76 > 0$$

$\therefore$ The given equation has two distinct real roots.

Now, from (1),

$$5x^2 - 6x - 2 = 0$$

$$\Rightarrow \quad x^2 - \frac{6}{5}x - \frac{2}{5} = 0 \qquad \text{[Dividing both sides by 5]}$$

$$\Rightarrow \quad x^2 - \frac{6}{5}x + \left(\frac{3}{5}\right)^2 - \left[\frac{2}{5} + \left(\frac{3}{5}\right)^2\right] = 0 \qquad \left[\text{Adding and subtracting } \left(\frac{b}{2a}\right)^2, \text{ i.e., } \left(\frac{6}{2 \times 5}\right)^2\right]$$

$$\Rightarrow \quad \left(x - \frac{3}{5}\right)^2 - \left(\frac{2}{5} + \frac{9}{25}\right) = 0$$

$$\Rightarrow \quad \left(x - \frac{3}{5}\right)^2 - \left(\frac{19}{25}\right) = 0$$

$$\Rightarrow \quad \left(x - \frac{3}{5}\right) = \pm\frac{\sqrt{19}}{5}$$

$$\Rightarrow \quad x = \frac{3}{5} \pm \frac{\sqrt{19}}{5} = \frac{3 \pm \sqrt{19}}{5}$$

$\therefore$ The roots of the given equation are $\frac{3+\sqrt{19}}{5}$ and $\frac{3-\sqrt{19}}{5}$.

Verification:

Putting $\frac{3+\sqrt{19}}{5}$ in L.H.S. of (1), we have

$$5\left(\frac{3+\sqrt{19}}{5}\right)^2 - 6\left(\frac{3+\sqrt{19}}{5}\right) - 2$$

$$\Rightarrow \quad = \frac{9+19+6\sqrt{19}}{5} - \left(\frac{18+6\sqrt{19}}{5}\right) - 2$$

$$\Rightarrow \quad = \frac{28+6\sqrt{19}-18-6\sqrt{19}-10}{5} = 0$$

Putting $\frac{3-\sqrt{19}}{5}$ in L.H.S. of (1), we have

$$5\left(\frac{3-\sqrt{19}}{5}\right)^2 - 6\left(\frac{3-\sqrt{19}}{5}\right) - 2$$

$$= \frac{9+19-6\sqrt{19}}{5} - \left(\frac{18-6\sqrt{19}}{5}\right) - 2 = \frac{28-6\sqrt{19}-18+6\sqrt{19}-10}{5} = 0$$

Hence, both the roots satisfy the given quadratic equation.

24. Here, the given A.P. consists of 37 terms. Therefore,

18th, 19th and 20th terms are the three middle most terms.

$\Rightarrow \quad a + 17d + a + 18d + a + 19d = 225$

$\Rightarrow \quad 3a + 54d = 225$

or $\quad a + 18d = 75$ ...(1)

Also, 35th, 36th and 37th terms are the last three terms.

$a + 34d + a + 35d + a + 36d = 429$

$\Rightarrow \quad 3a + 105d = 429$

or $\quad a + 35d = 143$ ...(2)

Subtracting (1) from (2), we have

$$\begin{array}{r} a + 35d = 143 \\ -a - 18d = -75 \\ \hline 17d = 68 \end{array} \quad \Rightarrow \quad d = 4$$

Putting the value of d in (1), we have

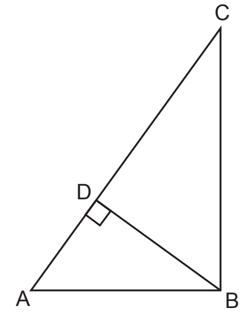
$a + 18 \times 4 = 75 \quad \Rightarrow \quad a + 72 = 75 \quad \Rightarrow \quad a = 3$

Hence, the given A.P. is 3, 7, 11, ...

25. Given: A right triangle ABC, right-angled at B.

To Prove: $AC^2 = AB^2 + BC^2$

Construction: Draw $BD \perp AC$.



In ΔADB and ΔABC,

$\angle ADB = \angle ABC$ [Each 90°]

$\angle BAD = \angle CAB$ [Common]

Proof: By AA similarity criterion, $\Delta ADB \sim \Delta ABC$

$\therefore \quad \frac{AD}{AB} = \frac{AB}{AC}$ [Corresponding sides of similar triangles are proportional.]

or $\quad AD \times AC = AB^2$ ...(1)

Similarly, $\Delta\,BDC \sim \Delta\,ABC$

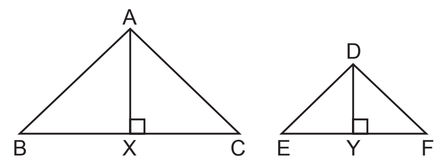
$$\therefore \quad \frac{DC}{BC} = \frac{BC}{AC}$$

or $DC \times AC = BC^2$ ...(2)

Adding (1) and (2) we get

$$AB^2 + BC^2 = AD \times AC + DC \times AC = (AD + DC) \times AC$$
$$= AC \times AC = AC^2.$$

OR



Given: $\Delta\,ABC$ is similar to $\Delta\,DEF$

i.e., $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF}$

To Prove: $\dfrac{ar(\Delta\,ABC)}{ar(\Delta\,DEF)} = \dfrac{AB^2}{DE^2} = \dfrac{BC^2}{EF^2} = \dfrac{AC^2}{DF^2}$

Construction: Draw $AX \perp BC$ and $DY \perp EF$.

Proof: In Δs ABX and DEY,

$\angle ABX = \angle DEY$ [$\because$ Δs ABC and DEF are similar.]

$\angle AXB = \angle DYE$ [Each is 90°.]

Hence, by AA similarity criterion, $\Delta\,ABX \sim \Delta\,DEY$.

$$\Rightarrow \quad \frac{AB}{DE} = \frac{AX}{DY} = \frac{BX}{EY} \qquad ...(1)$$

Now, $$\frac{ar(\Delta\,ABC)}{ar(\Delta\,DEF)} = \frac{\frac{1}{2}\cdot BC\cdot AX}{\frac{1}{2}\cdot EF\cdot DY} = \frac{BC}{EF}\cdot\frac{AX}{DY}$$

$$= \frac{BC}{EF}\cdot\frac{AB}{DE} \qquad \text{[Using (1)]}$$

$$= \frac{BC}{EF}\cdot\frac{BC}{EF} \qquad \text{[Given]}$$

$$= \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = \frac{AB^2}{DE^2}. \qquad \text{[In the same way]}$$

26. In ΔABC, we have

$BC = 7$ cm, $\angle B = 45°$ and $\angle A = 105°$

$\therefore$ $\angle C = 180° - (\angle A + \angle B)$ [By angle sum property of a triangle]

$= 180° - (105° - 45°)$

$= 180° - 150° = 30°$

Step of Construction:

(*i*) Draw a line segment BC = 7 cm.

(*ii*) Draw $\angle CBX = 45°$ at B and $\angle BCY = 30°$ at C to intersect each other at A to complete ΔABC.

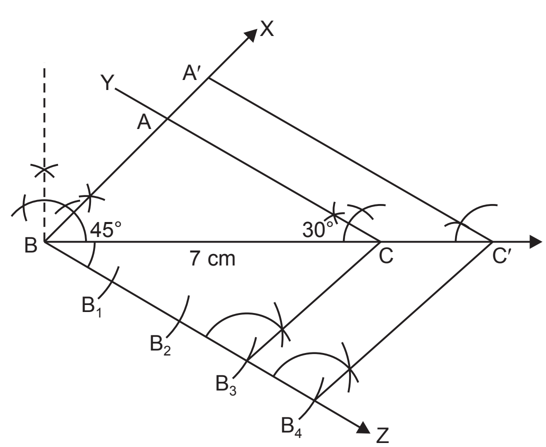
(*iii*) Draw an acute angle $\angle CBZ$ on other side of ΔABC.

(*iv*) Mark $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$ on ray BZ.

(*v*) Join B_3C and draw $B_4C' \parallel B_3C$ on BC produced.

(*vi*) Draw $C'A' \parallel CA$.

Thus, $\Delta A'BC'$ is the required triangle.



27. We have,

$$\text{L.H.S.} = \frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} \times \frac{\cos\theta + \sin\theta + 1}{\cos\theta + \sin\theta + 1}$$

$$= \frac{(\cos\theta+1)^2 - \sin^2\theta}{(\cos\theta+\sin\theta)^2 - (1)^2}$$

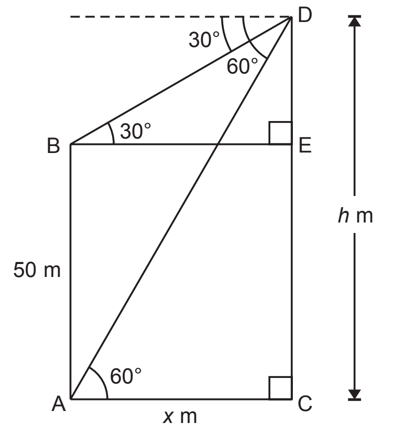
$$= \frac{\cos^2\theta + 1 + 2\cos\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1}$$

$$= \frac{2\cos\theta + 2\cos^2\theta}{2\sin\theta\cos\theta} \qquad [\because \; 1 - \sin^2\theta = \cos^2\theta]$$

$$= \frac{2\cos\theta(1+\cos\theta)}{2\sin\theta\cos\theta} = \frac{1+\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \text{cosec}\,\theta + \cot\theta = \text{R.H.S.}$$

28. In the figure, AB is the building and CD is the tower.



Let the height of tower CD be 'h' m and AC = BE = 'x' m.

Then, CE = AB = 50 m and DE = $(h - 50)$ m

∴ In right ΔACD, we have

$$\tan 60° = \frac{CD}{AC} \quad \Rightarrow \quad \sqrt{3} = \frac{h}{x} \quad \text{or} \quad h = \sqrt{3}x \qquad ...(1)$$

Also, in right ΔBED,

$$\tan 30° = \frac{DE}{BE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h-50}{x}$$

$$\Rightarrow \quad x = \sqrt{3}(h-50) \qquad ...(2)$$

Substituting the value of x in (1), we get

$$h = \sqrt{3}\left[\sqrt{3}(h-50)\right]$$

$$\Rightarrow \quad h = 3(h-50)$$

$$\Rightarrow \quad h = 3h - 150 \quad \Rightarrow \quad 2h = 150 \quad \text{or} \quad h = 75 \text{ m}$$

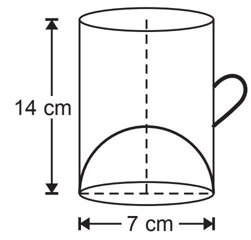
Substituting the value of h in (2), we get

$$x = \sqrt{3}(75-50) = 25\sqrt{3} \text{ m}$$

Thus, height of the tower is 75 m and distance between the building and tower is $25\sqrt{3}$ m.

29. Height of the mug, h = 14 cm

Radius of the mug, $r = \frac{7}{2}$ cm



∴ Actual quantity of milk in the mug

= Volume of cylindrical portion
– Volume of hemispherical bottom

$$= \pi r^2 h - \frac{2}{3}\pi r^3$$

$$= \pi r^2\left(h - \frac{2}{3}r\right) = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \left(14 - \frac{2}{3} \times \frac{7}{2}\right)$$

$$= \frac{22}{7} \times \frac{49}{4} \times \frac{35}{3} = \frac{2695}{6} \text{ cu cm} \quad \text{or} \quad \frac{2695}{6000} \text{ litres} \qquad [\because \; 1 \text{ litre} = 1000 \text{ cu cm}]$$

∴ Price of milk at the rate of ₹ 80 per litre

$$= ₹\ \frac{2695}{6000} \times 80 = ₹\ 35.93.$$

Value: The value exhibited by the dairy owner B is honesty.

30. For the given distribution, we prepare the following table:

Daily Pocket Allowance (in ₹)	*Mid-Value* (x_i)	*Number of Children* (f_i)	$u_i = \frac{x_i - A}{2}$	*Product* ($f_i u_i$)
11–13	12	3	–3	–9
13–15	14	6	–2	–12
15–17	16	9	–1	–9
17–19	18 = A	13	0	0
19–21	20	k	1	k
21–23	22	5	2	10
23–25	24	4	3	12
Total		$\Sigma f_i = 40 + k$		$\Sigma f_i u_i = k - 8$

$\therefore$ Mean, $\bar{x} = A + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$

$\Rightarrow$ $18 = 18 + \left(\frac{k-8}{40+k}\right) \times 2$ [$\because$ Mean = 18]

$\Rightarrow$ $\left(\frac{k-8}{40+k}\right) \times 2 = 0$

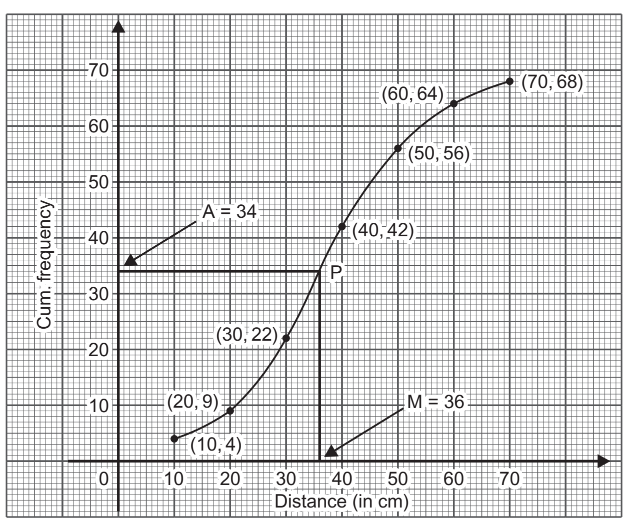
$\Rightarrow$ $k - 8 = 0$ or $k = 8$.

OR

From the given frequency distribution, we prepare the following cumulative frequency table:

Distance (in m)	*Number of Students (cf)*
Less than 10	4
Less than 20	9
Less than 30	22
Less than 40	42
Less than 50	56
Less than 60	64
Less than 70	68

Now, to draw a less than type ogive we plot the points (10, 4) (20, 9), (30, 22), ... taking upper limits of Distance (in m) on x-axis and corresponding cumulative frequencies on y-axis as shown in graph



To find the median, we draw a line parallel to x-axis from A $\left(= \frac{N}{2} = \frac{68}{2}\right) = 34$ to cut the ogive at P. From P, we draw a perpendicular to meet x-axis at M = 36.

Thus, the median distance of the given distribution is 36 m.

Model Test Paper (Solved)

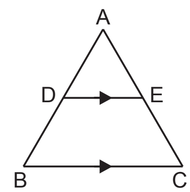
MATHEMATICS

Time Allowed: 3 Hours **Max. Marks:** 80

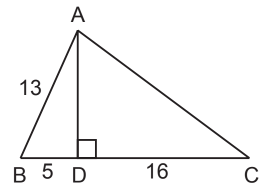
Section A

Questions from 1 to 6 carry 1 mark each.

1. If the HCF (65, 170) = 5, find the LCM (65, 170).
2. Check graphically, if the following system of linear equation has a unique solution.
   $2x - 3y - 6 = 0;\ \ 2x + y + 10 = 0.$
3. Show that $(a + b)^2$, $a^2 + b^2$ and $(a - b)^2$ are in A.P.
4. The distance between A(1, 3) and B(x, 7) is 5. Find the value of x.
5. In the given triangle ABC, DE || BC and $\frac{AE}{EC} = \frac{4}{3}$. If AB = 3.5 cm, find AD.



6. In the figure given, find (i) sin B (ii) tan C.



Section B

Questions from 7 to 12 carry 2 marks each.

7. Prove that if a positive integer is of the form $6q + 5$, then it is of the form $3m + 2$ for some integer m but not conversely.
8. Find the 40th term and the sum of the first 40 terms of the A.P.: 5, 0, –5, –10, ...
9. For what value(s) of K will the pair of linear equations
   $$Kx + 3y = K + 3$$
   $$12x + Ky = K$$
   (i) have a unique solution? (ii) no solution?
10. The line segment joining the points A(3, –4) and B(1, 2) is trisected at the points P and Q. If the coordinates of P and Q are (p, –2) and $\left(\frac{5}{3},\ q\right)$ respectively, find the values of p and q.
11. In a family, there are 3 children. Assuming that the chances of a child being a male or a female are equal, find the probability that
    (i) there is one girl in the family;
    (ii) there is at least one male child in the family.
12. Two different dice are tossed together. Find the probability that
    (i) the number on each dice is even;
    (ii) the sum of numbers appearing on the two dice is 5.

Section C

Questions from 13 to 22 carry 3 marks each.

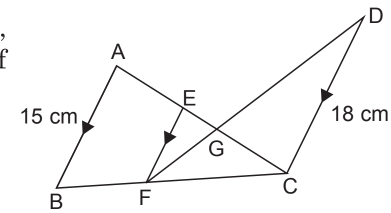
13. Using Euclid's Division Algorithm find the HCF of 4052 and 12576.
14. Find the zeros of the polynomial $p(x) = x^3 + 4x^2 + x - 6$, if it is given that the product of two of its zeros is 6.
15. Solve for x and y: $\frac{30}{x-y} + \frac{44}{x+y} = 10;\quad \frac{40}{x-y} + \frac{55}{x+y} = 13 \qquad (x - y \neq 0,\ x + y \neq 0)$

16. For what value of p are the points $(p, 2 - 2p)$, $(-p + 1, 2p)$ and $(-p - 4, -2p + 6)$ collinear?

OR

Three consecutive vertices of a parallelogram are (–2, –1), (1, 0) and (4, 3). Find the fourth vertex.

17. In given figure, AB, EF and DC are parallel lines. Given that EG = 5 cm, GC = 10 cm, AB = 15 cm and DC = 18 cm. Calculate the lengths of EF and AC.



OR

In Δ ABC, $\angle C = 90°$ and CD $\perp$ AB. Prove that $\dfrac{BC^2}{AC^2} = \dfrac{BD}{AD}$.

18. In two concentric circles, prove that all chords of the outer circle which touch the inner circle are of equal length.

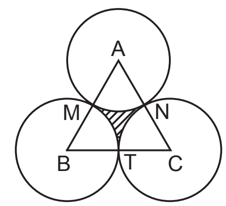
19. $4(\cos^4 60° + \sin^4 30°) - (\tan^2 60° + \cot^2 45°) + 3\,\text{cosec}^2\, 60°$.

OR

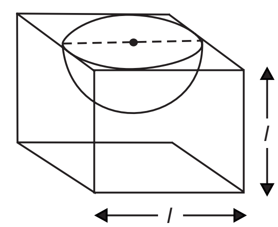
If θ is an acute angle and $\tan\theta + \cot\theta = 2$; find the value of $\tan^5\theta + \cot^5\theta$.

20. Three equal circles each of radius 4 cm touch one another as shown in the given figure. Find the area enclosed between them.

(Take $\pi = 3.14$ and $\sqrt{3} = 1.73$)



21. A hemispherical depression is cut out from one face of the cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.



OR

A sphere of diameter 12 cm is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

22. The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50. Compute the missing frequencies f_1 and f_2.

Class Interval	0–20	20–40	40–60	60–80	80–100	100–120	Total
Frequency	5	f_1	10	f_2	7	8	50

Section D

Questions from 23 to 30 carry 4 marks each.

23. Find the roots of the equation $\sqrt{\dfrac{x}{1-x}} + \sqrt{\dfrac{1-x}{x}} = \dfrac{13}{6}$, $(x \neq 0, x \neq 1)$.

OR

A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.

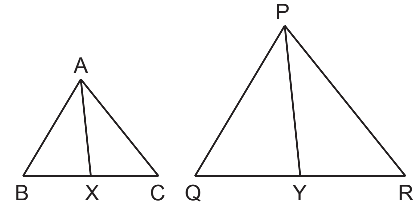
24. If the ratio of sums of p and q terms of an AP is $p^2 : q^2$, then find the ratio of its pth and qth terms.

25. Prove that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

OR

In the given figure, ABC and PQR are two triangles and AX and PY are their medians. If Δ ABC ~ Δ PQR, prove that

(*i*) Δ BXA ~ Δ QYP



(*ii*) $\frac{AX}{PY} = \frac{BC}{QR}$ *i.e.*, the ratio of corresponding sides is equal to the ratio of corresponding medians.

(*iii*) $\Delta AXC \sim \Delta PYR$

26. Construct a tangent to a circle with centre O and radius 4 cm from a point on the concentric circle of radius 6 cm and measure the length. Also, verify the measurement by actual calculation.

27. Solve the following equation:

$$\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3;\ 0° < \theta < 90°$$

28. A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of a 20 m high building, finds the angle of elevation of the same bird to be 45°. Both the boy and girl are on the opposite sides of the bird. Find the distance of the bird from the girl.

29. A vendor selling ice cream, wants to sell ice cream either in a paper cone of radius *r* cm and height *h* cm or in a cylindrical cup of base radius as half of the cone's, and depth 4/3 times of the cone's but charging the same amount. How do you judge him in terms of values?

30. A life insurance agent found the following data for distribution of ages of 100 policy-holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	*Number of Children*
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

OR

An agricultural research officer measured the heights (to the nearest centimetre) of 100 test plants after 2 months growth. The distribution of the heights is shown below:

Height (in cm)	1–5	6–10	11–15	16–20	21–25	26–30
Number of Plants	3	5	9	31	38	14

Construct both (*a*) less than type ogive and (*b*) more than type ogive for these data.

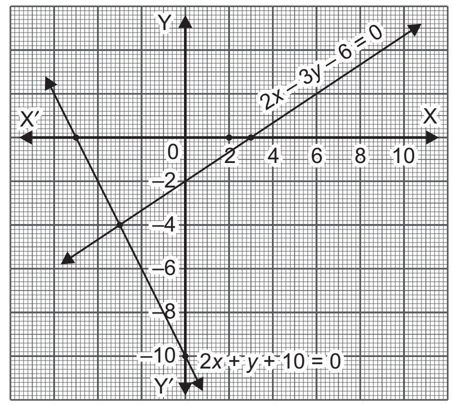
SOLUTIONS TO MODEL TEST PAPER

Section A

1. We know that

$$\text{HCF}(65, 170) \times \text{LCM}(65, 170) = 65 \times 170$$

$$\Rightarrow \quad \text{LCM}(65, 170) = \frac{65 \times 170}{5} = 13 \times 170 = 2210.$$



2. Since the two lines corresponding to the given linear equations, intersect at a point the system has a unique solution.

3. The given terms are $a^2 + b^2 + 2ab$, $a^2 + b^2$ and $a^2 + b^2 - 2ab$.

As 2nd term – 1st term = $-2ab$

and 3rd term – 2nd term = $-2ab$

Therefore, three terms are in A.P.

4. Here, $\text{AB} = \sqrt{(x-1)^2 + (7-3)^2} \Rightarrow \text{AB}^2 = (x-1)^2 + 16$

As AB = 5, we have $\text{AB}^2 = 25$

So, $(x-1)^2 + 16 = 25$

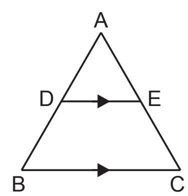
$\Rightarrow \quad (x-1)^2 = 9$

or $\quad x - 1 = \pm 3$

or $\quad x = 4$ or -2.

5. By B.P.T., we have

$$\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}} = \frac{4}{3}$$



$$\Rightarrow \quad \frac{\text{AD}}{\text{AB} - \text{AD}} = \frac{4}{3}$$

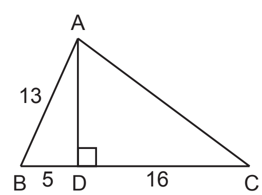
$$\Rightarrow \quad 4\text{AB} - 4\text{AD} = 3\text{AD}$$

$$\Rightarrow \quad 7\text{AD} = 4\text{AB}$$

$$\Rightarrow \quad \text{AD} = \frac{4}{7}\text{AB} = \frac{4}{7} \times 3.5 = 2$$

Thus, AD = 2 cm.

6. In the figure, $\text{AD} = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$



$$\therefore \quad \sin \text{B} = \frac{\text{AD}}{\text{AB}} = \frac{12}{13}; \text{ and } \tan \text{C} = \frac{\text{AD}}{\text{DC}} = \frac{12}{16}, \textit{ i.e.}, \frac{3}{4}.$$

Section B

7. The given integer = $6q + 5 = 3 \times 2q + 3 + 2 = 3 \times (2q + 1) + 2 = 3m + 2$, for some $m = 2q + 1$

Conversely, consider the integer 8.

Now, $\quad 8 = 6 + 2 = 3 \times 2 + 2$ *i.e.*, if is of the form $3m + 2$

But 8 cannot be written in the form $6q + 5$.

On the contrary, let it be possible. Then,

$$8 = 6q + 5$$

$\Rightarrow \quad 6q = 3, \text{ or } q = \frac{1}{2}$

which is a contradiction, since q is an integer.

8. Here, $a = 5$ and $d = -5$.

So, $a_{40} = a + 39d = 5 + 39(-5) = -190$

Now, $S_{40} = \frac{40}{2}\left[2\times 5+(40-1)(-5)\right] = 20[10-195]$

$= -3700.$

9. (*i*) The given pair of linear equation will have a unique solution when

$$\frac{K}{12} \neq \frac{3}{K}$$

i.e., $K^2 \neq 36$

Thus, the required values of K are all non-zero real numbers except 6 and –6.

(*ii*) The given pair of linear equation will have no solution when

$$\frac{K}{12} = \frac{3}{K} \neq \frac{K+3}{K}$$

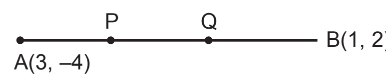
$\Rightarrow \quad K^2 = 36 \text{ and } 3K \neq K^2 + 3K$

$\Rightarrow \quad K = \pm 6 \text{ and } K \neq 0$

Hence for $x = 6$ or -6, the pair has no solution.

10. Here, P divides AB in the ratio 1 : 2

and Q divides AB in ratio 2 : 1.



So, $P\left(\frac{1+6}{1+2}, \frac{2-8}{1+2}\right)$, *i.e.*, $P\left(\frac{7}{3}, -2\right)$

$Q\left(\frac{2+3}{2+1}, \frac{4-4}{1+2}\right)$, *i.e.*, $Q\left(\frac{5}{3}, 0\right)$

Given the coordinates of P and Q as $(p, -2)$ and $\left(\frac{5}{3}, q\right)$, we have

$p = \frac{7}{3}$ and $q = 0$.

11. Here, the sample space S consists of

{BBB, BBG, BGG, GGG}

So, (*i*) Probability of one girl $= \frac{1}{4}$

(*ii*) Probability of at least one male child $= \frac{3}{4}$.

12. (*i*) Here, the sample space of events consists of 36 pairs of numbers 1 to 6. Out of these, following are pairs of even numbers.

(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4) and (6, 6)

So, required probability $= \frac{9}{36}$, *i.e.*, $\frac{1}{4}$.

(*ii*) Of the 36 pairs, following are pairs of two numbers with sum 5.

(1, 4), (2, 3), (3, 2), (4, 1)

So, required probability $= \frac{4}{36}$, *i.e.*, $\frac{1}{9}$.

Section C

13. Using Euclid's Division Algorithm, we have

$$12576 = 3 \times 4052 + 420$$

Further,
$$4052 = 9 \times 420 + 272$$
$$420 = 1 \times 272 + 148$$
$$272 = 1 \times 148 + 124$$
$$148 = 1 \times 124 + 24$$
$$124 = 5 \times 24 + 4$$
$$24 = 6 \times 4 + 0$$

Hence, HCF of 4052 and 12576 is 4.

14. Let α, β, γ be the zeros of the given polynomial. Then

$$\alpha + \beta + \gamma = -4, \quad \alpha\beta + \beta\gamma + \gamma\alpha = 1 \quad \text{and} \quad \alpha\beta\gamma = 6$$

Since it is given that $\alpha\beta = 6$, we have $\gamma = 1$

$\Rightarrow$ $\alpha + \beta = -5$ and $\alpha\beta + \beta + \alpha = 1$

$\Rightarrow$ $\alpha + \beta = -5$ and $\alpha\beta = 6$

$\Rightarrow$ $\alpha = -3, \quad \beta = -2$

Thus, the three zeros are 1, –3 and –2.

15. Let $\dfrac{1}{x-y} = p$ and $\dfrac{1}{x+y} = q$. Then the two given equations turn into

$$30p + 44q - 10 = 0$$

and
$$40p + 55q - 13 = 0$$

By cross multiplication method, we have

$$\frac{p}{-572+550} = \frac{q}{-400+390} = \frac{1}{1650-1760}$$

i.e.,
$$\frac{p}{-22} = \frac{q}{-10} = \frac{1}{-110}$$

$$p = \frac{1}{5} \quad \text{and} \quad q = \frac{1}{11}$$

Hence,
$$\frac{1}{x-y} = \frac{1}{5} \quad \text{and} \quad \frac{1}{x+y} = \frac{1}{11}$$

or
$$x - y = 5$$

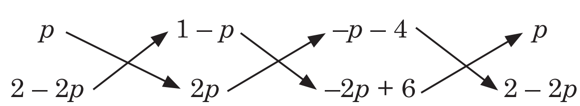
and
$$x + y = 11$$

Solving these two equations simultaneously, we get

$$x = 8, y = 3.$$

16. For the given points to be collinear, the area of the triangle having these points as vertices must be zero.

So,



$\therefore$ $\{2p^2 + (1-p)(-2p+6) + (-p-4)(2-2p)\} - \{(2-2p)(1-p) + 2p(-p-4) + p(-2p+6)\} = 0$

$\Rightarrow$ $\{2p^2 + 6 - 2p + 2p^2 - 6p - (8 - 6p - 2p^2)\} - \{2 - 4p + 2p^2 - 2p^2 - 8p - 2p^2 + 6p\} = 0$

$\Rightarrow$ $(6p^2 - 2p - 2) - (-2p^2 - 6p + 2) = 0$

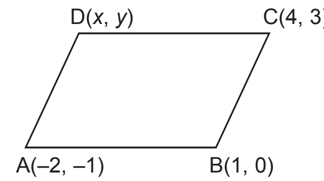
$\Rightarrow$ $8p^2 + 4p - 4 = 0$

$\Rightarrow$ $2p^2 + p - 1 = 0$

$\Rightarrow$ $p = -1$ and $\dfrac{1}{2}$.

OR

Let the fourth vertex be D(x, y). Then,



$$\text{Mid-point of AC} = \left(\frac{4-2}{2}, \frac{3-1}{2}\right), \textit{ i.e., } (1, 1)$$

$$\text{Mid-point of BD} = \left(\frac{x+1}{2}, \frac{y+0}{2}\right), \textit{ i.e., } \left(\frac{x+1}{2}, \frac{y}{2}\right)$$

Since the mid-point of AC is the same as the mid-point of BD, in case of a parallelogram, we have

$$\frac{x+1}{2} = 1 \quad \text{and} \quad \frac{y}{2} = 1$$

$$x = 1 \quad \text{and} \quad y = 2$$

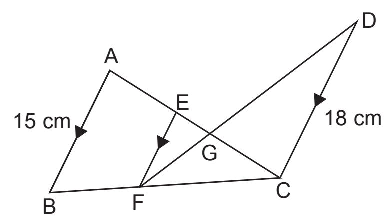
Thus, the fourth vertex is (1, 2).

17. In Δs EFG and CDG,

$\angle F = \angle D$ [Since EF || DC and FD is a transversal.]

$\angle EGF = \angle CGD$ [Vertically opposite angles]

∴ By AA similarity criterion, we have ΔEFG ~ ΔCDG



$$\Rightarrow \quad \frac{EF}{EG} = \frac{CD}{CG}$$

$$\Rightarrow \quad \frac{EF}{5} = \frac{18}{10}$$

$$\Rightarrow \quad EF = 9 \text{ cm}$$

Further, in Δs CAB and CEF,

$\angle C = \angle C$ [Common angle]

$\angle A = \angle E$ [Since AB || EF and AC is a transversal.]

∴ By AA similarity criterion, we have ΔCAB ~ ΔCEF

$$\Rightarrow \quad \frac{AC}{EC} = \frac{AB}{EF} \quad \Rightarrow \quad \frac{AC}{15} = \frac{15}{9}$$ [Since EF = 9 cm]

$$\Rightarrow \quad AC = 25 \text{ cm.}$$

OR

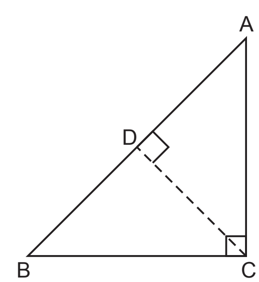
In Δs ABC and ACD,

$\angle A = \angle A$ [Common]

$\angle ACB = \angle ADC$ [Each is 90°.]

∴ By AA criterion of similarity, we have

ΔABC ~ ΔACD



$$\Rightarrow \quad \frac{AC}{AD} = \frac{AB}{AC}$$

$$\Rightarrow \quad AC^2 = AB \times AD \qquad ...(1)$$

In the same way,

ΔBAC ~ ΔBCD

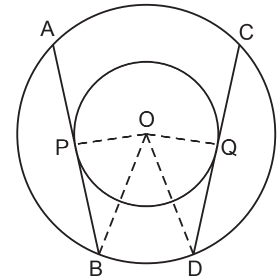
$$\Rightarrow \quad \frac{BC}{BD} = \frac{BA}{BC}$$

$$\Rightarrow \quad BC^2 = BA \times BD = AB \times BD \qquad ...(2)$$

From Eq. (1) and Eq. (2), we have

$$\frac{BC^2}{AC^2} = \frac{BD}{AD}.$$

18. Let AB and CD be two chords of the outer circle which touch the inner circle at P and Q respectively.



To show: AB = CD

As AB and CD touch the inner circle

OP = OQ = Radius of the smaller circle

Also, $\angle OPB = \angle OQD = 90°$.

Now, $OP \perp AB \Rightarrow$ P bisects the chord AB.

$\Rightarrow \quad AP = PB = \frac{1}{2}AB \quad$ ...(1)

Similarly Q bisects chord CD.

$\Rightarrow \quad CQ = QD = \frac{1}{2}CD \quad$ ...(2)

In right-angled triangles OPB and OQD

$OP = OQ$

$OB = OD$

$\Rightarrow \quad \Delta OPB \cong \Delta OQD \quad$ [By R.H.S. criterion of congruence]

$\Rightarrow \quad PB = QD$

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}CD \quad$ [By (1) and (2)]

$\Rightarrow \quad AB = CD.$

19. Here, $4(\cos^4 60° + \sin^4 30°) - (\tan^2 60° + \cot^2 45°) + 3\,\text{cosec}^2\, 60°$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - \left[\left(\sqrt{3}\right)^2 + 1^2\right] + 3\left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - [3+1] + 3\frac{4}{3}$$

$$= 4 \times \frac{1}{8} - 4 + 4 = \frac{1}{2}.$$

OR

Here, $\tan\theta + \cot\theta = 2$

$\Rightarrow \quad \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \quad \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow \quad (\tan\theta - 1)^2 = 0$

$\Rightarrow \quad \tan\theta = 1 = \tan 45°$

$\Rightarrow \quad \theta = 45°$

$\therefore \tan^5\theta + \cot^5\theta = \tan^5 45° + \cot^5 45°$

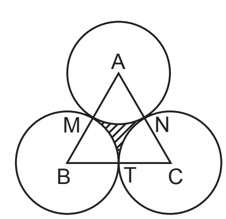
$= (1)^5 + (1)^5$

$= 1 + 1 = 2.$

20. Obviously, ΔABC is an equilateral triangle with side 8 cm.

$\therefore \quad$ Area of $\Delta ABC = \frac{\sqrt{3}}{4} \times (8)^2$ sq cm

$= 16\sqrt{3}$ sq cm



Also, area of sector AMN = Area of sector BMT = Area of sector CNT

$$= \frac{60°}{360°}\pi r^2 = \frac{60°}{360°} \times \pi \times 16 \text{ sq cm} = \frac{8\pi}{3} \text{ sq cm}$$

$\therefore$ Area of the space enclosed between the circles

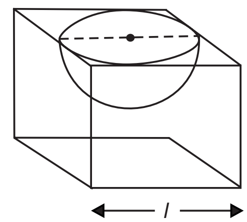
$$= \text{Area of } \Delta ABC - \text{Area of (sector AMN + sector BMT + sector CNT)}$$

$$= \left(16\sqrt{3} - 3 \times \frac{8\pi}{3}\right) \text{ sq cm} = \left(16\sqrt{3} - 8\pi\right) \text{ sq cm}$$

$$= 8(2 \times 1.73 - 3.14) \text{ sq cm} = 2.56 \text{ sq cm.}$$

21. Here $\frac{l}{2}$ is the radius of the hemisphere cut-out from the top face of the cubical wooden block.

$\therefore$ Surface area of the remaining solid = (Surface area of the cubical wooden box of length l) – (Area of the top of the hemispherical part) + Curved surface area of the hemispherical part

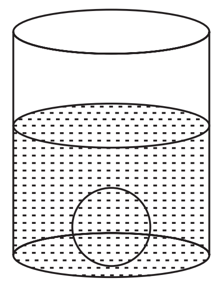


$$= 6l^2 - \pi \times \left(\frac{l}{2}\right)^2 + 2\pi\left(\frac{l}{2}\right)^2$$

$$= 6l^2 + \pi\frac{l^2}{4} = \frac{l^2}{4}(24 + \pi) \text{ sq units.}$$

OR

Let the radius of the base of the right circular cylindrical vessel be r cm and let h cm be the height of the water level in the vessel at the beginning. Then



$$\pi r^2\left(h + \frac{32}{9}\right) - \pi r^2 h = \frac{4}{3}\pi(6)^3$$

$$\Rightarrow \quad \pi r^2\left(\frac{32}{9}\right) = \frac{4}{3}\pi(216)$$

$$\Rightarrow \quad r^2 = \frac{4}{3} \times 216 \times \frac{9}{32}$$

$$= 81$$

$$\Rightarrow \quad r = 9 \text{ cm}$$

Thus, the diameter of the cylindrical vessel is 18 cm.

22. The frequency distribution table for the given data is as follows:

Class Interval	*Frequency* (f_i)	*Class Mark* (x_i)	*Product* ($f_i x_i$)
0–20	5	10	50
20–40	f_1	30	$30f_1$
40–60	10	50	500
60–80	f_2	70	$70f_2$
80–100	7	90	630
100–120	8	110	880
Total	$\Sigma f_i = 50$		$\Sigma f_i x_i = 2060 + 30f_1 + 70f_2$

Now, $$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad 62.8 = \frac{2060 + 30f_1 + 70f_2}{50}$$

$$\Rightarrow \quad 2060 + 30f_1 + 70f_2 = 3140$$

$$\Rightarrow \quad 3f_1 + 7f_2 = 108 \quad \ldots(1)$$

Also, we have

$$30 + f_1 + f_2 = 50$$

$$\Rightarrow \quad f_1 + f_2 = 20 \quad \ldots(2)$$

Solving Eq. (1) and Eq. (2) simultaneously, we get

$$f_1 = 8 \text{ and } f_2 = 12$$

Thus, the missing frequencies f_1 and f_2 are 8 and 12 respectively.

Section D

23. Taking $\sqrt{\frac{x}{1-x}}$ as y in the given equation, we have

$$y+\frac{1}{y} = \frac{13}{6}$$

$$\frac{y^2+1}{y} = \frac{13}{6}$$

$\Rightarrow$ $6y^2 + 6 = 13y$

$\Rightarrow$ $6y^2 - 13y + 6 = 0$

$\Rightarrow$ $6y^2 - 9y - 4y + 6 = 0$

$\Rightarrow$ $3y(2y - 3) - 2(2y - 3) = 0$

$\Rightarrow$ $(3y - 2)(2y - 3) = 0$

$\Rightarrow$ $3y - 2 = 0$ or $2y - 3 = 0$

$\Rightarrow$ $y = \frac{2}{3}$ or $y = \frac{3}{2}$

$\therefore$ $\sqrt{\frac{x}{1-x}} = \frac{2}{3}$ or $\sqrt{\frac{x}{1-x}} = \frac{3}{2}$

$\Rightarrow$ $\frac{x}{1-x} = \frac{4}{9}$ or $\frac{x}{1-x} = \frac{9}{4}$

$\Rightarrow$ $9x = 4 - 4x$ or $4x = 9 - 9x$

$\Rightarrow$ $13x = 4$ or $13x = 9$

$\Rightarrow$ $x = \frac{4}{13}$ or $x = \frac{9}{13}$.

Thus, $\frac{4}{13}$ and $\frac{9}{13}$ are the two roots of the given equation.

OR

Suppose, B alone takes x days to finish the work and A alone can finish it in $(x - 10)$ days.

Then, (A's 1 day work) + (B's 1 day work) = $\frac{1}{x}+\frac{1}{x-10}$

Given (A + B)'s 1 day work = $\frac{1}{12}$

$\therefore$ $\frac{1}{x}+\frac{1}{x-10} = \frac{1}{12}$

$\Rightarrow$ $\frac{(x-10)+x}{x(x-10)} = \frac{1}{12}$

$\Rightarrow$ $12(2x - 10) = x(x - 10)$ $\Rightarrow$ $24x - 120 = x^2 - 10x$

$\Rightarrow$ $x^2 - 34x + 120 = 0$ $\Rightarrow$ $x^2 - 4x - 30x + 120 = 0$

$\Rightarrow$ $x(x - 4) - 30(x - 4) = 0$ $\Rightarrow$ $(x - 30)(x - 4) = 0$

$\Rightarrow$ $x - 30 = 0$ or $x - 4 = 0$

$\Rightarrow$ $x = 30$ or $x = 4$

Since x cannot be less than 10, the value of x is 30.

Thus, B alone can finish the work in 30 days.

24. Let a and d be, respectively, the first term and common difference of the given A.P. Then

$$S_p = \frac{p}{2}[2a + (p-1)d] \text{ and } S_q = \frac{q}{2}[2a + (q-1)d]$$

Given: $$\frac{S_p}{S_q} = \frac{p^2}{q^2}$$

$\therefore$ $$\frac{\frac{p}{2}\left[2a+(p-1)d\right]}{\frac{q}{2}\left[2a+(q-1)d\right]} = \frac{p^2}{q^2}$$

$\Rightarrow$ $$[2a + (p-1)d]q = p[2a + (q-1)d]$$

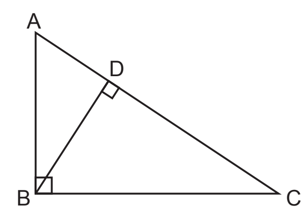
$\Rightarrow$ $$2a(q-p) = (q-p)d$$

$\Rightarrow$ $$d = 2a \qquad [\because \quad p \neq q]$$

$\therefore$ $$\frac{t_p}{t_q} = \frac{a+(p-1)d}{a+(q-1)d}$$

$$= \frac{a+(p-1)2a}{a+(q-1)2a} = \frac{2p-1}{2q-1}.$$

25. Given: A right-angled ΔABC, right-angled at B and $BD \perp AC$.



To Prove: (*i*) ΔADB is similar to ΔABC.

(*ii*) ΔBDC is similar to ΔABC.

(*iii*) ΔADB is similar to ΔBDC.

Construction: (*i*) In Δs ADB and ABC,

$$\angle A = \angle A,$$
$$\angle ADB = \angle ABC \qquad \text{[Each is 90°.]}$$

$\therefore$ By AA criterion of similar triangles, ΔADB is similar to ΔABC.

(*ii*) In Δs BDC and ABC,

$$\angle C = \angle C,$$
$$\angle BDC = \angle ABC \qquad \text{[Each is 90°.]}$$

$\therefore$ By AA criterion of similar triangles, ΔBDC is similar to ΔABC.

(*iii*) We have $\angle ABD + \angle DBC = 90°$

Also, $\angle C + \angle DBC = 90°$

So, $\angle C = \angle ABD$ ...(1)

Now, in Δs ADB and BDC

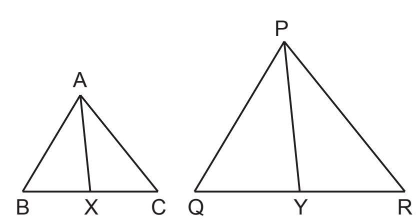
$$\angle ABD = \angle C \qquad \text{[From (1)]}$$
$$\angle ADB = \angle BDC \qquad \text{[Each is 90°.]}$$

$\therefore$ By AA criterion of similar triangles, ΔADB is similar to ΔBDC.

OR

Since AX and PY are medians, X and Y are the mid-points of BC and QR respectively.

$\Rightarrow$ $BX = XC$ and $QY = YR$ ...(1)



Since $\Delta ABC \sim \Delta PQR$,

$\Rightarrow$ $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

and $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$ ...(2)

$\Rightarrow$ $$\frac{AB}{PQ} = \frac{2BX}{2QY} \text{ and } \frac{2XC}{2YR} = \frac{CA}{RP}$$

$\Rightarrow$ $$\frac{AB}{PQ} = \frac{BX}{QY} \text{ and } \frac{XC}{YR} = \frac{CA}{RP} \qquad ...(3)$$

From Eq. (2) and Eq. (3), we have

$$\angle B = \angle Q \text{ and } \frac{AB}{PQ} = \frac{BX}{QY}$$

$\Rightarrow$ $\Delta ABX \sim \Delta PQY$ or $\Delta BXA \sim \Delta QYP$ [By SAS similarity criterion]

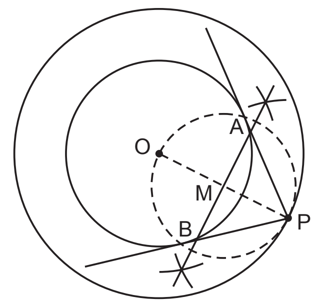
Hence, $\frac{AX}{PY} = \frac{AB}{PQ} = \frac{BC}{QR}$

Also, $\angle C = \angle R$ and $\frac{XC}{YR} = \frac{CA}{RP}$

$\Rightarrow$ $\Delta AXC \sim \Delta PYR$ [By SAS similarity criterion]

26. Steps of Construction:

1. Draw two concentric circles having radii 4 cm and 6 cm. O is the centre of the circles.
2. Take any point P on the bigger circle.
3. Join OP and mark mid-point M of OP.
4. Taking M as centre and radius = OM, draw a dotted circle which intersects the smaller circle at A and B.
5. Join PA and PB [See figure].



Then, PA and PB are the two required tangents.

By measurement, the length of each tangent is 4.4 cm (approx).

By calculation, the length of each tangent is

$\sqrt{OP^2 - OA^2} = \sqrt{6^2 - 4^2} = \sqrt{36-16} = \sqrt{20} = 4.47\text{cm}$ (approx).

27. The given equation is

$$\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3, \ 0° < \theta < 90°$$

$\Rightarrow$ $\frac{\cos^2\theta}{\frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta} = 3$ $\Rightarrow$ $\frac{\cos^2\theta\sin^2\theta}{\cos^2\theta\left(1-\sin^2\theta\right)} = 3$

$\Rightarrow$ $\frac{\cos^2\theta\sin^2\theta}{\cos^4\theta} = 3$ $\Rightarrow$ $\frac{\sin^2\theta}{\cos^2\theta} = 3$

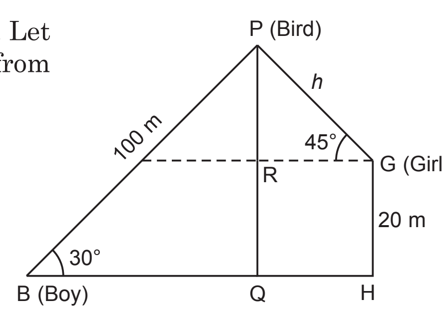
$\Rightarrow$ $\tan^2\theta = 3$

$\Rightarrow$ $\tan\theta = \sqrt{3}$ [$\because$ $\tan\theta \neq -\sqrt{3}$ as $0° < \theta < 90°$]

$\Rightarrow$ $\theta = 60°$

Thus, $\theta = 60°$ is the solution of the given equation.

28. Let B and G be the positions of the boy and the girl respectively. Let P be the position of the bird. We need to determine GP(*h* metres) from figure.



In right Δ BQP,

$$\frac{PQ}{BP} = \sin 30° = \frac{1}{2}$$

$\Rightarrow$ $PQ = \frac{1}{2}BP = \frac{1}{2} \times 100 \text{ m} = 50 \text{ m}$

$\Rightarrow$ $PR = PQ - RQ$

$= PQ - GH = 50 \text{ m} - 20 \text{ m} = 30 \text{ m}$

In right Δ PRG,

$$\frac{PR}{GP} = \sin 45°$$

$\Rightarrow$ $$GP = PR \text{ cosec } 45° = 30 \times \sqrt{2} \text{ m}$$

Thus, the distance of the bird from the girl is $30\sqrt{2}$ m.

29. Volume of ice cream in the cone

$$= \frac{1}{3}\pi r^2 h$$

Volume of ice cream in the cylindrical cup

$$= \pi\left(\frac{r}{2}\right)^2\left(\frac{4}{3}h\right)$$

$$= \frac{1}{3}\pi r^2 h$$

Since the amount of ice cream in the cone and cylindrical cup is the same and he is charging the same amount, it shows the honesty of vendor.

30. Here, the given distribution is a cumulative frequency distribution of less than type, so we first prepare the corresponding frequency table, which is as under:

Class	15–20	20–25	25–30	30–35	35–40	40–45	45–50	50–55	55–60
Frequency	2	4	18	21	33	11	3	6	2
Cumulative Frequency	2	6	24	45	78	89	92	98	100

Here, $$\frac{n}{2} = \frac{100}{2} = 50.$$

So, the median class is 35–40.

Thus, $l = 35$, $h = 5$, $f = 33$ and $c = 45$.

$\therefore$ $$\text{Median} = l + \left(\frac{\frac{n}{2} - c}{f}\right) \times h$$

$$= 35 + \left(\frac{50 - 45}{33}\right) \times 5$$

$$= 35 + \frac{25}{33} = 35.76 \text{ years}$$

Hence, the median age is 35.76 years.

OR

Since the data given is in the inclusive classes, we convert it into the exclusive classes.

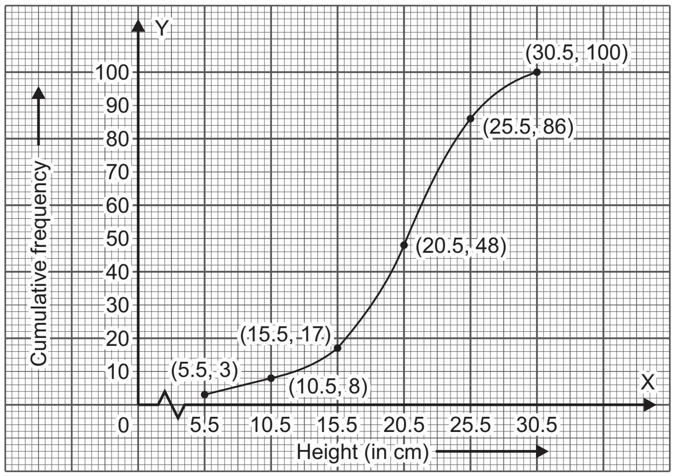
Height (in cm)	0.5–5.5	5.5–10.5	10.5–15.5	15.5–20.5	20.5–25.5	25.5–30.5
Number of Plants	3	5	9	31	38	14

(*a*) **Less than type Ogive:** For this type of ogive, we prepare the cumulative frequency table of the data as given below:

Height (in cm)	*Cumulative Frequency*
Less than 5.5	3
Less than 10.5	8
Less than 15.5	17
Less than 20.5	48
Less than 25.5	86
Less than 30.5	100

Upper Limits	*Cumulative Frequency*
5.5	3
10.5	8
15.5	17
20.5	48
25.5	86
30.5	100

Plotting the points (5.5, 3), (10.5, 8), (15.5, 17), (20.5, 48), (25.5, 86) and (30.5, 100) and joining these points in order by a freehand smooth curve, we get less than type ogive as shown in given figure.



(*b*) **More than type Ogive:** For 'more than type ogive', we make the cumulative frequency table as given below:

Height (in cm)	*Cumulative Frequency*
0.5 or more	100
5.5 or more	97
10.5 or more	92
15.5 or more	83
20.5 or more	52
25.5 or more	14
30.5 or more	0

Lower Limits	0.5	5.5	10.5	15.5	20.5	25.5	30.5
Cumulative Frequency	100	97	92	83	52	14	0

Plotting the points (0.5, 100), (5.5, 97), (10.5, 92), (15.5, 83), (20.5, 52), (25.5, 14) and (30.5, 0) and joining them by a freehand smooth curve, we get more than type ogive, as shown in given figure.